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August 5, 2013

Practical Reasoning in DL (Introduction)

What's in this course?

1. Introduction (today)
   - Origins, syntax and semantics of basic DLs ($\mathcal{ALC}$)
   - Basic reasoning problems, inter-reducibility
   - Anatomy of a reasoner
2. Tableau algorithms
3. Classification and realization (Bijan)
4. Consequence-based reasoning for lightweight DL ($\mathcal{EL}$)
5. Dealing with data (ontology-based access to databases, Bijan)

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We'll pay special attention to practicality

- Essential optimization techniques
- Performance evaluation techniques

Welcome!

Let us know if you

- have questions; do ask them at any time
- have difficulties understanding
- find this course too slow/boring
- find this course too fast/difficult

In this course, we will:

- ask you to think a lot
- ask you to work through numerous examples

Origins of Description Logics

Basics: Syntax, Semantics, Reasoning Problems

Anatomy of a Reasoner

DLs: Where They Come From

DLs are logic-based knowledge representation (KR) formalisms

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- Common perception: logic is difficult for human conception
  - e.g., how long does it take you to read $\forall x \exists y \forall z((r(x, y) \wedge s(y, z)) \Rightarrow (\neg s(a, y) \vee r(x, z)))$
  - or check that it is equivalent to $\forall x \exists y \forall z(r(x, z) \vee \neg r(x, y) \vee \neg s(y, z) \vee \neg s(a, y))$

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- Also not-so-easy for machines (only semi-decidable)

Are there better suited alternatives?

- Can we help users learn/speak/interact with logic?
- Or perhaps not use logic at all?

Early KR Formalisms

Were mostly graphical because pictures are:

- easier to grasp:

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Most graphical KR formalisms represent knowledge as graphs with

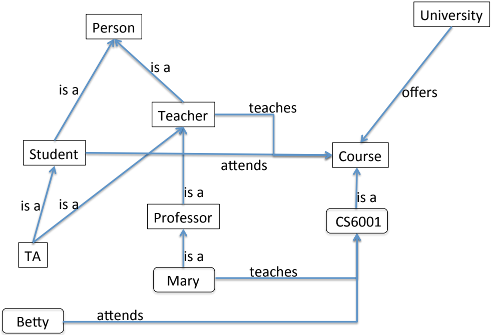
- labeled nodes

  mostly representing concepts, classes, individuals etc.

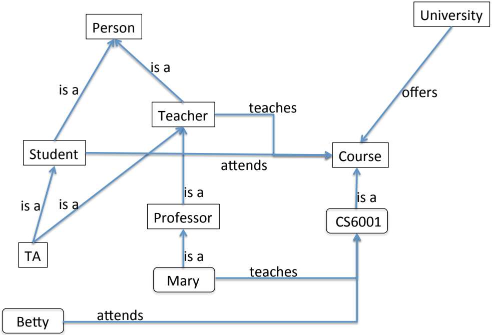
- labeled edges

  mostly representing properties, relationships etc.

Semantic Networks



Semantic Networks



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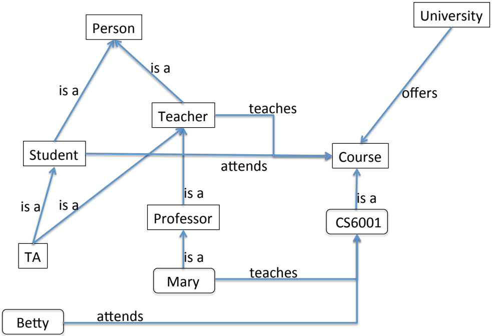
Semantic Networks



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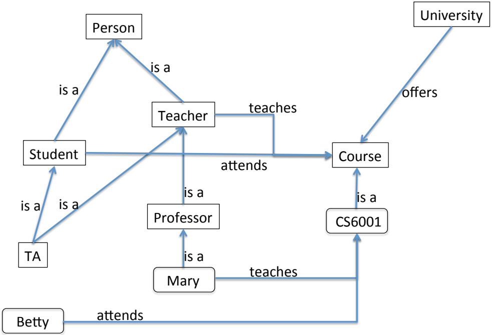

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Problem: it is unclear. Semantics is missing or implicit.

