(3)* 4

Sei $\alpha := \frac{(d-1)p}{d-p} > 1$.

wird diff'bar da $\alpha > 1$

Dann gilt $\alpha \cdot \frac{d}{d-1} = p^*$ und $\partial_j \left(|f|^\alpha \right) = \alpha |f|^{\alpha-1} \cdot \partial_j f$

$$= \frac{(\alpha-1)p}{p-1}$$

$$\Longrightarrow \left(\int_{\mathbb{R}^d} |f|^{\alpha \frac{d}{d-1}} \right)^{\frac{d-1}{d}} \leq 1 \cdot \alpha \int_{\mathbb{R}^d} |f|^{\alpha-1} |\nabla f| \, dx$$

$$\leq \alpha \left(\int_{\mathbb{R}^d} |f|^{\frac{(\alpha-1)p}{p-1}} \right)^{\frac{p-1}{p}} \left(\int_{\mathbb{R}^d} |\nabla f|^p \right)^{1/p}$$

$$\frac{1}{q} = 1 - \frac{1}{p} = \frac{p-1}{p}$$

Konstante wird so natürlich nicht getroffen.

Also $\left(\int |f|^{p^*} \right)^{\frac{d-1}{d}} \leq \alpha \left(\int |f|^{p^*} \right)^{\frac{p-1}{p}} \|\nabla f\|_p$

Da $\frac{d-1}{d} - \frac{p-1}{p} = \frac{d-p}{dp} = \frac{1}{p^*}$ folgt die Beh.