Practical Reasoning in DL
(Introduction)
What’s in this course?

1. Introduction (today)
   - Origins, syntax and semantics of basic DLs (\textit{ALC})
   - Basic reasoning problems, inter-reducibility
   - Anatomy of a reasoner

2. Tableau algorithms

3. Classification and realization (Bijan)

4. Consequence-based reasoning for lightweight DL (\textit{EL})

5. Dealing with data (ontology-based access to databases, Bijan)
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We’ll pay special attention to practicality

- Essential optimization techniques
- Performance evaluation techniques
Welcome!

Let us know if you

- have questions; do ask them at any time
- have difficulties understanding
- find this course too slow/boring
- find this course too fast/difficult

In this course, we will:

- ask you to think a lot
- ask you to work through numerous examples
Origins of Description Logics

Basics: Syntax, Semantics, Reasoning Problems

Anatomy of a Reasoner
DLs: Where They Come From

DLs are logic-based knowledge representation (KR) formalisms
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- Common perception: logic is difficult for human conception
  - e.g., how long does it take you to read
    \[
    \forall x \exists y \forall z ((r(x, y) \land s(y, z)) \Rightarrow (\neg s(a, y) \lor r(x, z)))
    \]
  - or check that it is equivalent to
    \[
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▶ Also not-so-easy for machines (only semi-decidable)
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- Also not-so-easy for machines (only semi-decidable)

Are there better suited alternatives?

- Can we help users learn/speak/interact with logic?
- Or perhaps not use logic at all?
Early KR Formalisms

Were mostly **graphical** because pictures are:

- easier to grasp:
  “A picture says more than a thousand words.”

- close to how knowledge is represented in human beings (?)
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Most graphical KR formalisms represent knowledge as **graphs** with

- labeled nodes
  
  mostly representing concepts, classes, individuals etc.

- labeled edges
  
  mostly representing properties, relationships etc.
What does this graph say exactly? Is Betty a Student?

Problem: it is unclear. Semantics is missing or implicit.

Remedy: base your picture on logic or use logic directly.
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Semantic Networks

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“I think we must begin with the realization that there is currently no theory of semantic networks.”
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17 distinct interpretations of IS-A!
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Instead we need a KR formalism which
- has a well-defined semantics
- is reasonably accessible to users
- balances expressivity and computational practicality
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Instead we need a KR formalism which

▶ has a well-defined semantics
▶ is reasonably accessible to users
▶ balances expressivity and computational practicality

DLs came out of the effort to design such formalism
Terminological Knowledge and Facts

**DLs**: designed to represent **terminological** or **conceptual** knowledge
Terminological Knowledge and Facts

**DLs:** designed to represent *terminological* or *conceptual* knowledge

**Goals:**

- Formalise basic *terminology* of an application domain;
- Enable reasoning about *concepts*:
  - Can there be *Mammals*?
  - Is every *Mammal* an *Animal*?
  - Are *Frogs* *Reptiles*?
Terminological Knowledge and Facts

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- Enable reasoning about **concepts:**
  - Can there be **Mammals**?
  - Is every **Mammal** an **Animal**?
  - Are **Frogs** **Reptiles**?
- Represent facts about individuals
- Enable reasoning about **individuals** and **concepts**:
  - Are my facts **consistent** with my terminology?
  - Is **Kermit** a **Frog**?
Reasoning

By “reasoning” we understand deductive inference

▶ From general knowledge to specific conclusions
▶ All results are necessarily true
  If $\alpha$ follows from $\Sigma$, then $\neg\alpha$ is inconsistent with $\Sigma$
Reasoning

By “reasoning” we understand *deductive* inference

- From general knowledge to specific conclusions
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  If $\alpha$ follows from $\Sigma$, then $\neg \alpha$ is inconsistent with $\Sigma$

$\Sigma :$ *ESSLLI attendees are students.* *John attends ESSLLI.*

$\alpha :$ *John is a student*
Reasoning

By “reasoning” we understand deductive inference

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  If $\alpha$ follows from $\Sigma$, then $\neg \alpha$ is inconsistent with $\Sigma$

$\Sigma :$ ESSLLI attendants are students. John attends ESSLLI.

$\alpha :$ John is a student

We never induce relationships from examples

John attends ESSLLI, Mary attends ESSLLI, Jim attends ESSLLI, Maria attends ESSLLI, ...

