Electron dose dependence of signal-to-noise ratio, atom contrast and resolution in transmission electron microscope images


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**Abstract**

In order to achieve the highest resolution in aberration-corrected (AC) high-resolution transmission electron microscopy (HRTEM) images, high electron doses are required which only a few samples can withstand. In this paper we perform dose-dependent AC-HRTEM image calculations, and study the dependence of the signal-to-noise ratio, atom contrast and resolution on electron dose and sampling. We introduce dose-dependent contrast, which can be used to evaluate the visibility of objects under different dose conditions. Based on our calculations, we determine optimum samplings for high and low electron dose imaging conditions.

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Signal-to-noise ratio (SNR)
Contrast
Resolution
Low-voltage G/Ge-correction

1. Introduction

The instrumental resolution of transmission electron microscopes (TEMs) has dramatically improved during the last decade, mainly due to the introduction and practical realisation of hardware aberration correction [1–3]. As a result, materials can now be imaged and identified down to single atomic columns [4,5] or, in the case of the new class of two-dimensional materials, even single atoms [6,7]. With the aim of reducing radiation damage induced by the imaging electrons, low-voltage aberration-corrected TEMs, down to voltages of 20 kV [8,9], 30 kV [10], and 40 kV [11], are currently under development. A voltage-tunable fully-corrected (that is, corrected for higher-order geometrical aberrations as well as chromatic aberrations of the imaging lenses [12]) TEM seems close to becoming reality [13].

Achieving the improved resolution of an aberration-corrected TEM requires, however, an infinite electron dose on the studied specimen, and only few materials can withstand very high, let alone infinite doses. Materials can be damaged via the knock-on damage mechanism, where atoms are displaced by direct impacts of the imaging electrons, and in such cases lowering of the electron energy below a material specific threshold is desirable [8,14–17]. On the other hand, the electron–electron (inelastic) scattering cross section increases at lower electron energies and, depending on the material, ionization can become the dominating damage mechanism [17]. Effective ways of reducing ionization damage may be cooling of the specimen [18] or conductive coating [19]. As extreme examples of the latter, samples have been enclosed within carbon nanotubes [9], or between graphene layers [20], greatly reducing radiation damage during imaging. Such approaches are not always feasible, however, and images thus need to be acquired with limited electron doses.

The stability of the microscope is another factor limiting the electron dose in a single image. The microscope tends to drift away from the corrected state, and as a result images can be acquired only within a small time window before resolution is deteriorated [21–23]. Also all kinds of instabilities including electrostatic and magnetic field noise [24] and instabilities caused by the sample stage can lead to blurring of the images, if long exposure times are used.

With all this in mind, microscopists have to develop strategies for limited electron dose imaging. Thus, having a robust framework for estimating the effects of this limitation is necessary. Here, we address this issue, by exploring the influence of the electron dose and sampling on the signal-to-noise ratio (SNR), the atom or lattice contrast, as well as the resolution, with the help of dose-dependent image simulations. We introduce a modified definition for the image contrast, which takes the electron dose into account. The dose related noise is treated as stochastic fluctuations around the ideal electron count at each image pixel, instead of the previously used additive noise [25–30]. Using these tools, we determine the optimal sampling for achieving atomic resolution images of graphene, as a function of the information limit and magnification of the microscope, as well as determine the required electron dose based on the calculated atom contrast. Graphene is used as the example material due to the simplicity of its structure, which allows straightforward interpretation of the results.
2. Theoretical and experimental methods

2.1. Image simulation with finite electron dose

The structural information of the sample is carried primarily by elastically scattered electrons, which is distorted by an electromagnetic lens during the propagation process in the microscope. The distorted information is transferred to the detector and the average number of electrons collected by each detector pixel is determined by the electron dose, the sampling and the probability of the electron to be found on each pixel, which is the squared modulus of the image wave. The actual number of electrons collected by each detector pixel is governed by Poisson statistics.