Remedy: base your picture on logic or use logic directly

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17 distinct interpretations of IS-A!

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DLs came out of the effort to design such formalism

Terminological Knowledge and Facts

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Goals:

- Formalise basic terminology of an application domain;
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  - Can there be Mammals?
  - Is every Mammal an Animal?
  - Are Frogs Reptiles?

Terminological Knowledge and Facts

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- Enable reasoning about concepts:
  - Can there be Mammals?
  - Is every Mammal an Animal?
  - Are Frogs Reptiles?
- Represent facts about individuals
- Enable reasoning about individuals and concepts:
  - Are my facts consistent with my terminology?
  - Is *Kermit* a Frog?

Reasoning

By "reasoning" we understand deductive inference

- From general knowledge to specific conclusions
- All results are necessarily true
  If α follows from Σ, then $\neg\alpha$ is inconsistent with Σ

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α : *John is a student*

We never induce relationships from examples

John attends ESSLLI, Mary attends ESSLLI, Jim attends ESSLLI, Maria attends ESSLLI,...

α : *ESSLLI attendants are students*

Applications of Description Logics

Medical Informatics

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Medical Informatics

- SNOMED CT

  Systematized Nomenclature of Medicine – Clinical Terms
  clinical terminology (used for EHR, clinical DSS, etc.)
  $>$300,000 classes (diseases, conditions, etc.)

- NCI Thesaurus (NCI = National Cancer Institute of the USA)

  vocabulary for clinical care, translational and basic research, public information, administrative activities
  Information on $>$10,000 cancers

- ICD 11 (International Classification of Diseases)

  used worldwide for health statistics
  when someone dies, there's always a code from ICD 11

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  controlled vocabulary of terms for gene product characteristics and gene product annotation data

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Semantic Web

- Supply meaning to (linked open) data
- Use TBox when querying data (Lecture 5 will cover this)
  - ontology-based data access
  - data intergration

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- Missing conclusions:
  "Flu is not inferred to be a sub-concept of ViralDisease"

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Reasoning enables definition-oriented development

- User does not assert relations, only writes definitions
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Reasoning can be used to explain the errors

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Syntax and Semantics

What you should remember from your logic class!

Any logic has two key components:

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Why?

- Syntax: machines can parse/reject formulas
  specified using a formal grammar (EBNF, etc)
- Semantics: machines can understand them
  specified using the language of the Set Theory
- Programming language analogy:
  Syntax and semantics specified in the Standard (e.g., C++)

Concept Language

Core part of a DL: its concept language, e.g.:

$$\mathsf{Animal} \sqcap \exists \mathsf{hasPart}.\mathsf{Feather}$$

describes all animals that are related via hasPart to a feather

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Syntactic components of a concept language:

- Concept names: stand for sets of elements, e.g., Animal
- Role names: stand for binary relations between elements, e.g., hasPart
- Constructors: used to build concept expressions: $\sqcap, \sqcup, \exists, \forall$

Syntax of $\mathcal{ALC}$

$\mathcal{ALC}$ is one of the basic/earliest description logics

- properly contains propositional logic
- enough expressivity for conceptual graphs
- notational variant of well-studied modal logic K_N

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The set of concepts in $\mathcal{ALC}$ is defined recursively as follows:

- Every concept name is a concept
- $\top, \bot$ are concepts (pronounced "top" and "bottom")
- $C \sqcap D$, $C \sqcup D$, and $\neg C$ are concepts if C and D are
- Role restrictions are concepts if C is a concept and R is a role

  $\exists \mathsf{R}.C$ existential restriction

  $\forall \mathsf{R}.C$ universal restriction

Semantics of $\mathcal{ALC}$

Semantics given via interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$, where:

