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STUDIENBRIEF

SIGNALS AND SYSTEMS

Weiterbildender Masterstudiengang „Sensorsystemtechnik“
der Fakultät für Ingenieurwissenschaften und Informatik
mit dem Abschluss „Master of Science (M. Sc.)“
an der Universität Ulm

Kürzel / Nummer:	SuS
Englischer Titel:	Signals and Systems
Leistungspunkte:	6 ECTS
Semesterwochenstunden:	?
Sprache:	English
Turnus / Dauer:	jedes Wintersemester / 1 Semester
Modulverantwortlicher:	Dr. Werner Teich
Dozenten:	Dr. Werner Teich
Einordnung des Moduls in Studiengänge:	Sensorsystemtechnik, M.Sc., Wahlpflichtmodul,
Voraussetzungen (inhaltlich):	Advanced Mathematics
Lernziele:	The concepts of signals and systems are powerful tools for any engineer dealing with information bearing, measurable physical quantitites. Areas of applications include, among others, communications engineering, signal processing, control engineering, and systems engineering. The students will be able to classify, interpret, and compare signals and systems with respect to their characteristic properties. They can explain and apply analytical and numerical methods to analyze and synthesize signals and systems in time and frequency domain. Suitable signal transformations can be chosen and calculated with the help of transformation tables. The students are able to recognize stochastic signals and analyze them based on their characteristic properties. They can calculate and interpret the influence of linear time-invariant systems on stochastic signals.
Inhalt:	- basic properties of discrete-time and continuous-time systems - z-transformation - basic properties of discrete-time and continuous-time systems - linear time-invariant systems, convolution integral - Fourier transformation, discrete Fourier transformation, Fourier series - sampling theorem - probability theory, random variables and stochastic processes - stochastic signals and linear time-invariant systems
Literatur:	- Alan V. Oppenheim and Alan S. Willsky: Signals and Systems, Prentice Hall 1996 - Mrinal Mandal and Amir Asif: Continuous and Discrete Time Signals and Systems, Cambridge University Press, 2007 - Athanasios Papoulis and S. Unnikrishna Pillai: Probability, Random Variables, and Stochastic Processes, McGraw-Hill, 2002 - Thomas Frey und Martin Bossert: Signal- und Systemtheorie, B.G. Teubner Verlag, 2004 - Jens Ohm und Hans Dieter Lüke: Signalübertragung, Springer Verlag 2010

Lehrveranstaltungen und Lehrformen:	Präsenzveranstaltungen: Einführungsveranstaltung: 2 h Vertiefende Übungen: 8 h Seminar zur Prüfungsvorbereitung: 8 h Modulprüfung: 4 h E-Learning: Webinar: 24 h Online-Gruppenarbeit: 40 h Selbststudium: 86 h Chat zur Prüfungsvorbereitung: 8 h
Abschätzung des Arbeitsaufwands:	Vermittlung des Unterrichtsstoffs: 45 h Vor- und Nachbereitung, Übungen, Anwendung: 127 h Sonstiges: 4 h Modulprüfung: 4 h Summe: 180 h
Leistungsnachweis und Prüfungen:	Oral exam or written exam with a duration of 120 min
Voraussetzungen (formal):	None
Notenbildung:	Grade of the modul is based on the result of the modul exam

1 Introduction

This chapter introduces and motivates the concepts of signals and systems.

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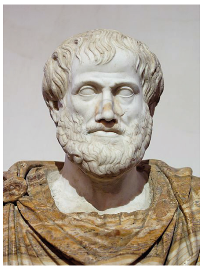
The concepts of signals and systems are powerful tools for any engineer dealing with information bearing, measurable physical quantitites. Areas of applications include, among others, communications engineering, signal processing, control engineering, and systems engineering. The concepts are also very useful in many other fields such as natural sciences (e.g., physics or biology) or even in economics.