In this paper we utilize the elastic model based on [31] in the image calculation for graphene at low voltages ranging from 20 kV to 80 kV on a C/C-corrected microscope. Inelastic scattering is not included for the following reason:

On a C/corrected microscope, the contrast delocalization caused by inelastic scattering in a single layer of graphene at voltages as low as 20 kV is negligible. In this case, the influence of inelastic scattering on the image is mainly the decrease of the voltages as low as 20 kV is negligible. In this case, the inelastic scattered electrons, which is distorted by an electromagnetic field emitter, an electrostatic filter. The TEM-platform has been optimized to achieve the highest objective lens correction was applied. The size of the energy window of the camera. For a phase object, the intensity Iy of the final image is a convolution between the calculated image I [31] and the damping functions:

\[ I_y = \int I_x(q) E_u(q) E_p(q) MTF(q) e^{i2\pi q \cdot y + i2\pi q \cdot y} dq. \]

(1)

Image spread is caused by all kinds of noise causing a random deflection of the image. The origin of these zeroth-order aberrations is vibrations and drift of the stage, parasitic time-dependent fields resulting from instabilities of the lens currents and magnetic fields resulting from eddy currents in the material of the lenses as shown recently by Uhlemann et al. [24]. The combined effect of these disturbances on the image contrast is given by the envelope function [12]

\[ E_u(q) = \exp(-\frac{1}{2} (2\sigma(C_u))^2 q^2). \]

(2)

The spatial frequency q = \theta/\lambda depends on the scattering angle \theta and the wavelength \lambda of the electrons; \sigma(C_u) denotes the standard deviation of the image spread. The residual focus spread is caused by the movement of the stage and all the electromagnetic lenses in the z-direction and by the parasitic first-order aberrations of the focusing elements. The effect of the focus spread on the image contrast is expressed by the envelope function [12]

\[ E_f(q) = \exp(-\frac{1}{2} (\lambda \sigma(C_f))^2 q^2). \]

(3)

Here \sigma(C_f) denotes the standard deviation of the focus spread. In most cases, TEM images are recorded using charge-coupled detectors (CCD) with a fiber-optics coupled scintillator. In the ideal case, each electron is only detected by one of the detector pixels. The number of detected electrons varies from pixel to pixel, resulting in different gray levels. In practice, a single imaging electron can cause signals in more than one pixel because of multiple scattering within the scintillator material and the creation of an excitation volume. This effect is described by the point-spread function (PSF) of the detector, and its Fourier transform is the modulation-transfer function (MTF) [33–35].

The average number of electrons \(N_j\) collected by the jth detector pixel is

\[ N_j = D \delta^2 l_j, \]

(4)

where D represents the electron dose, \delta denotes the sampling (pixel size) and \(l_j\) is the probability of the electron hitting the jth pixel.

2. The actual number of electrons collected by each detector pixel is generated with random Poisson distribution, indicating that the standard deviation of the number of electrons collected by the jth detector pixel is \(\sqrt{N_j}\).

The SNR of the whole image is evaluated as [36]

\[ \text{SNR} = \frac{N}{\sigma(N)} \]

(5)

where the average number of electrons per image pixel \(\overline{N}\) is defined as

\[ \overline{N} = \frac{1}{J} \sum_{j=1}^{J} N_j = D \delta^2 \sum_{j=1}^{J} l_j = D \delta^2 T. \]

(6)

Here J is the total number of pixels and \(T\) is the average image intensity. \(\sigma(N)\) is the standard deviation of the number of electrons collected by each pixel. We define the actual number of electrons collected by the jth image pixel as \(Pois(D \delta^2 l_j)\), then

\[ \sigma(N) = \sqrt{\frac{1}{J} \sum_{j=1}^{J} [Pois(D \delta^2 l_j) - Pois(D \delta^2 l_j)]^2}. \]

(7)

If \(l_j = T\) for any image pixel, indicating the probability of the electrons hitting each image pixel is the same, then \(\sigma(N) = \sqrt{D \delta^2} = \sqrt{\overline{N}} \) and \(\text{SNR} = \overline{N}/\sqrt{\overline{N}}\).