- $\Delta^{\mathcal{I}}$ is a non-empty set (the domain),
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Constructor	Syntax	Example	Semantics
concept name	A	Human	$\mathsf{A}^\mathcal{I} \subseteq \Delta^\mathcal{I}$
role name	R	likes	$\mathsf{R}^\mathcal{I} \subseteq \Delta^\mathcal{I} \times \Delta^\mathcal{I}$
Top concept	$\top$		$\top^\mathcal{I} \equiv \Delta^\mathcal{I}$
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Top concept	$\top$		$\top^{\mathcal{I}} \equiv \Delta^{\mathcal{I}}$
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conjunction	$C \sqcap D$	Human $\sqcap$ Male	$C^{\mathcal{I}} \cap D^{\mathcal{I}}$
disjunction	$C \sqcup D$	Nice $\sqcup$ Rich	$C^{\mathcal{I}} \cup D^{\mathcal{I}}$
negation	$\neg C$	$\neg$Meat	$\Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$
restrictions:			
existential	$\exists \mathsf{R}.C$	$\exists$hasChild.Human	$\{x \mid \exists y.(x,y) \in \mathsf{R}^{\mathcal{I}} \wedge y \in C^{\mathcal{I}}\}$
universal	$\forall \mathsf{R}.C$	$\forall$hasChild.Blond	$\{x \mid \forall y.(x,y) \in \mathsf{R}^{\mathcal{I}} \Rightarrow y \in C^{\mathcal{I}}\}$

The Griffin Family

Look at interpretations on some real example

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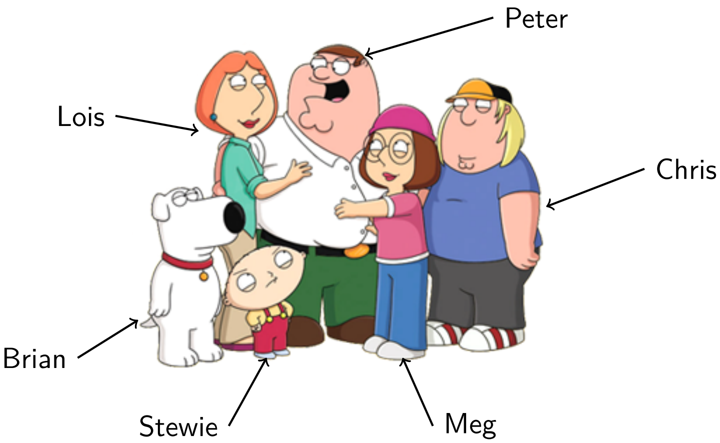

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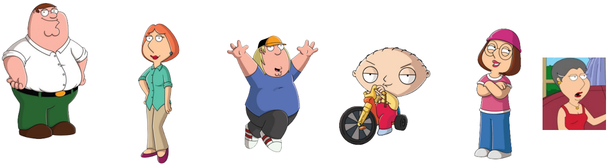

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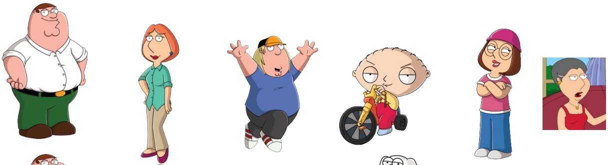
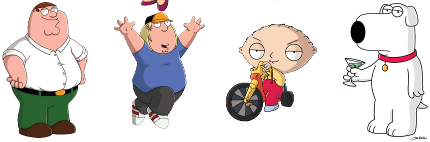


$\mathsf{Male}^{\mathcal{I}}$:



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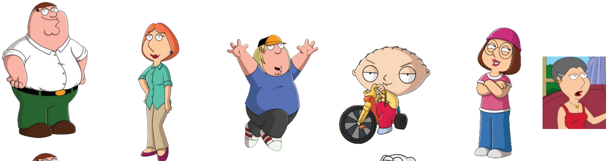

