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Energy Derivatives

Exercise Sheet 1

Discussion: Tuesday 05/19/2009

1. **(Background knowledge - Energy markets)**
   - Average annual electricity demand in Germany, US, China (total, households)
   - Sources of electricity production in Germany (gas, coal, oil, nuclear, water, solar)
   - Please list the major activities contributing to the residential use of electricity
   - Largest utility companies in Germany
   - Exchanges for energy products in Europe
   - Other things that you want to know about energy markets

2. **Futures price**
   The current price of silver is 9 Euro per ounce. The storage costs are 0.24 Euro per ounce per year (payable quarterly in advance). Assume that the interest rates of all maturities are equal to 10% per annum. Compute the futures price of silver for delivery in nine months using continuous compounding.

3. **(Storage costs)**
   Suppose that $f(0, t_1)$ and $f(0, t_2)$ are futures prices for contracts on the same commodity with maturity dates t_1 and t_2, respectively where $t_2 > t_1 \geq 0$. Prove that

$$f(0, t_2) \leq [f(0, t_1) + U]\, e^{-r(t_2 - t_1)}$$

   where r is the risk-free interest rate (assumed to be constant) and U is the cost of storing the commodity between t_1 and t_2 discounted to time t_1 at the risk-free rate. For the purpose of this problem, assume that a futures contract is the same as a forward contract.

4. **(Put-Call-Parity)**
   Assume that the discounted stock price $e^{-\rho t} S_t$ is a martingale under $\mathbb{Q}$
   (a) Show that the put-call parity for European options is

$$C_t^{(S)} - P_t^{(S)} = S_t e^{-\rho(t_0 - t)} - c e^{-\rho(t_0 - t)}$$

   where $C_t = \mathbb{E}_{\mathbb{Q}}\left(e^{-\rho(t_0 - t)}(S_{t_0} - c)^+ | \mathcal{F}_t\right)$ is the price of a European call option with strike c and maturity t_0. P_t is the price of a European put option with the same strike and maturity.
   (b) Assume that the futures price is given by $G(t, T) = \mathbb{E}_{\mathbb{Q}}\left(S_T | \mathcal{F}_t\right)$ where $t \leq T$. Show that the put-call parity for European options on futures is

$$C_t^{(G)} - P_t^{(G)} = G(t, t_0) e^{-\rho(t_0 - t)} - c e^{-\rho(t_0 - t)}$$

5. **(Calibration)**

(a) Let $dS_t = S_t(\mu dt + \sigma dW_t)$. Using Ito's lemma derive the dynamics of $\ln(S_t)$

(b) Let $dX_t = -\gamma(X_t - \alpha)dt + \sigma dW_t$ be an Ornstein-Uhlenbeck process.

(i) Derive

$$\mathbb{E}(X_t|\mathcal{F}_s) = X_s e^{-\gamma(t-s)} + \alpha\left(1 - e^{-\gamma(t-s)}\right)$$
$$Var(X_t|\mathcal{F}_s) = \frac{\sigma^2}{2\gamma}\left(1 - e^{-2\gamma(t-s)}\right)$$

(ii) Show how the result from (i) can be used for calibration of data to an Ornstein-Uhlenbeck process *Hint: Regression.*

6. **(Futures option)** Consider a European put futures option on crude oil. The time to maturity is four months, the current futures price is 40 Euro, the exercise price is 40 Euro, the risk-free interest rate is 5 % per annum, and the volatility of the futures price is 25% per annum. Determine the arbitrage-free put price.