2.2. Experimental setup for TEM imaging

Our modeling is based on the characteristics of the prototype SALVE II microscope. This system is based on a Zeiss Libra 200 MC equipped with a Schottky field emitter, an electrostatic \(\omega\)-type monochromator and an \(\omega\)-type in-column energy-filter. The TEM-platform has been optimized to achieve the highest objective lens current stability (\(\Delta I/I < 10^{-7}\)) within the voltage range between 20 and 80 kV [8]. The monochromator was used with the largest slit of 60 \(\mu\)m (basically no effect of monochromation) as \(C_c\) correction was applied. The size of the energy window of the filter was not limited by an energy-selecting slit but only by the physical limitations such as fixed apertures and tube diameters (about 130 eV at 40 kV). The SALVE II prototype is equipped with an aberration corrector to achieve atomic resolution even at low voltages of 20 and 40 kV, based on the design proposed by Rose [37]. Geometric axial aberrations are corrected up to the 5th order except for \(C_5\), which was designed to be around 4 mm to obtain an optimized phase contrast transfer function. Off-axis aberrations are corrected up to the 3rd order for larger fields of view. The CMOS (complementary metal oxide semiconductor) based camera is the type TVIPS T416 (4k detector, 16 \times 16 \mu m^2 pixel size). The fiber-optics coupled scintillator was optimized to obtain large conversion rates for high sensitivity and very small thickness for high resolution.
3. Results

In order to study the dependence of SNR, atom contrast and specimen resolution on electron dose and sampling, we have simulated images of graphene obtained with the \( C_1/C_0 \)-corrected SALVE II microscope operated at an accelerating voltage of 80 kV (Fig. 1). Subsequently, we have determined the optimum sampling based on the previous study for 20 kV, 40 kV, 60 kV and 80 kV.

The MTF data (Fig. A1) for the SALVE microscope at 20 kV, 40 kV, 60 kV and 80 kV were previously measured using averaged single electron events detected by the T416 camera [38,39]. Parameters used for the image simulations of the SALVE II microscope are listed in Table 1. The defocus \( \Delta f \) and the coefficient \( C_3 \) of the third-order spherical aberration are free parameters; \( C_5 = 4 \) mm, \( C_7 = 0 \) and residual focus spread \( \sigma(C_5) = 5 \) Å are fixed for all accelerating voltages.

In the following subsections we are evaluating step by step the dependence of SNR (Section 3.1), atom contrast (Section 3.2) and specimen resolution (Section 3.3) on dose and sampling. Based on these studies, we will determine the optimum sampling for imaging condition with finite dose in Section 3.4.

### 3.1. Dependence of SNR on dose and sampling

Each row and each column in Fig. 1a show the evolution of the object visibility with respect to the sampling and dose, respectively. As the sampling gets finer from left to right, the object visibility ameliorates first, then declines. The object visibility increases with the dose (Fig. 1 every column from bottom to top). For example, the camera MTF can be ignored for sampling finer than 0.15 Å/pixel (green lines in CTF plot), since in these cases the MTF is always 1 within the range of spatial frequency marked. The damping of the image spread (yellow line) is much stronger than the focus spread (red line) at 80 kV for the SALVE II microscope.

Fig. 2a shows that the SNRs of the images calculated for different samplings (Fig. 1a) increase with the dose linearly at a logarithmic scale for doses smaller than \( 5 \times 10^6 \) e\(^{-}\)/nm\(^2\), and converge gradually towards the maximum SNRs obtained for infinite dose. The maximum SNR increases with the sampling. The behavior in Fig. 2a can be actually understood by discussing Eqs. (5) and (7). In the low-dose cases, since the number of electrons on each image pixel \( \Delta \delta^2 I_j \) is close to the average \( \Delta \delta^2 I \), we can replace \( \text{Pois}(\Delta \delta^2 I_j) \) with \( \text{Pois}(\Delta \delta^2 I) \) and obtain

\[
\sigma(N) = \sqrt{\sum_{j=1}^{N} \left[ \frac{1}{j} \sum_{i=1}^{j} \text{Pois}(\Delta \delta^2 I_j) - \text{Pois}(\Delta \delta^2 I) \right]^2} = \sqrt{\Delta \delta^2 I} = \sqrt{N},
\]  

where \( \Delta \delta \) represents the standard deviation of image spread.
Contrast originates from the brightness difference between the object and the background. For the measurement of the experimental atom contrast, the intensities of the peak and the valley: \( I_b \) and \( I_a \) are determined from line profile drawn through the atoms, and the atom contrast is judged with either the modified Weber formula [31]

\[
C_W = \frac{I_b - I_a}{I_b}
\]