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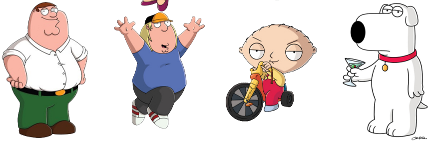

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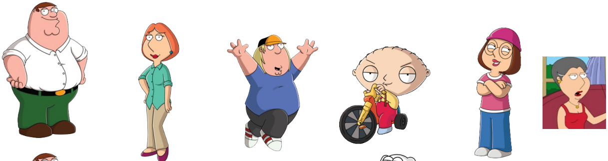

Male$^{\mathcal{I}}$:

Parent$^{\mathcal{I}}$:

hasChild$^{\mathcal{I}}$: {(Barbara,Lois), (Peter, Chris), (Peter, Meg),
(Peter, Stewie), (Lois, Chris), (Lois, Meg), (Lois, Stewie)}

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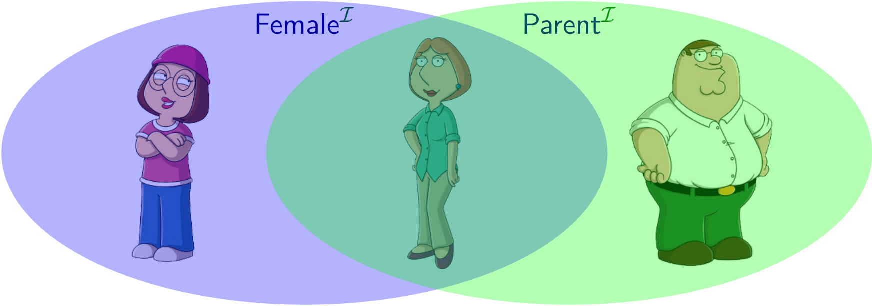
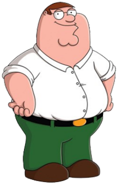


Lois is an instance of $(\mathsf{Female} \sqcap \mathsf{Parent})^{\mathcal{I}}$

Existential Restrictions

$\exists$hasChild.Male means the set of those who are in hasChild$^{\mathcal{I}}$ relation with an instance of Male$^{\mathcal{I}}$



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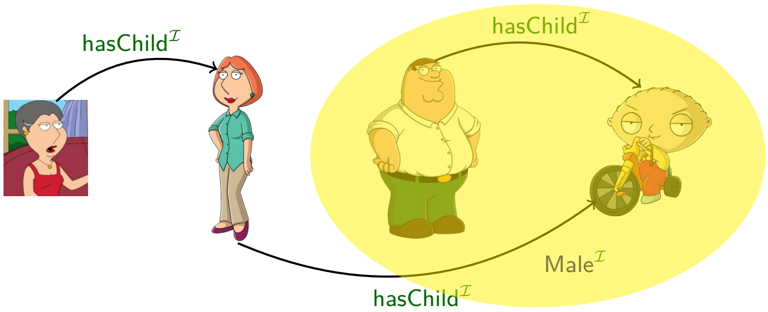

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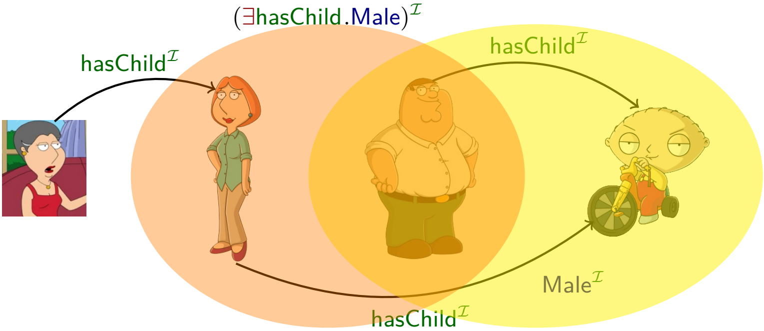

