Musterlösung Blatt 11

1.

$$\text{Bogenlänge} = \int_{-1}^{1} \sqrt{1 + (\cosh' x)^2}\, dx = \int_{-1}^{1} \frac{1}{2}e^x + \frac{1}{2}e^{-x}\, dx = e - 1/e$$

2. a)

$$\int_0^1 \sin(2x)\, dx = 1$$

b)

$$\text{Sehnen-Trapez-Regel} = \pi/4(\sin\pi + \sin 0) = 0$$

$$\text{Fass-Regel} = \pi/12(\sin 0 + 4\sin\pi/2 + \sin\pi) = \pi/3$$

c) n :=(Anzahl Unterteilungen)

$$\text{Sehnen-Trapez-Regel}: \quad |\frac{n}{12}(\frac{\pi}{2n})^3(-4\sin(2\xi))| < 10^{-4} \quad \Rightarrow n \geq 114$$

$$\text{Fass-Regel}: \quad |\frac{n}{2880}(\frac{\pi}{2n})^5(16\sin(2\xi))| < 10^{-4} \quad \Rightarrow n \geq 4$$

3. a)

$$\int e^{-x}\sin x\, dx = -\frac{1}{2}e^{-x}(\sin x + \cos x) \quad \Rightarrow \int_0^\infty e^{-x}\sin x\, dx = \frac{1}{2}$$

b)

$$\int_0^1 \frac{e^x}{x}\, dx \geq \int_0^1 \frac{1}{x}\, dx \quad \Rightarrow \text{Divergenz}$$

c)

$$\int \frac{dx}{\sqrt{x}(1+x)} = \int \frac{2\,dy}{1+y^2} = 2\arctan y = 2\arctan\sqrt{x} \quad \Rightarrow \int_0^\infty \frac{dx}{\sqrt{x}(1+x)} = \pi$$

d)

$$\int_1^\infty \frac{dx}{1+\ln x} \geq \int_1^\infty \frac{dx}{x} \quad \Rightarrow \text{Divergenz}$$

e)

$$\int xe^{-x^2}\, dx = -\frac{1}{2}e^{-x^2} \quad \Rightarrow \int_0^\infty xe^{-x^2}\, dx = \frac{1}{2}$$

f)

$$\int \frac{dx}{1+x^2} = \arctan x \quad \Rightarrow \int_{-\infty}^\infty \frac{dx}{1+x^2} = \pi$$

4. a)

$$\int_1^\infty \frac{\cos(5x)}{x^{3/2}}\, dx \leq \int_1^\infty \frac{1}{x^{3/2}}\, dx \quad \Rightarrow \text{ Integtral existiert}$$

b) c) d)

$$\lim_{x\to 0} \frac{\cos(x) - 1}{x^2} = -\frac{1}{2} \quad \Rightarrow \text{Die Integrale in b) und c) existieren, das in d) divergiert.}$$