$\alpha :$ ESSLLI attendants are students
Applications of Description Logics

Medical Informatics
Applications of Description Logics

Medical Informatics

- **SNOMED CT**
  Systematized Nomenclature of Medicine – Clinical Terms
  clinical terminology (used for EHR, clinical DSS, etc.)
  >300,000 classes (diseases, conditions, etc.)

- **NCI Thesaurus** (NCI = National Cancer Institute of the USA)
  vocabulary for clinical care, translational and basic research,
  public information, administrative activities
  Information on >10,000 cancers

- **ICD 11** (International Classification of Diseases)
  used worldwide for health statistics
  when someone dies, there’s always a code from ICD 11
Applications of Description Logics

Bioinformatics

- **GO** (Gene Ontology)
  
  *controlled* vocabulary of terms for gene product characteristics and gene product annotation data

- **Bioportal**
  
  REST/Web UI access to 255 bio-health ontologies
Applications of Description Logics

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Semantic Web

- Supply **meaning** to (linked open) data
- Use **TBox** when **querying** data (Lecture 5 will cover this)
  - ontology-based data access
  - data integration
Why Reasoning is Important?

Helps to avoid or fix errors during ontology development or use:

- Inconsistencies: “protein P1 is located in L1, protein P2 is located in L2 disjoint with L1, interaction I was recorded between P1 and P2”
- Wrong conclusions: “Flu is inferred to be a sub-concept of Cancer”
- Missing conclusions: “Flu is not inferred to be a sub-concept of ViralDisease”

Reasoning enables definition-oriented development

- User does not assert relations, only writes definitions
- Reasoning infers the concept hierarchy

Reasoning can be used to explain the errors
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Why Reasoning is Important?

Helps to avoid or fix errors during ontology development or use:

- **Inconsistencies**: “protein P₁ is located in L₁, protein P₂ is located in L₂ disjoint with L₁, interaction I was recorded between P₁ and P₂”

- **Wrong conclusions**: “Flu is inferred to be a sub-concept of Cancer”
Why Reasoning is Important?

Helps to avoid or fix errors during ontology development or use:

- **Inconsistencies**: “protein $P_1$ is located in $L_1$, protein $P_2$ is located in $L_2$ disjoint with $L_1$, interaction $I$ was recorded between $P_1$ and $P_2$”

- **Wrong conclusions**: “Flu is inferred to be a sub-concept of Cancer”

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Origins of Description Logics

Basics: Syntax, Semantics, Reasoning Problems

Anatomy of a Reasoner
Syntax and Semantics

What you should remember from your logic class!

Any logic has two key components:

▶ **Syntax**: formal language used to write formulas of the logic
▶ **Semantics**: specifies how to interpret those formulas
Syntax and Semantics

What you should remember from your logic class!

Any logic has two key components:

- **Syntax:** formal language used to write formulas of the logic
- **Semantics:** specifies how to interpret those formulas

Why?

- **Syntax:** machines can parse/reject formulas specified using a formal grammar (EBNF, etc)
- **Semantics:** machines can understand them specified using the language of the Set Theory
- Programming language analogy:
  Syntax and semantics specified in the Standard (e.g., C++)
Concept Language

Core part of a DL: its concept language, e.g.:

\[ \text{Animal} \sqcap \exists \text{hasPart} . \text{Feather} \]

describes all animals that are related via hasPart to a feather
Concept Language

Core part of a DL: its concept language, e.g.:

\[ \text{Animal} \sqcap \exists \text{hasPart}.\text{Feather} \]

describes all animals that are related via hasPart to a feather

Syntactic components of a concept language:

- **Concept** names: stand for sets of elements, e.g., Animal
- **Role** names: stand for binary relations between elements, e.g., hasPart
- **Constructors**: used to build concept expressions:
  \[ \sqcap, \sqcup, \exists, \forall \]
Syntax of \textit{ALC}

\textit{ALC} is one of the basic/earliest description logics

- properly contains propositional logic
- enough expressivity for conceptual graphs
- notational variant of well-studied modal logic $K_N$
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$\text{ALC}$ is one of the basic/earliest description logics

- properly contains propositional logic
- enough expressivity for conceptual graphs
- notational variant of well-studied modal logic $K_N$

The set of concepts in $\text{ALC}$ is defined recursively as follows:

- Every concept name is a concept
- $\top, \bot$ are concepts (pronounced “top” and “bottom”)
- $C \sqcap D, C \sqcup D, \text{ and } \neg C$ are concepts if $C$ and $D$ are
- Role restrictions are concepts if $C$ is a concept and $R$ is a role
  - $\exists R. C$ existential restriction
  - $\forall R. C$ universal restriction
Semantics of $\mathcal{ALC}$

Semantics given via interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$, where:

- $\Delta^{\mathcal{I}}$ is a non-empty set (the domain),
- $\cdot^{\mathcal{I}}$ is a mapping (the interpretation function)
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<tr>
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<td>conjunction</td>
<td>$C \sqcap D$</td>
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<td>disjunction</td>
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<td>Nice $\sqcup$ Rich</td>
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<td>negation</td>
<td>$\neg C$</td>
<td>$\neg$Meat</td>
<td>$\Delta^\mathcal{I} \setminus C^\mathcal{I}$</td>
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<tr>
<td>restrictions:</td>
<td></td>
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<td>$\exists R. C$</td>
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<td>${ x</td>
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<tr>
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The Griffin Family

Look at interpretations on some real example
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We pick some $\mathcal{I}$. It fixes the interpretation of base terms:
The Griffin Family

We pick some $\mathcal{I}$. It fixes the interpretation of base terms: $\text{Human}^\mathcal{I}$:
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We pick some $\mathcal{I}$. It fixes the interpretation of base terms:

**Human$^\mathcal{I}$**: 

**Male$^\mathcal{I}$**: 

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**Parent**$^\mathcal{I}$:
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We pick some $\mathcal{I}$. It fixes the interpretation of base terms:

Human$^\mathcal{I}$:

Male$^\mathcal{I}$:

Parent$^\mathcal{I}$:

hasChild$^\mathcal{I}$: $\{(\text{Barbara}, \text{Lois}), (\text{Peter}, \text{Chris}), (\text{Peter}, \text{Meg}), (\text{Peter}, \text{Stewie}), (\text{Lois}, \text{Chris}), (\text{Lois}, \text{Meg}), (\text{Lois}, \text{Stewie})\}$
Boolean Connectives Are Easy

\[ \mathcal{ALC} \text{ is a propositionally complete language} \]
Boolean Connectives Are Easy

\textbf{ALC} is a propositionally complete language

Lois is an instance of \((\text{Female} \sqcap \text{Parent})^\mathcal{I}\)
Existential Restrictions

$\exists \text{hasChild}.\text{Male}$ means the set of those who are in $\text{hasChild}^I$ relation with an instance of $\text{Male}^I$
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$\exists \text{hasChild} . \text{Male}$ means the set of those who are in $\text{hasChild}^\mathcal{I}$ relation with an instance of $\text{Male}^\mathcal{I}$
**Existential Restrictions**

∃hasChild.Male means the set of those who are in hasChild<sup>I</sup> relation with an instance of Male<sup>I</sup>
Universal Restrictions

\( \forall \text{hasChild}.\text{Female} \) means the set of those who are in \( \text{hasChild}^{I} \) relation with only instances of \( \text{Female}^{I} \):
\[
\{ x \mid \forall y. (x, y) \in \text{hasChild}^{I} \Rightarrow y \in \text{Female}^{I} \}
\]

Question: what are the instances of \( (\forall \text{hasChild}.\text{Female})^{I} \)
Universal Restrictions

∀hasChild.Female means the set of those who are in hasChild\(^I\) relation with only instances of Female\(^I\):
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**$\mathcal{ALC}$** Concept Interpretations

$C^\mathcal{I}$ can be visualized as a labeled graph $\mathcal{G}_{C^\mathcal{I}} = \langle V, E \rangle$, where

- $V$ is a non-empty set of domain elements where $v_0 \in C^\mathcal{I}$
- $D \in L(v)$ if $v \in D^\mathcal{I}$
- $(x, y)$ is a $R$-labeled edge if $(x, y) \in R^\mathcal{I}$
**ALC** Concept Interpretations

$C^I$ can be visualized as a labeled graph $\mathcal{G}_{C^I} = \langle V, E \rangle$, where

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**Example:** Male $\sqcap \exists$hasChild.$(\text{Nerd} \sqcap \exists$hasSibling.$\text{Female})$
**ALC Concept Interpretations**

$C^\mathcal{I}$ can be visualized as a *labeled* graph $G_{C^\mathcal{I}} = \langle V, E \rangle$, where