(12)

or the Michelson formula [40]

\[
C_M = \frac{I_b - I_a}{I_b + I_a}
\]

(13)

In the case of very high electron dose, these formulas are correct because the SNRs of the images are high enough so that the disturbance introduced by the electron noise to the image contrast is negligible. However, in an experimental image recorded with low dose, due to the strong statistical fluctuation of the electrons counts in the neighbor pixels, the recognition of the object becomes difficult. This statistical influence is not included in Eq. (12) or (13), and the contrast value calculated based on either Eq. (12) or (13) can result in similar contrast values for both high-dose and low-dose images.

Fig. 3 shows an example. Fig. 3a and b are extracted from Fig. 1 for the sampling of 0.038 A/\text{pixel}. The intensity difference between the atoms and the background in Fig. 3a (calculated with the dose of \( 1 \times 10^6 \) e\(^-\)/\text{nm}^2) is lower than in Fig. 3b (calculated with infinite dose), and the atom visibility in Fig. 3a is therefore lower than in Fig. 3b; however, the contrast evaluation based on the corresponding line profiles Fig. 3c and Fig. 3d with Eq. (12) or (13) produces similar results for both images, which is counter to our visual cognition.

In order to describe the statistical fluctuation in the experimental images, we introduce the term \( \sigma(N)/[\sigma(i)I] \). Here \( \sigma(N) \) is the standard deviation of the image calculated with finite electron dose, and \( \sigma(i)I \) is the standard deviation of the image \( I \) calculated with infinite dose (\( I \approx 1 \)) scaled with the average number of electrons per pixel \( N \). Based on the discussion in Section 3.1, \( \sigma(N)/[\sigma(i)I] \) is approximately proportional to \( 1/[\sqrt{D}\sigma(i)I] \) under low-dose imaging conditions, and converges towards 1 for high-dose imaging conditions. The trend can be confirmed by Fig. 4. As a result, the influence of the statistical fluctuation is smaller at high dose than at low dose, and smaller for coarse samplings than for fine samplings, generally.

We evaluate the dose-dependent contrast by making one change to Eq. (12).

Our new dose-dependent contrast is defined as

\[
C_d = \frac{I_b - I_a}{\sigma(N)/[\sigma(i)I]_b} = \frac{I_b - I_a}{\sigma(N)/[\sigma(i)I]_b} = C_W[\text{SNR} \cdot \sigma(i)]^\top
\]

(14)

A factor \( \sigma(N)/[\sigma(i)I]_b \) is introduced to the denominator. Here \( \gamma \) is positive factor to be determined, and based on Appendix B one can take \( \gamma = 2 \) for all samplings. The introduction of this term is based on the relation between \( \sigma(N)/[\sigma(i)I] \) and atom contrast (Fig. 1a and Fig. 4). The term \( \sigma(N)/[\sigma(i)I] \) decreases with respect to the dose, resulting in increased atom contrast. For images calculated with very high dose, \( \sigma(N)/[\sigma(i)I] \) converges to 1 and the atom contrast converges to Eq. (12), which is the contrast calculated with infinite dose. For low dose, the term \( \sigma(N)/[\sigma(i)I] \) is approximately proportional to \( 1/[\sqrt{D}\sigma(i)I] \), and \( C_d \) approaches to 0 when \( N \) or the dose \( D \)
approaches 0. The trend is consistent with what we observe in Fig. 1a.

Fig. 5 shows the atom contrast calculated with Eq. (14) for different samplings and electron doses. The atom contrast increases with respect to the electron dose until it reaches the upper limit. For coarse sampling, the maximum atom contrast as well as the dose required to reach this maximum value is smaller than for fine sampling. The maximum contrast is 9.4% for the sampling of 0.3 Å/pixel, 14.7% for the sampling of 0.15 Å/pixel, 16.6% for the sampling of 0.075 Å/pixel and 17.2% for the samplings finer than 0.038 Å/pixel. Low-pass filtering (Fig. 5b) can enhance the atom contrast (compared with Fig. 5a) and the effect is especially remarkable for fine samplings.