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instances of $(\forall\text{hasChild.Female})^{\mathcal{I}}$




Question: what are the instances of $(\forall\text{hasChild.Female})^{\mathcal{I}}$

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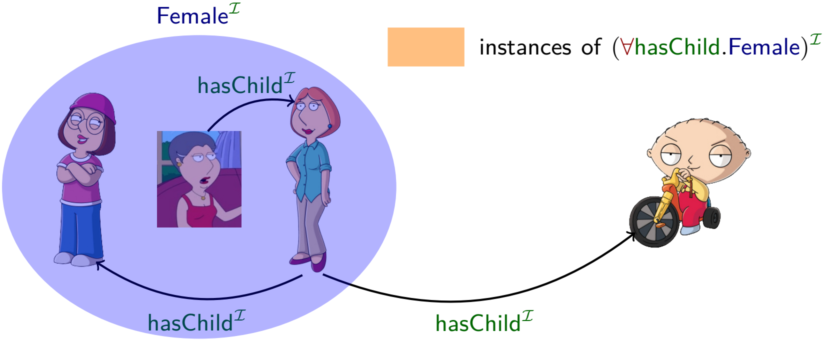

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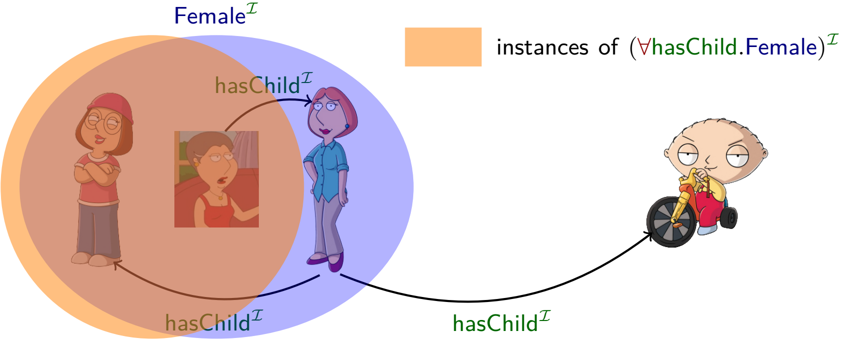

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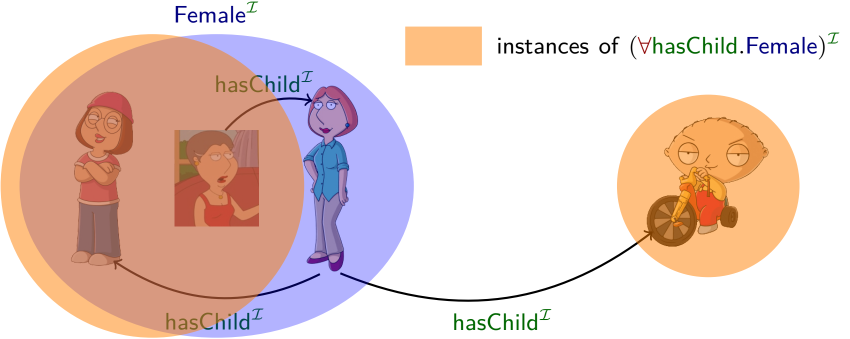

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$\mathcal{ALC}$ Concept Interpretations

$C^{\mathcal{I}}$ can be visualized as a labeled graph $\mathcal{G}_{C^{\mathcal{I}}} = \langle V, E \rangle$, where

- V is a non-empty set of domain elements where $v_0 \in C^{\mathcal{I}}$
- $D \in L(v)$ if $v \in D^{\mathcal{I}}$
- (x, y) is a R-labeled edge if $(x, y) \in \mathsf{R}^{\mathcal{I}}$

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Example: Male $\sqcap$ $\exists$hasChild.(Nerd $\sqcap$ $\exists$hasSibling.Female)