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**Example:** Male $\sqcap \exists \text{hasChild}.(\text{Nerd} \sqcap \exists \text{hasSibling}.\text{Female})$

```
Peter: Male $\sqcap \exists \text{hasChild}.(\text{Nerd} \sqcap \exists \text{hasSibling}.\text{Female})$
```

```
Stewie: Nerd $\sqcap \exists \text{hasSibling}.\text{Female}$
```

```
Meg: Female
```

```
\text{hasChild}
```

```
\text{hasSibling}
```

Basic Reasoning Problems

Definition: let $C, D$ be $\mathcal{ALC}$ concepts. We say that

- $C$ is satisfiable if there exists some $\mathcal{I}$ such that $C^\mathcal{I} \neq \emptyset$.
- $C$ is subsumed by $D$ (written $\emptyset \models C^\mathcal{I} \subseteq D^\mathcal{I}$) if for every interpretation $\mathcal{I}$, it is true that $C^\mathcal{I} \subseteq D^\mathcal{I}$.
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**Question:** Which of the following concepts is satisfiable? Which is subsumed by which?

(1) $\exists R. (A \cap B)$  (2) $\exists R. (A \cup B)$
(3) $\forall R. (A \cap B)$  (4) $\exists R. (A \cap \neg A)$
(5) $\exists R. A \cap \forall R. B$  (6) $\exists R. A$
(7) $\exists R. A \cap \forall R. \neg A$  (8) $\exists R. A \cap \forall S. \neg A$
The TBox (Terminology)

Definition

- A general concept inclusion (GCI) is a statement of the form $C \sqsubseteq D$, where $C$, $D$ are (possibly complex) concepts.
- A (general) TBox is a finite set of GCIs: $\mathcal{T} = \{ C_i \sqsubseteq D_i \mid 1 \leq i \leq n \}$.
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  $$\mathcal{T} = \{ C_i \sqsubseteq D_i \mid 1 \leq i \leq n \}$$
- $\mathcal{I}$ satisfies $C \sqsubseteq D$ if $C^\mathcal{I} \subseteq D^\mathcal{I}$ (written $\mathcal{I} \models C \sqsubseteq D$)
- $\mathcal{I}$ is a model of TBox $\mathcal{T}$ if $\mathcal{I}$ satisfies every $C_i \sqsubseteq D_i$
- We use $C \equiv D$ to abbreviate $C \sqsubseteq D$, $D \sqsubseteq C$
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Example:
\[
\{ \text{Father} \equiv \text{Man} \sqcap \exists \text{hasChild.Human}, \\
\text{Human} \equiv \text{Mammal} \sqcap \forall \text{hasParent.Human}, \\
\exists \text{favourite.Brewery} \sqsubseteq \exists \text{drinks.Beer} \}
\]
**ALC TBox Interpretations**

\( T^I \) can be visualized just as \( C^I \), as \( G_{T^I} = \langle V, E \rangle \), where

- \( D \in L(v) \) if \( v \in D^I \)
- \( (x, y) \) is a \( R \)-labeled edge if \( (x, y) \in R^I \)
- one may start with one node for each concept name in \( T \)

Example:

\[
\begin{align*}
\text{Father} & \equiv \text{Man} \sqcap \exists \text{hasChild}. \text{Human} \\
\text{Human} & \equiv \text{Mammal} \sqcap \forall \text{hasParent}. \text{Human} \\
\exists \text{favourite}. \text{Brewery} & \sqsubseteq \exists \text{drinks}. \text{Beer}
\end{align*}
\]

Exercise:

1. Draw one model of this TBox
2. Draw one non-model of this TBox
\textbf{ALC} TBox Interpretations

\( \mathcal{T}^I \) can be visualized just as \( \mathcal{C}^I \), as \( \mathcal{G}_{\mathcal{T}^I} = \langle V, E \rangle \), where

- \( D \in L(v) \) if \( v \in D^I \)
- \( (x, y) \) is a \( R \)-labeled edge if \( (x, y) \in R^I \)
- one may start with one node for each concept name in \( \mathcal{T} \)

Example:
\[
\{ \text{Father} \equiv \text{Man} \sqcap \exists \text{hasChild.Human} , \\
\text{Human} \equiv \text{Mammal} \sqcap \forall \text{hasParent.Human} , \\
\exists \text{favourite.Brewery} \sqsubseteq \exists \text{drinks.Beer} \}
\]