The distance between two neighbor atoms in graphene is 1.4 Å, and this resolution can be obtained with the lowest dose of $5 \times 10^6$ e$^-$/nm$^2$ at the sampling of 0.15 Å/pixel and 0.075 Å/pixel (Fig. 6a). The dashed line in Fig. 5a shows the maximum atom contrast of 8.6% obtained under this dose condition. Low-pass filtering not only results in atomic resolution at fine samplings (Fig. 6b), but also improves the atom contrast (Fig. 5b). At the sampling of 0.15 Å/pixel under the dose of $5 \times 10^6$ e$^-$/nm$^2$, the contrast is improved to 13%.

3.3. Dependence of specimen resolution on dose and sampling

The attainable specimen resolution $d_s$ is dependent on the electron dose $D$, SNR and the dose-dependent contrast $C_D$. We adapt Eq. 3.1 of [15] to include the new dose-dependent contrast and express the specimen resolution as

$$d_s = \sqrt{d_i^2 + \frac{(SNR)^2}{C_D^2D}}.$$  

Here $d_i$ is the instrumental resolution, which is mainly defined by the wavelength $\lambda$ and the usable aperture $UA$: $d_i = \lambda / UA$.

Eq. (15) indicates that specimen resolution is a function of the electron dose $D$ and sampling, since SNR and $C$ are functions of the two quantities. The specimen resolution $d_s$ improves as the electron dose increases and reaches the limit of instrumental resolution for infinite dose. Based on the previous calculations on SNR and contrast, we plot the specimen resolution with respect to dose and sampling in
Fig. 6. The specimen resolution approaches infinity for very low doses, at which the atom contrast is also close to 0 (Fig. 5). The specimen resolution improves as the dose increases and converges to 0.8 Å which is equal to the instrumental resolution at 80 kV on the SALVE II prototype.

Low-pass filtering effectively improves the specimen resolution for fine samplings under low-dose conditions. For single-layer graphene, the atomic resolution can be achieved at 1.4 Å, marked by the dashed lines in Fig. 6a and b. The lowest dose to achieve this resolution is $5 \times 10^6 \text{ e}^-/\text{nm}^2$ for an unfiltered image, and $2 \times 10^6 \text{ e}^-/\text{nm}^2$ for a low-pass filtered image. The calculation is performed for the accelerating voltage of 80 kV.

3.4. Determination of the optimum sampling

In order to obtain good object visibility at low doses, it is necessary to choose a suitable sampling. Coarse sampling causes both contrast and resolution loss and hyperfine sampling results in low contrast (Figs. 5 and 6).

Generally, it is enough to preserve the structure information up to the required specimen resolution $d_s$. The highest spatial frequency corresponding to the specimen resolution is $1/d_s$, which is smaller than or equal to the information limit defined by the instrument $1/d_i$. It should be at least guaranteed that the Nyquist frequency $q_n \geq 1/d_s$, so that the sampling $\delta$ satisfies $\delta = 1/(2q_n) \leq d_i/2$, based on the sampling theorem for discrete Fourier transform [41].

On the other hand, the damping of the MTF at the Nyquist frequency $q = q_n$ is so strong that the contrast corresponding to this frequency is only 10–15% compared with the case without any MTF damping (Fig. A1). Coarse sampling results in strong damping of the MTF, and accordingly weak atom contrast. This effect can be confirmed in the case of high electron dose ($> 5 \times 10^8 \text{ e}^-/\text{nm}^2$), shown in Fig. B2, which is an extended version of Fig. 5a by exhibiting a broader range of electron dose. The maximum atom contrast decreases when the sampling gets coarse. At the sampling of 0.6 Å/pixel, the atom contrast obtained even with infinite electron dose is only about 1%.