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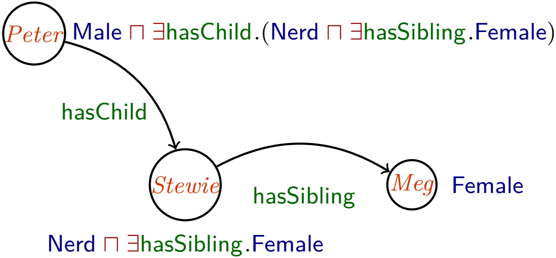

Basic Reasoning Problems

Definition: let C, D be $\mathcal{ALC}$ concepts. We say that

- C is satisfiable if there exists some $\mathcal{I}$ such that $C^{\mathcal{I}} \neq \emptyset$.
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Question: Which of the following concepts is satisfiable? Which is subsumed by which?

(1) $\exists \mathsf{R}.(\mathsf{A} \sqcap \mathsf{B})$ (2) $\exists \mathsf{R}.(\mathsf{A} \sqcup \mathsf{B})$

(3) $\forall \mathsf{R}.(\mathsf{A} \sqcap \mathsf{B})$ (4) $\exists \mathsf{R}.(\mathsf{A} \sqcap \neg \mathsf{A})$

(5) $\exists \mathsf{R}.\mathsf{A} \sqcap \forall \mathsf{R}.\mathsf{B}$ (6) $\exists \mathsf{R}.\mathsf{A}$

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The TBox (Terminology)

Definition

- A general concept inclusion (GCI) is a statement of the form $C \sqsubseteq D$, where C, D are (possibly complex) concepts
- A (general) TBox is a finite set of GCIs: $\mathcal{T} = \{C_i \sqsubseteq D_i \mid 1 \leqslant i \leqslant n\}$

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Example:

$$\begin{aligned} \{ \ \mathsf{Father} &\equiv \mathsf{Man} \sqcap \exists \mathsf{hasChild}.\mathsf{Human}, \\ \mathsf{Human} &\equiv \mathsf{Mammal} \sqcap \forall \mathsf{hasParent}.\mathsf{Human}, \\ \exists \mathsf{favourite}.\mathsf{Brewery} &\sqsubseteq \exists \mathsf{drinks}.\mathsf{Beer} \ \} \end{aligned}$$

$\mathcal{ALC}$ TBox Interpretations

$\mathcal{T}^{\mathcal{I}}$ can be visualized just as $C^{\mathcal{I}}$, as $\mathcal{G}_{\mathcal{T}^{\mathcal{I}}} = \langle V, E \rangle$, where

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Exercise:

1. Draw one model of this TBox
2. Draw one non-model of this TBox

Reasoning Problems w.r.t. TBox

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- C is satisfiable w.r.t. $\mathcal{T}$
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A pair of a TBox $\mathcal{T}$ and an ABox $\mathcal{A}$ is called ontology

$\mathcal{O} = (\mathcal{T}, \mathcal{A})$

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Semantics: an interpretation $\mathcal{I}$

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Answer: no non-trivial role assertion entailments in $\mathcal{ALC}$! $\ddot\smile$

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Example:

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Question: can you see any entailments?

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This has big practical impact for reasoners

- Schema is often small
- Data is often large

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OWL is W3C-standardized Web Ontology Language

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OWL includes more stuff:

- Datatypes: strings, integers, dates, etc.
  a.k.a. concrete domains in some DLs
- Non-logical stuff: annotations

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Each profile trades some expressivity for computational guarantees:

- OWL EL: PTime classification
- OWL QL: scalable query answering
- OWL RL: completeness w.r.t. rule systems

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Question: do we need 4 different algorithms for these?

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Theorem: Let $\mathcal{O}$ be an ontology and a a fresh individual. Then:

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This does not mean that the naive reduction is practical!