Signals

Colloquially speaking, the terms *signal* and *system* are used in a versatile and diverse way. Generally, if we talk about a signal in everyday life, we imply a change in a physical quantity in order to attract attention and to transfer meaning. Examples are the whistle blowing of the conductor in a train station calling attention and indicating the departure of the train. Other examples are the signal horn (note, that the term *signal* is already included in the name) and the flashing light of a firetruck alerting and forcing to give the right of way to the emergency vehicle or the traffic lights at a street crossing, attracting attention and signaling the right of way.

Generally, we talk about a *system* if an entity is composed of smaller parts and the behaviour of the system is not only given by the individual functions of the constituting parts but also by the interconnections between them. Aristotle has put this in a nutshell by saying "The whole is more than the sum of its parts (in Greek: '...μὴ ἔστιν οἷον σωρὸς τὸ πᾶν ἀλλ᾽ ἔστι τι τὸ ὅλον παρὰ τὰ μόρια...', in Aristotle, Metaphysics, 7.1041b).



Aristotle
(384-322 B.C.)

Examples can be taken from many areas of life, but typically are taken from the technical domain. E.g., the system *car* reacts to excitations, such as the steering angle or the accelerator position and external perturbances such as roughness of the roadway, with a time-variant course of position and velocity. Financial markets, on the other hand, react with fluctuating stock prices on information given by companies and analysts. Applying a particular voltage characteristic at the input, an electrical network reacts with a specific temporal characteristic of the output voltage.

System Theory

System theory is concerned with the mathematical description and calculation of such systems. To do so, we get rid of the physical units of the system excitation or reaction and model them mathematically as functions of independent variables, mostly time, but often also space etc.. Excitations of systems are than called *input signals* and reactions *output signals*. The specification of the system itself is on the same abstract level. A proper mathematical description of a system could be, e.g., a differential equation. Complex systems like the car or the financial market are difficult to capture perfectly. The problem is, that the interrelation between all the input and output parameters is either not known or can not be quantified properly. However, an adequate modelling is necessary for a successful application of system theory.

We limit ourselves in this course to basic systems, such as electric networks or digital filters. Two important issues must then be distinguished: *system analysis*, e.g., determining the tranfer characteristic of an optical fibre or *system synthesis*, e.g., the design of a digital filter. Note, that the last problem, system synthesis, is a typical engineering task.

System Analysis versus System Synthesis

2 Introduction to Signals

This chapter introduces the concept of *signals* and the basic *mathematical operations* on signals. The students will get an in-depth understanding about signals in general.
Based on that they will be able to explain what a signal is and give examples for the usage of this mathematical concept. The students will be able to recall elementary signals and how they relate to each other in both, their formular representations and their graphs. The students will be also able to classify different types of signals regarding distinguishing features. They will also get the ability to apply the signal operations presented in this chapter to the introduced signals but also to solve further problems of signal theory.

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A *signal* is an abstract mathematical description of a variable quantity. It is modelled as a function of one or more independent variables. Often signals describe the temporal behavior of a quantity. In this case, the independent variable is time. In image processing applications, signals can also be functions of the spatial dimension (e.g. brightness distribution on a semiconductor sensor). In this course we limit ourselves to signals as scalar functions of time.

Definition Signal

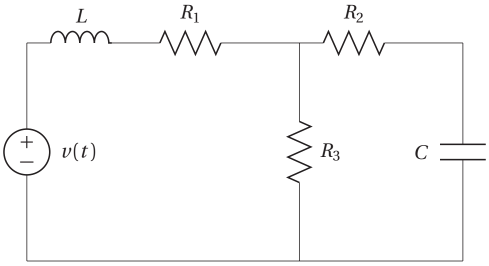
Definition 2.1 (Signal)**.** A signal is a function of one or more independent variables. It has a meaning or represents information. Often a signal stands for an abstract and normalized description of a physical quantity.