Exercise:

1. Draw one model of this TBox
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Reasoning Problems w.r.t. TBox

**Definition:** let $C, D$ be concepts, $\mathcal{T}$ a TBox. We say that

- $C$ is **satisfiable w.r.t.** $\mathcal{T}$ if there is a model $\mathcal{I}$ of $\mathcal{T}$ with $C^\mathcal{I} \neq \emptyset$

- $C$ is **subsumed by** $D$ w.r.t. $\mathcal{T}$ (written $\mathcal{T} \models C \sqsubseteq D$) if, for every model $\mathcal{I}$ of $\mathcal{T}$, we have $C^\mathcal{I} \subseteq D^\mathcal{I}$
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**Example:**

$$\mathcal{T} = \{ \ A \subseteq B \cap \exists R. C, \ \exists R. T \subseteq \neg A \}$$

**Questions:** Does $\mathcal{T}$ have a model?
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$$\mathcal{T} = \{ \text{A} \sqsubseteq \text{B} \sqcap \exists \text{R.C}, \exists \text{R.T} \sqsubseteq \neg \text{A} \}$$

**Questions:** Does $\mathcal{T}$ have a model? Are all concept names in $\mathcal{T}$ satisfiable?
TBox + ABox ≡ Ontology

- TBox captures knowledge on a general, conceptual level
- contains concept def.s + general axioms about concepts
TBox + ABox ≡ Ontology

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Think databases: TBox defines schema. Where’s data?
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- captures knowledge on an individual level
- is a finite set of
  - concept assertions $a : C$ e.g., John : Man,
  - role assertions $(a, b) : R$ e.g., (John, Mary) : hasChild
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A pair of a TBox \( T \) and an ABox \( A \) is called ontology

\( O = (T, A) \)
$\textbf{ALC}$ ABox Interpretations

Semantics: an interpretation $\mathcal{I}$

- maps each individual name $e$ to some $e^\mathcal{I} \in \Delta^\mathcal{I}$
**ALC** ABox Interpretations

**Semantics:** an interpretation $\mathcal{I}$

- maps each **individual name** $e$ to some $e^\mathcal{I} \in \Delta^\mathcal{I}$
- satisfies a concept assertion $a : C$ if $a^\mathcal{I} \in C^\mathcal{I}$

**Question:** why do I not define entailments of role assertions?

**Answer:** no non-trivial role assertion entailments in ALC!
\textbf{\textit{ALC}} ABox Interpretations

\textbf{Semantics: an interpretation $\mathcal{I}$}

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\( \mathcal{A} = \{ a : B \sqcap \exists R. C, b : A \sqcap \neg D \sqcap \forall S. \forall R. F, (b, a) : S \} \)

Question: can you see any entailments?
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Combined interpretation for TBox and ABox
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\exists R.\top \sqsubseteq \neg A , \quad (a, b) : R \}$

Questions: Does $\mathcal{O}$ have a model?
Can you see any entailments?
What about $\mathcal{O} \cup \{ b : A \}$?
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This has big practical impact for reasoners

- Schema is often small
- Data is often large
Description Logics and OWL

OWL is W3C-standardized **Web Ontology Language**

- If you publish your ontology, it **should** be in OWL
- If you don’t, then it **better** be in OWL
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OWL is basically a DL + a common syntax
  ▶ Syntax: OWL/XML, Functional, Manchester, RDF-based
  ▶ Semantics: DL model theory
    any reasoning in OWL is reduced to reasoning in DL
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OWL includes more stuff:
  ▶ Datatypes: strings, integers, dates, etc.
    a.k.a. concrete domains in some DLs
  ▶ Non-logical stuff: annotations
OWL Profiles

OWL is a family of languages designed for specific scenarios
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## OWL Profiles

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Each profile trades some expressivity for computational guarantees:

- **OWL EL**: \( \text{PTIME} \) classification
- **OWL QL**: scalable query answering
- **OWL RL**: completeness w.r.t. rule systems
Reducibility of DL Reasoning Problems

Given an ontology $\mathcal{O} = (\mathcal{T}, \mathcal{A})$:

- is $\mathcal{O}$ consistent?

\[ \mathcal{O} \models \top \sqsubseteq \bot? \]
Reducibility of DL Reasoning Problems

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Question: do we need 4 different algorithms for these?
Consistency Suffices