Origins of Description Logics

Basics: Syntax, Semantics, Reasoning Problems

Anatomy of a Reasoner

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  individual to concepts mapping for realization

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Reasoner is more than just implementations of algorithms

- It has to interact with the world (suitable APIs, load data,...)
- It has to convert the input into a suitable form
- It has to invoke the right algorithm at the right time
- It has to manage optimizations

Reasoner: the Main Layers

Interface Layer

Parsing | API bindings | Protégé plugin

Internal Data Model

(Terms, $\sqcap, \sqcup, \exists, \forall$)

Intermediate Layer

Pre-processing, indexing, caching, taxonomy
Incomplete reasoning

Reasoning Layer

Classification
Realization

Core reasoning procedure

Query answering

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APIs usually provide parsers, serializers, and the data model. . .

. . . but reasoners often support their own for efficiency

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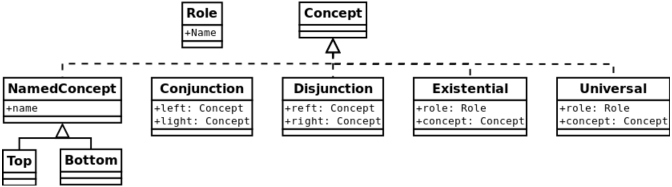
Example: Object-Oriented API for $\mathcal{ALC}$

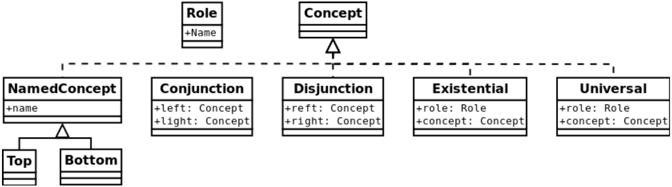


Internal Data Model

Internal Data Model is representation of loaded knowledge

- Optimized for reasoning tasks
- Covers supported features of the language

Example: Object-Oriented API for $\mathcal{ALC}$



IDM does not have to mirror the language model

- Can opt for the minimal sufficient set of constructors
- ELK does not store axioms, only rules

Intermediate Layer: Pre-processing and Indexing

Pre-processing: massaging data before sending to reasoning layer

Purely syntactic axioms/concept rewriting:

- Normalization: $\neg(C \sqcap D) \rightsquigarrow \neg C \sqcup \neg D$
- Simplification: $\exists \mathsf{R}.\mathsf{A} \sqcap \exists \mathsf{R}.(\mathsf{A} \sqcap \mathsf{B}) \rightsquigarrow \exists \mathsf{R}.(\mathsf{A} \sqcap \mathsf{B})$
- Absorption: $A \sqcap C \sqsubseteq D \rightsquigarrow \mathsf{A} \sqsubseteq \neg C \sqcup D$

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- $\mathsf{A} \mapsto$ set of told subsumers
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Practical hint: both can be done in parallel with parsing/loading

Intermediate Layer: Reductions

Reasoner reduces the input problem to the one for which the core procedure is optimized

- Tableau: $\mathcal{O} \stackrel{?}{\models} \alpha \rightsquigarrow$ is $\mathcal{O} \cup \{\neg\alpha\}$ consistent?
- Consequence-based algorithms: $\mathcal{O} \stackrel{?}{\models} \neg C \sqcup D \rightsquigarrow \mathcal{O} \stackrel{?}{\models} C \sqsubseteq D$ (CB algorithms compute subsumers in a goal-directed way)

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Reasoner can also reduce the problem to one previously solved

- $\mathcal{O} \overset{?}{\models} \neg C \sqcup D \sqsubseteq \bot \rightsquigarrow \mathcal{O} \overset{?}{\models} C \sqsubseteq D$
  if subsumers for C have been computed

Intermediate Layer: Caching

- Single shot reasoning: just answer one query, e.g., $\mathcal{O} \stackrel{?}{\models} C \sqsubseteq D$, and discard everything
- Multiple reasoning: save and re-use intermediate results

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Example: >24M inferences when classifying SNOMED CT (“only” 300K concepts)

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Other stuff, e.g., complex subsumers, can be re-used later.
This layer decides:

- what to save
- what to discard (w.r.t. which policy)
- how to look things up

Caching

Example: the reasoner inferred $A \sqsubseteq C \sqcap D$
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Next task: compute subsumers of A. It can immediately:

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Caches may be cleared when the ontology is changed

. . . or not! Incremental reasoning algorithms exist

Deletions are particularly tricky. Why?