The influence of the MTF damping on the image contrast can also be confirmed for low-pass filtered images (Fig. 5b). The filtered contrast for coarse sampling is lower than for fine sampling under the same dose condition.
For both unfiltered and filtered images, fine sampling ($\leq 0.075$ Å/pixel) is necessary in order to obtain high atom contrast. An ideal MTF profile (MTF $\approx 1$) should cover the spatial frequency up to the required specimen resolution, indicating that $(1/d_{\text{sam}})/(2q_{\text{D}}) \leq 0.1$. Therefore the sampling should be

$$\delta = 1/(2q_{\text{D}}) \leq 0.1d_{\text{s}}.$$  \hspace{0.5cm} (16)

The maximum sampling also defines the lowest magnification ratio allowed:

$$M = \frac{\delta}{\delta} \geq 10\delta_{\text{p}}/d_{\text{s}}.$$  \hspace{0.5cm} (17)

Here $\delta_{\text{p}}$ is the physical size of the detector pixel. If one wants to obtain an image with magnification ratio larger than $10\delta_{\text{p}}/d_{\text{s}}$, then the sampling $\delta$ should satisfy the relation $\delta \leq \delta_{\text{p}}/M$.

In order to obtain the maximum atom contrast for raw experimental images, the sampling should be finer than 0.075 Å/pixel and the required dose should be at least $5 \times 10^8$ e$^-$/nm$^2$ according to Fig. B2. However, such high electron dose is not realistic for beam-sensitive materials in TEM, which also means that it is barely possible to obtain an experimental image with the maximum atom contrast. For imaging with finite dose, the atom contrast obtained with fine sampling is not necessarily higher than obtained with coarse sampling even under the same dose conditions. An example is shown in Figs. 5 and B2, where for the electron doses between $5 \times 10^6$ e$^-$/nm$^2$ and $1 \times 10^8$ e$^-$/nm$^2$, the atom contrast at the sampling of 0.15 Å/pixel is higher than at finer samplings $\leq 0.075$ Å/pixel.

In order to find out the best sampling under low-dose conditions, we substitute $C_{\text{p}} = C_{\text{W}}[\text{SNR} \cdot \sigma(l)]^2$ (Eq. (14)) and the approximation $\text{SNR} \approx \sqrt{N}$ (Eq. (9)) into the second term in Eq. (15). We obtain

$$\frac{\langle \text{SNR}^2 \rangle}{C_{\text{p}}D} = \frac{1}{C_{\text{W}}\sigma^2(l)} \frac{1}{N\text{D}} \left[ \frac{1}{\delta C_{\text{W}}\sigma^2(l)} \right]^2 \frac{1}{D^2}$$  \hspace{0.5cm} (18)

We define

$$G(\delta) = \left[ \frac{1}{\delta C_{\text{W}}\sigma^2(l)} \right]^2,$$  \hspace{0.5cm} (19)

and since $C_{\text{W}}$ as well as $\sigma(l)$ depends on the sampling $\delta$, $G(\delta)$ is a function of $\delta$ only. In order to achieve the required specimen resolution $d_{\text{s}}$ with low electron dose $D$, $G(\delta)$ must be small. If $G(\delta)$ has a minimum value, then the required dose $D$ can also reach the minimum. In our case for graphene imaged with the SALVE II microscope, $G(\delta)$ is plotted with respect to the sampling $\delta$ in Fig. 7 for different accelerating voltages. Fig. 7 shows that $G(\delta)$ reaches the minimum for all accelerating voltages at about 0.2 Å/pixel, indicating that the required electron dose to reach a specific resolution at this sampling is the lowest with the current experimental settings. The closest sampling in our examples is 0.15 Å/pixel. On the other hand, a fine specimen resolution under a given dose condition also indicates a large contrast value $C_{\text{p}}$ (Eq. (15)). In the case of graphene, the distance between two neighbor atoms is 1.4 Å. As shown in Fig. 6a, the lowest electron dose to achieve this atomic resolution for graphene is $5 \times 10^6$ e$^-$/nm$^2$ at the sampling of 0.15 Å/pixel or 0.075 Å/pixel, and the maximum contrast of about 8.6% under this dose condition is obtained at the sampling of 0.15 Å/pixel, which is consistent with the analysis above.