**Theorem:** Let $\mathcal{O}$ be an ontology and $a$ a fresh individual. Then:

1. $C$ is satisfiable w.r.t. $\mathcal{O}$ iff $\mathcal{O} \cup \{a : C\}$ is consistent
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Answer: a decision procedure to solve consistency decides all standard DL reasoning problems
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This does not mean that the naive reduction is practical!
Origins of Description Logics

Basics: Syntax, Semantics, Reasoning Problems

Anatomy of a Reasoner
What is Reasoner

Reasoner is a system that solves DL reasoning problems
What is Reasoner

Reasoner is a system that solves DL reasoning problems

- **Input**: ontology (+ an axiom, e.g., $C \sqsubseteq D$)
- **Output**: yes/no (consistency, entailment), concept hierarchy (classification)
  - yes/no for consistency, entailment, satisfiability
  - concept hierarchy for classification
  - individual to concepts mapping for realization
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Reasoner is more than just implementations of algorithms

- It has to interact with the world (suitable APIs, load data, . . . )
- It has to convert the input into a suitable form
- It has to invoke the right algorithm at the right time
- It has to manage optimizations
Reasoner: the Main Layers

**Interface Layer**
- Parsing
- API bindings
- Protégé plugin

**Internal Data Model**
- (Terms, $\cap$, $\cup$, $\exists$, $\forall$)

**Intermediate Layer**
- Pre-processing, indexing, caching, taxonomy
- Incomplete reasoning

**Reasoning Layer**
- Classification
- Realization
- Core reasoning procedure
- Query answering
Interface Layer

Reasoner must be able to interact with world

- Load ontologies and export the results
- Interact with software via standard APIs (OWL API)
- Be useable in ontology editors (Protégé)
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APIs usually provide parsers, serializers, and the data model... but reasoners often support their own for efficiency
Internal Data Model

Internal Data Model is representation of loaded knowledge

- Optimized for reasoning tasks
- Covers supported features of the language
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Example: Object-Oriented API for $\mathcal{ALC}$
Internal Data Model

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Example: Object-Oriented API for $\mathcal{ALC}$

IDM does not have to mirror the language model
- Can opt for the minimal sufficient set of constructors
- ELK does not store axioms, only rules
Intermediate Layer: Pre-processing and Indexing

Pre-processing: massaging data before sending to reasoning layer

Purely syntactic axioms/concept rewriting:

- **Normalization**: \( \neg (C \sqcap D) \leadsto \neg C \sqcup \neg D \)
- **Simplification**: \( \exists R. A \sqcap \exists R. (A \sqcap B) \leadsto \exists R. (A \sqcap B) \)
- **Absorption**: \( A \sqcap C \sqsubseteq D \leadsto A \sqsubseteq \neg C \sqcup D \)
Intermediate Layer: Pre-processing and Indexing

Pre-processing: massaging data before sending to reasoning layer

Purely syntactic axioms/concept rewriting:

- **Normalization:** \( \neg (C \cap D) \leadsto \neg C \sqcup \neg D \)
- **Simplification:** \( \exists R. A \sqcap \exists R. (A \sqcap B) \leadsto \exists R. (A \sqcap B) \)
- **Absorption:** \( A \sqcap C \sqsubseteq D \leadsto A \sqsubseteq \neg C \sqcup D \)

Indexing: extra data structure for faster look-ups

- \( A \mapsto \) set of told subsumers
- \( A \mapsto \) set of told disjoint concepts
- \( \ldots \)
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Indexing: extra data structure for faster look-ups

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- ...  

Practical hint: both can be done in parallel with parsing/loading
Intermediate Layer: Reductions

Reasoner **reduces** the input problem to the one for which the **core procedure** is optimized

- **Tableau:** \( \mathcal{O} \models \alpha \leadsto \) is \( \mathcal{O} \cup \{\neg \alpha\} \) consistent?