Intermediate Layer: Incomplete Reasoning

Main reasoning algorithms are nearly always expensive

Often there are cheaper ways to get the answer

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- $\mathcal{EL}$: if $\perp$ does not occur in $\mathcal{O}$, it cannot be inconsistent
- Detecting obvious conflicts, e.g., $\exists \mathsf{R}.\neg C$ and $\forall \mathsf{R}.C$

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Approximations can help too

If $\mathcal{O}' \subseteq \mathcal{O}$ and $\mathcal{O}' \models \alpha$, then $\mathcal{O} \models \alpha$ (monotonicity)

$\mathcal{O}'$ can fit into a simpler language $\Rightarrow$ easier to reason with

Reasoning Layer: Core Reasoning Procedure

Implementation of the main reasoning algorithm

- Expressive DLs: usually tableau algorithm for consistency (L2)
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- New language features
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Should be compact and extensible to:

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Should be reusable for higher level tasks:

- Explanations and debugging
  (find all reasons why $\mathcal{O} \models C \sqsubseteq \bot$ happens)
- Query answering
- Incremental reasoning

Reasoning Layer: Classification and Realization

Classification: compute $\mathcal{O} \models \mathsf{A} \sqsubseteq \mathsf{B}$ for all concept names in $\mathcal{O}$

Realization: compute $\mathcal{O} \models a\colon \mathsf{A}$ for all individuals in $\mathcal{O}$

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Once the ontology is classified, many tasks are easier

Taxonomy Construction

Users want to see only direct subsumptions

- A is directly subsumed by B if $\mathcal{O} \models A \sqsubseteq B$ and
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Transitive reduction algorithms can

- maintain the reduction as the ontologies is classified
- compute the reduction post factum

Example: Pellet

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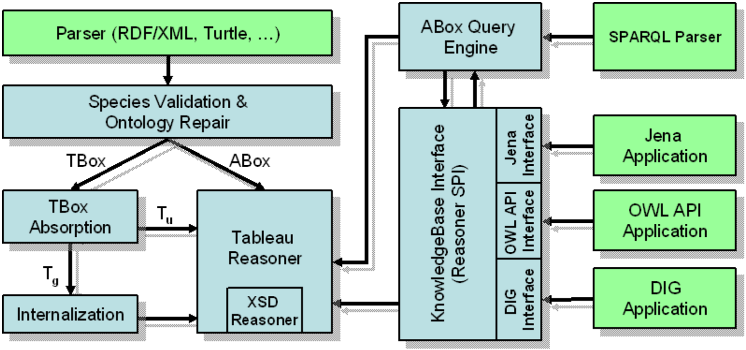

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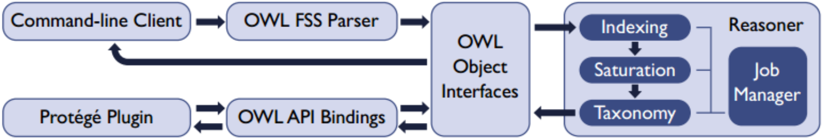

Distinctive features:

- Pipelining: loading | indexing, classificiation | taxonomy
- Concurrency: all concepts are classified in parallel

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Brief history of Description Logics

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- TBox and ABox a.k.a. schema and data
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  - Concept satisfiability, entailment, ontology consistency
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Tomorrow: how reasoning is actually done (tableau algorithms)