The Chasm at Depth Four, and Tensor Rank: Old results, new insights

Proving good lower bounds on the size of the arithmetic formulas computing an explicit polynomial is one of the central themes in the study of algebraic complexity. But the best known lower bound on the size of an arithmetic formula computing an explicit polynomial is quadratic in the number of variables, due to Kalorkoti [SIAM J. Comp. 1985]. There hasn't been any progress on that front in the last 32 years.

But in a surprising result, Raz [J. ACM 2013] showed that for any \( n \) and \( d \) where \( d \) is sub-logarithmic in \( n \), constructing explicit tensors of high enough rank would imply superpolynomial lower bounds for general arithmetic formulas. We first give a new and arguably simpler proof of this connection. We do this by showing a structured depth reduction to depth four for arithmetic formulas. This is a simpler proof of the depth reduction results of Agrawal and Vinay [FOCS 2008], Koiran [TCS 2012], and Tavenas [MFCS 2013]. We also extend the result of Raz to homogeneous formulas. In this setting, we show that such a connection holds for any \( d \) that is sub-polynomial in \( n \). As a corollary of our extension, we show that for any \( n \) and \( d \) where \( d \) is \( O(\log n) \) (as opposed to the sub-logarithmic in \( n \) as in Raz's result), constructing explicit tensors of high enough rank would imply superpolynomial lower bounds for general arithmetic formulas.

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