Examples for signals are:

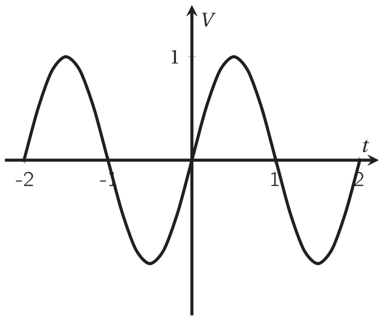
- brightness distribution on a screen
- sound pressure fluctuations (i.e. speech or music signal)
- air temperature at noon
- stock prices or stock indices (e.g. Dow Jones)
- voltage or current fluctuations at or in an electronic device (resistor, capacitor, inductor, etc.).

The following figures show illustrations of typical signals and their generation processes, see also [1]:

- voltage in an electrical circuit, cf. Fig 2.1,
- speech signal (analog signal), cf. Fig 2.2,

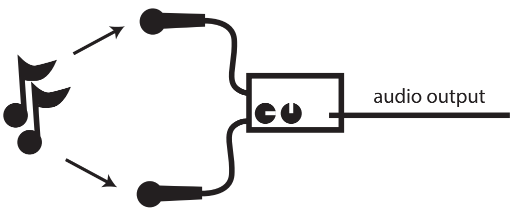


(a) Circuit

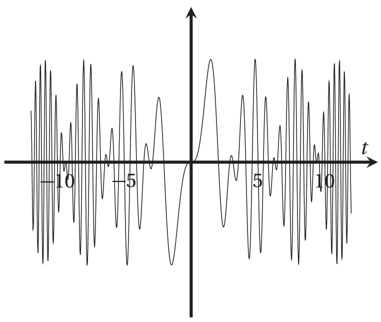


(b) Voltage Signal

Figure 2.1: Eletrical Circuit



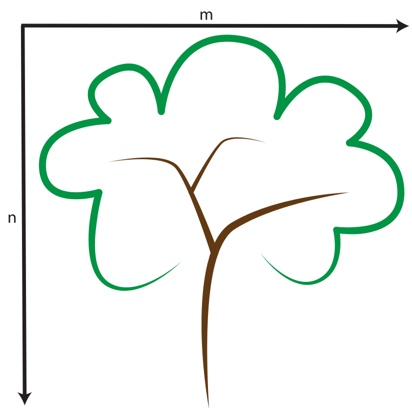
(a) Microphone



(b) Audio Signal Waveform as Generated Output Signal

Figure 2.2: Audio Recording

(a) A Digital Camera



(b) Image as Generated Output Signal

Figure 2.3: Image Recording

- stock indices (discrete-time signal), cf. Fig **??**,
- picture, cf. Fig 2.3.

3 Introduction to Systems

This chapter introduces the concept of *systems*. The students will get an understanding about systems in general and their interaction with signals. This allows the students to explain what a system is and to give examples for the usage of the system concept. They can also recall various types of systems, their basic properties and basic components complex systems can consist of. They will be further able to analyze and classify new systems according to these basic properties.
Another skill the students will acquire is designing of new systems by using basic building blocks. It also enables them to use both, the graphical representation in terms of block diagrams on the one hand and mathematical expressions on the other hand for system composition. They will be also able to transfer descriptions of larger composed systems from one representation to the other one.

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In general terms a *system* represents a more or less complex structure which reacts on an external exitation or influence in a specific way. In system theory, a system is the mathematical description of a process, structure or device that results in the transformation of signals. Some examples of systems from a wide range of fields are :

System Theory

Examples for Systems

- electrical networks
- transmission media (radio channel, cable, fibre optics, etc.)
- digital filter
- predator-prey relationship (biological systems)
- foreign exchange market (economical system)

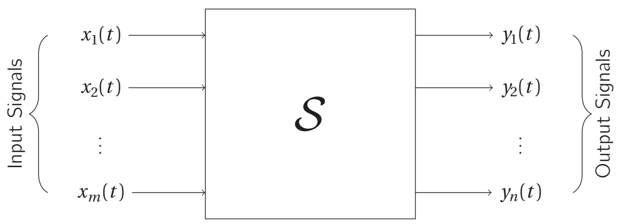

Figure 3.1: Continuous-Time Multiple-Input, Multiple-Output (MIMO) System with m Inputs and n Outputs