- **Consequence-based algorithms:** \( \mathcal{O} \models \neg C \sqcup D \leadsto \mathcal{O} \models C \sqsubseteq D \)  
  (CB algorithms compute subsumers in a **goal-directed** way)
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- **Consequence-based algorithms:** $\mathcal{O} \models \neg C \sqcup D \equiv \mathcal{O} \models C \sqsubseteq D$
  
  (CB algorithms compute subsumers in a goal-directed way)

Reasoner can also reduce the problem to one previously solved

- $\mathcal{O} \models \neg C \sqcup D \sqsubseteq \bot \equiv \mathcal{O} \models C \sqsubseteq D$

  if subsumers for $C$ have been computed
Intermediate Layer: Caching

- **Single shot reasoning**: just answer one query, e.g.,
  \[
  \mathcal{O} \models C \sqsubseteq D, \text{ and discard everything}
  \]
- **Multiple reasoning**: save and re-use intermediate results
Intermediate Layer: Caching

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Reasoner infers a lot more than it shows to the user

**Example:** >24M inferences when classifying SNOMED CT
(“only” 300K concepts)
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**Example:** >24M inferences when classifying SNOMED CT ("only" 300K concepts)

Other stuff, e.g., complex subsumers, can be re-used later.

This layer decides:

- what to save
- what to discard (w.r.t. which policy)
- how to look things up
Caching

Example: the reasoner inferred $A \sqsubseteq C \sqcap D$ while checking satisfiability of $A$
Caching

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while checking satisfiability of $A$

**Next task:** compute subsumers of $A$. It can immediately:

- ignore concepts disjoint with $C$ or $D$, if known
- take subsumers of $C$ or $D$, if known
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Caches may be cleared when the ontology is changed... or not! Incremental reasoning algorithms exist

Deletions are particularly tricky. Why?
Intermediate Layer: Incomplete Reasoning

Main reasoning algorithms are nearly always expensive.

Often there are cheaper ways to get the answer:

- looking up in the cache
- by probing instead of searching systematically
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Examples:

- \( \mathcal{EL} \): if \( \bot \) does not occur in \( O \), it cannot be inconsistent
- Detecting obvious conflicts, e.g., \( \exists R. \neg C \) and \( \forall R. C \)
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Approximations can help too
If $\mathcal{O}' \subseteq \mathcal{O}$ and $\mathcal{O}' \models \alpha$, then $\mathcal{O} \models \alpha$ (monotonicity)
$\mathcal{O}'$ can fit into a simpler language $\Rightarrow$ easier to reason with
Reasoning Layer: Core Reasoning Procedure

Implementation of the main reasoning algorithm

- **Expressive DLs**: usually tableau algorithm for consistency (L2)
- **Lightweight DLs**: rule-based saturation algorithm (L4)
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Should be **compact and extensible** to:

- New language features
- New optimizations
Reasoning Layer: Core Reasoning Procedure

Implementation of the main reasoning algorithm

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Should be **compact** and **extensible** to:

- New language features
- New optimizations

Should be **reusable** for higher level tasks:

- Explanations and debugging  
  (find all *reasons* why $\mathcal{O} \models C \sqsubseteq \bot$ happens)
- Query answering
- Incremental reasoning
Reasoning Layer: Classification and Realization

Classification: compute $\mathcal{O} \models A \sqsubseteq B$ for all concept names in $\mathcal{O}$

Realization: compute $\mathcal{O} \models a : A$ for all individuals in $\mathcal{O}$
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Once the ontology is classified, many tasks are easier
Taxonomy Construction

Users want to see only direct subsumptions

- A is directly subsumed by B if $\mathcal{O} \models A \sqsubseteq B$ and
- There is no C s.t. $\mathcal{O} \models A \sqsubseteq C$ and $\mathcal{O} \models C \sqsubseteq B$
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Class taxonomy is transitively reduced graph of subsumptions

Non-trivial, the complexity is $O(n^k)$ where $k > 2$
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Transitive reduction algorithms can
- maintain the reduction as the ontologies is classified
- compute the reduction post factum
Example: Pellet

Pellet – one of the earliest complete reasoners for expressive DLs
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Example: ELK

**ELK** – state-of-the-art consequence-based reasoner for $\mathcal{EL}$-family
Example: ELK

**ELK** – state-of-the-art **consequence-based** reasoner for $\mathcal{EL}$-family

- **Pipelining:** loading, indexing, classification, taxonomy
- **Concurrency:** all concepts are classified in parallel
Stuff Seen Today
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Brief history of Description Logics

- Concept languages originating from semantic networks
- Have formal first-order semantics
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**ALC** – a basic propositionally complete DL

- Syntax and semantics (concept constructors, model theory)
- TBox and ABox a.k.a. schema and data
- Reasoning problems
  - Concept satisfiability, entailment, ontology consistency
  - Inter-reducibility

What's inside a modern DL reasoner

Tomorrow: how reasoning is actually done (tableau algorithms)
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