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Summer Term 2015

INSTITUTE OF MATHEMATICAL FINANCE

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Financial Mathematics II

Exercise Sheet 10

Discussion: Thursday 09/07/2015, 16:00-17:30, He18, E60,
and Friday 10/07/2015, 08:15-09:45, He18, 120.
Handing in: Thursday 09/07/2015, beginning of the lecture.

Exercise 10.1

Let $(\Omega, \mathcal{F}, \mathbb{F} = (\mathcal{F}_t)_{t\in[0,T]}, P)$ be a stochastic basis satisfying Convention 5.9 and $(B_t)_{t\in[0,T]}$ be a standard Brownian motion with respect to $\mathbb{F}$. Find with use of Itô's formula stochastic differential equations (SDE) for the following processes

(1) $X_t = B_t^3\,, \quad t \in [0,T]\,,$

(2) $Y_t = tB_t\,, \quad t \in [0,T]\,.$

Find further a representation of $[X,Y]$ as SDE.

Exercise 10.2

Let $(\Omega, \mathcal{F}, \mathbb{F} = (\mathcal{F}_t)_{t\in[0,T]}, P)$ be a stochastic basis satisfying Convention 5.9. Let $H = (H_t)_{t\in[0,T]}$ be a càglàd adapted process which is bounded in $L^2(\Omega)$, i.e.

$$K := \left\| \sup_{t\in[0,T]} |H_t| \right\|_2^2 < \infty\,.$$

Let $X = (X_t)_{t\in[0,T]}$ be a càdlàg square integrable martingale. Show that then

$$Y_t = \int_0^t H_s \,\mathrm{d}X_s\,, \quad t \in [0,T]\,,$$

is a martingale.

Exercise 10.3

Let $(\Omega, \mathcal{F}, \mathbb{F} = (\mathcal{F}_t)_{t\in[0,T]}, P)$ be a stochastic basis satisfying Convention 5.9 and $(B_t)_{t\in[0,T]}$ be a standard Brownian motion with respect to $\mathbb{F}$. Define

$$X_t = \int_0^t s^2 \,\mathrm{d}B_s \quad \text{and} \quad Y_t = \int_0^t (1-s)\,\mathrm{d}B_s\,.$$

(a) Define $h(t) = \frac{1}{5}t^5 + \frac{t^3}{3} - t^2 + t$ and show that $\mathbf{E}(X_t^2 + Y_t^2) = h(t)$.

(b) Show that $M = (M_t)_{t\in[0,T]}$ defined by $M_t = X_t^2 + Y_t^2 - h(t)$ is a martingale.

(c) Find the set of all $t \in [0,T]$ such that X and Y are negative correlated, i.e. $\mathbf{Cov}(X_t, Y_t) \le 0$