In mathematical terms a system corresponds to the *mapping* of one or more input signals to one or more output signals. As for signals (cf. Sec. 2.1.1) we distinguish between *continuous-time* and *discrete-time* systems. If all input

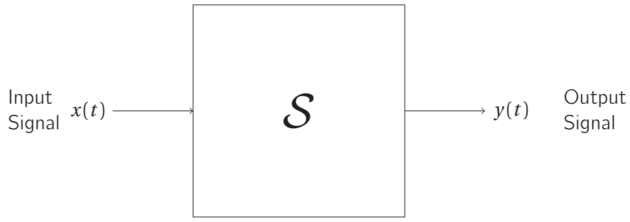

Figure 3.2: Continuous-Time Single-Input, Single-Output (SISO) System

and output signals are continuous-time signals (cf. Def. 2.2), we refer to it as a continuous-time system. On the other hand, if all input and output signals are discrete-time signals (cf. Def. 2.3), we refer to it as a discrete-time system. An example for a continuous-time signal is an electrical network of resistors, capacitors and inductors (cf. Fig. 2.1). A digital filter is an example for a discrete-time system. The general case of a system with several input and output signals is usually modelled as a multiple-input multiple-output (MIMO) system. A schematic represenation of such a MIMO system is shown in Fig. 3.1. Considering the advantages of transmission systems with several transmit and receive antennas (spatial diversity, increased bandwidth efficiency), MIMO systems have become very popular in Communications Engineering. However, due to time limitations, we restrict ourselves throughout this course to systems with a single input and a single output signal, sometimes referred to as single-input single-output (SISO) systems. Note, that the principles we derive to analyze and to synthesize SISO systems can be generalized also to MIMO systems. This brings us to the following definition of a continuous-time (discrete-time) system with a single input signal $x(t)$ ($x[k]$) and a single output signal $y(t)$ ($y[k]$):

Continuous-Time and Discrete-Time Systems

Multiple-Input, Multiple-Output System

Single-Input, Single-Output System

Definition 3.1 (System)**.** A continuous-time system is a mapping $\mathcal{S}$ which relates the continuous-time output signal $y(t)$ to the continuous-time input signal $x(t)$:

$$y(t) = \mathcal{S}\{x(t)\}$$

A discrete-time system is a mapping $\mathcal{S}$ which relates the discrete-time output signal $y[k]$ to the continuous-time input signal $x[k]$:

$$y[k] = \mathcal{S}\{x[k]\}$$

Definition System

A schematic represenation of a SISO system is given in Fig. 3.2. Most continuous-time systems can be modelled by *differential equations*. Discrete-time systems, on the other hand, are specified by *difference equations*. Note, that this is an abstract description of a system which is independent of the specific realization. Therefore different realizations or different problem settings can lead to the same mathematical description of a system.

Modelling by Differential Equations

4 Continuous-Time LTI Systems

This chapter gives in-depth information about the concept of *linear time-invariant (LTI) systems*. In the previous chapter we have already seen that LTI systems constitute the most important class of systems. Therefore, the students will be able to describe and analyze *continuous-time* LTI systems as well in time domain as in frequency domain.
The students get to know the concept of the *impulse response* of an LTI system. This allows them to describe an LTI system's behaviour in the time domain. After the introduction of the *convolution integral* the students can describe the relation between the impulse response and the convolution operation. They will be also able to calculate the output signal of an LTI system with a known impulse response.
Further, the students learn about different types of LTI systems. This allows them to classify these systems based on their impulse reponses.