As a summary, the optimum sampling and the lowest magnification ratio for raw image recorded with high dose are determined by the required specimen resolution $d_{\text{s}}$. The optimum sampling is determined by

$$\delta = \min \left[ 0.1d_{\text{s}}, \frac{d_{\text{s}}}{M} \right] \left( d_{\text{s}} \geq d_{\text{f}}, \ M \geq 10\delta_{\text{p}}/d_{\text{s}} \right).$$  \hspace{0.5cm} (20)

4. Summary

In this paper we have studied the influence of electron dose and sampling on the SNR, dose-dependent contrast and resolution using dose-dependent HRTEM image calculations. All three quantities improve with increasing electron dose, converging towards their values obtained at infinite dose. As sampling gets coarse, the SNR increases and the resolution deteriorates; the atom contrast improves as long as the damping of MTF is negligible. We have determined optimum sampling under high-dose and low-dose conditions. Under high-dose conditions, the optimum sampling depends mainly on the required specimen resolution. Under low-dose conditions, the best sampling is determined by our criteria that the required specimen resolution should be achieved with the minimal electron dose.

Acknowledgments

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Appendix A. The fitting of MTFs for the SALVE II microscope

The MTF data (Fig. A1) for the SALVE microscope at 20 kV, 40 kV, 60 kV and 80 kV were previously measured using averaged single electron events detected by the T416 camera [38,39].

The measured MTFs (Fig. A1) are fitted by Eq. (A.1) and the coefficients $a_1$, $a_2$, $a_3$, $a_4$ as well as the coefficient of determination $R^2$ are listed in Table A1. The magnitude of $R^2$ shows the correlation between the original MTF profile and the fitted function. Large $R^2$ value indicates good fitting.

$$MTF(q) = \frac{a_1 - a_2}{1 + \frac{q}{2a_3q_N}} + a_4$$  \hspace{1cm} (A.1)

Here $q_N$ is the Nyquist frequency related to the sampling $\delta$ by $\delta = 1/(2q_N)$.

Usually the $x$-axis of the MTF profile represents the frequency corresponding to a certain feature sampled with $n$ pixels in real space, and $1/n$ has the unit of pixel$^{-1}$. Here we have used another notation in order to facilitate the image calculation. By multiplying both the numerator and the denominator with the sampling $\delta$, we obtain

$$\frac{1}{n} = \frac{\delta}{n\delta} = \frac{1}{1/\delta}$$  \hspace{1cm} (A.2)

The denominator is the reciprocal of the sampling $\delta$, which equals two times of the Nyquist frequency $q_N$. The numerator represents the spatial frequency $q$ corresponding to the feature sampled with $n$ pixels in real space. We obtain

$$\frac{1}{n} = q\delta = q \frac{\delta}{2q_N}$$  \hspace{1cm} (A.3)

![Fig. A1. Experimentally measured MTFs for the SALVE II microscope at 20 kV, 40 kV, 60 kV and 80 kV. At Nyquist frequency where $q/(2\delta) = 0.5$, the MTF is between 0.1 and 0.15, indicating that the image contrast corresponding to this frequency has a loss of 85–90%.]

Table A1

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<th>$a_3$</th>
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Appendix B. Determination of the factor $\gamma$ in Eq. (14)

We calculated the coefficient of determination $R^2$ showing the correlation between the images calculated with finite electron doses and the ones calculated with infinite electron dose (Fig. B1). If we define

$$S_{12} = \sum_{j=1}^{I} [\text{Pois}(D\delta^2 I_j) - \text{Pois}(D\delta^2 I_b)[I_j - I_b]$$  \hspace{1cm} (B.1)

$$S_{11} = \sum_{j=1}^{I} [\text{Pois}(D\delta^2 I_j) - \text{Pois}(D\delta^2 I_b)^2]$$  \hspace{1cm} (B.2)

$$S_{22} = \sum_{j=1}^{I} [I_j - I_b]^2$$  \hspace{1cm} (B.3)

Then the coefficient of determination is

$$R^2 = \frac{S_{12}^2}{S_{11}S_{22}}$$  \hspace{1cm} (B.4)