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4.1 Time-Domain Analysis of Continuous-Time LTI Systems

In time domain, continuous-time LTI systems are completely characterized by the impulse response $h(t)$ of the LTI system:

Definition 4.1 (Impulse Response)**.** The *impulse response* $h(t)$ of a continuous-time linear time-invariant system $\mathcal{S}$ is defined as the output of the system which results, if the system is stimulated by a Dirac delta impulse $\delta(t)$ (cf. Sec. 2.2.12):

$$h(t) = \mathcal{S}\{\delta(t)\}$$

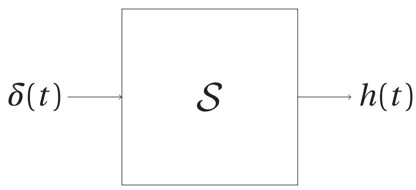

Figure 4.1: Graphical Illustration of Def. 4.1. $\mathcal{S}$ is assumed to be an LTI system.

Fig. 4.1 shows the graphical illustration of this definition. Note, according to system theory, the impulse response $h(t)$ is not a signal, but it is a function which characterizes the behaviour of an LTI system. The definition given above basically states that we can measure the impulse response $h(t)$ of any LTI system by applying a Dirac delta impulse at the input of the system. We are now going to show how we can use the impulse response $h(t)$ to calculate the output signal $y(t)$ of an LTI system if an arbitrary input signal $x(t)$ is applied to the system. To do so, we use Eq. 2.52 to rewrite the input signal $x(t)$ as follows:

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\mathrm{d}\tau \tag{4.1}$$

Eq. 4.1 implies that any continuous-time function $x(t)$ can be represented as a weighted superposition of an infinite number of time-shifted Dirac delta functions. This superposition is visualized in Fig. 4.2 by representing the

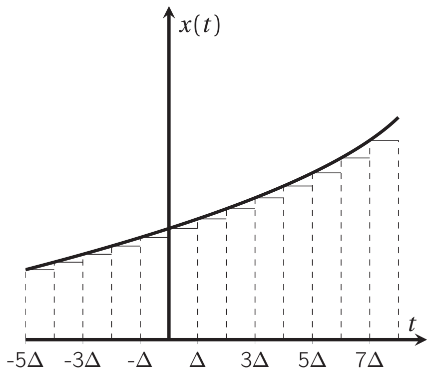

Figure 4.2: Continuous-Time Signal $x(t)$ and its approximation shown with the staircase function

Dirac delta function δ by a rect-function with small width T and unit area, cf. Eq. 2.53 and Fig. 2.19:

$$\delta(t) = \lim_{T\to\infty} \left\{ \frac{1}{T} \mathrm{rect}\left(\frac{t}{T}\right) \right\} \tag{4.2}$$

The output signal $y(t)$ can thus be rewritten in the following form:

$$y(t) = \mathcal{S}\{x(t)\} = \mathcal{S}\left\{ \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\mathrm{d}\tau \right\}$$

For LTI systems the *principle of superposition* (cf. Sec. 3.1.1) holds. Thus we obtain:

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\mathcal{S}\{\delta(t-\tau)\mathrm{d}\tau\}$$

Using the property of *time-invariance* (cf. 3.3) and the definition of the impulse response (cf. Def. 4.1), we finally obtain for the output signal:

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\mathrm{d}\tau \tag{4.3}$$

Ansprechpartner

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Mod:Master
Sensorsystemtechnik

Das Studienangebot „Sensorsystemtechnik" wurde entwickelt im Projekt Mod:Master, das aus Mitteln des Bundesministeriums für Bildung und Forschung gefördert und aus dem Europäischen Sozialfonds der Europäischen Union kofinanziert wird (Förderkennzeichen: 16OH11027, Projektnummer WOH11012). Dabei handelt es sich um ein Vorhaben im Programm „Aufstieg durch Bildung: offene Hochschulen".

AUFSTIEG DURCH BILDUNG OFFENE HOCHSCHULEN

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GEFÖRDERT VOM
Bundesministerium für Bildung und Forschung