The coefficient $R^2$ lies in the range of [0, 1], where the number 1 indicates a perfect match and 0 indicates no match at all. $R^2$ increases smoothly with respect to the electron dose. When $R^2=1$, the atom contrast equals the value obtained with infinite dose $C_D = (I_b - I_0)/I_b$; and when $R^2=0$, the atom contrast is $C_D=0$. Therefore we can define the dose-dependent contrast by scaling $R^2$ with the Weber contrast $C_D = (I_b - I_0)/I_b$, resulting in Fig. B2. One can see that for coarse sampling, the maximum atom contrast is smaller than for fine sampling. This trend is consistent with the evolution of atom contrast with respect to the electron dose (Fig. 1), and we can use Eq. (14) to fit the curves in Fig. B2 in the following way:

$$\frac{R^2 I_b - I_0}{I_b} = \left(\frac{\sigma(N)}{\bar{\sigma}(I)}\right)^{-\gamma} \frac{I_b - I_0}{I_b}$$  \hspace{1cm} (B.5)

results in

$$\ln \frac{1}{R^2} = \gamma \ln \frac{\sigma(N)}{\bar{\sigma}(I)}$$  \hspace{1cm} (B.6)

Here all the terms in the logarithms are larger than 0. By linear fitting of $\ln(\sigma(N)/\bar{\sigma}(I))$ and $\ln(1/R^2)$, we can obtain the coefficient $\gamma$ which is the slope of the line. The fitting is displayed in Fig. B3, where all the lines show similar slope.

The corresponding $\gamma$ values for different samplings obtained through linear fitting in Fig. B3 is listed in Table B1. Since the

![Fig. B1. The coefficient of determination $R^2$ shows the correlation between the images calculated with finite and infinite electron doses at different samplings. The calculation is performed for the accelerating voltage of 80 kV.](image-url)
results are very close, we can take $\gamma=2$ for all samplings. This result $\gamma=2$ for all samplings is not surprising. In the case of high dose, $\text{Poiss}(D\delta^2 I_j) \approx D\delta^2 I_j$ and $\text{Poiss}(D\delta^2 I_j') \approx D\delta^2 T_j$. Therefore

$$S_{12} = \frac{1}{s} \sum_{j=1}^{s} \left[ \text{Poiss}(D\delta^2 I_j) - \text{Poiss}(D\delta^2 I_j') \right] [I_j - T_j]$$

$$\approx D\delta^2 \sum_{j=1}^{s} [I_j - T_j]^2 = D\delta^2 S_{22},$$

(B.7)

The relation between $R$ and $\overline{\sigma/N}(\sigma/N)$ is then clarified. Here the relation $\gamma=1$ has been used.

**References**


**Table B1**

The $\gamma$ values fitted for different samplings based on Eq. (B.6).

<table>
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<tr>
<th>Sampling (Å/pixel)</th>
<th>$\gamma$</th>
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<tr>
<td>0.6</td>
<td>1.981</td>
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<tr>
<td>0.3</td>
<td>1.987</td>
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<tr>
<td>0.15</td>
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<td>0.075</td>
<td>2.003</td>
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<td>0.019</td>
<td>2.006</td>
</tr>
<tr>
<td>0.01</td>
<td>2.000</td>
</tr>
</tbody>
</table>

**Fig. B2.** The atom contrast obtained by scaling the coefficient $R^2$ with the contrast achieved under infinite dose, namely $R^2(\text{I}_0 - \text{I}_a)/\text{I}_a$. At coarse samplings, the maximum atom contrast is lower than at fine samplings. The calculation is performed for the accelerating voltage of 80 kV.

**Fig. B3.** The linear fitting of $\ln (\sigma(N)/\overline{\sigma}(N))$ and $\ln (1/R^2)$. The slope of the line is the coefficient $\gamma$ and the slope is very close for different samplings. The calculation is performed for the accelerating voltage of 80 kV.

$$R^2 = S_{12}^2 \sum_{j=1}^{s} \left[ \text{Poiss}(D\delta^2 I_j) - \text{Poiss}(D\delta^2 I_j') \right] [I_j - T_j]$$

$$\approx D\delta^2 \sum_{j=1}^{s} [I_j - T_j]^2 = D\delta^2 S_{22},$$

(B.7)