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Exercise to Energy Derivatives

Sheet 1

Due Thursday, November 15, 2007

Exercise 1: The current price of silver is 9 € per ounce. The storage costs are 0.24 € per ounce per year payable quarterly in advance. Assuming that interest rates of all maturities equal 10 % per annum with continuous compounding, calculate the futures price of silver for delivery in nine month.

Exercise 2: Suppose that $f(0, t_1)$ and $f(0, t_2)$ are futures prices for contracts on the same commodity with maturity dates of t_1 and t_2 and $t_2 > t_1 \geq t$. Prove that

$$f(0, t_2) \leq (f(0, t_1) + U)e^{r(t_2 - t_1)},$$

where r is the risk-free interest rate (assumed to be constant) and U is the cost of storing the commodity between times t_1 and t_2 discounted to time t_1 at the risk-free rate. For the purpose of this problem, assume that a futures contract is the same as a forward contract.

Exercise 3: Show that the put-call parity relationship for European futures options is

$$C + Ke^{-rT} = P + f(0, T)e^{-rT},$$

where C is the price of a European call option, P is the price of a European put option, and both options have strike price K and maturity T.

Exercise 4: Show that if C is the price of an American call with strike price K and maturity T on a stock providing a dividend yield of q, and P is the price of an American put on the same stock with the same stock with the same strike price and exercise date,

$$S_0 e^{-qT} - K \leq C - P \leq S_0 - Ke^{-rT},$$

where S_0 is the stock price, r is the risk-free interest rate, and $r > 0$.

Hint: To obtain the first half of the inequality, consider possible values of

- Portfolio A: a European Call option plus an amount K invested at the risk-free rate
- Portfolio B: an American put option plus e^{-qT} of stock, with dividends being reinvested in the stock.

To obtain the second half of the inequality, consider possible values of

- Portfolio C: an American call option plus an amount $Ke^{-r(T-t)}$ invested at the risk-free rate
- Portfolio D: a European put option plus one stock, with dividends being reinvested in the stock.

Exercise 5: Show that if C is the price of an American call option on a futures contract when the strike price is K and the maturity is T, and P is the price of an American put on the same futures contract with the same strike and exercise date,

$$f(0,T)e^{-rT} - K \leq C - P \leq f(0,T) - Ke^{-rT},$$

where $f(0,T)$ is the futures price and r is the risk-free interest rate. Assume that $r > 0$ and that there is no difference between forward and futures contracts.

Hint: Use an analogous approach to that indicated for Exercise 4.

Exercise 6: Show that the put-call parity relationship for European index options is

$$C + Ke^{-rT} = P + S_0 e^{-qT},$$

where q is the dividend yield on the index, C is the price of a European call option, P is the price of a European put option, and both options have strike price K and maturity T.

Exercise 7: Consider a European put futures option on crude oil. The time to maturity is four months, the current futures price is 20 €, the exercise price is 20 €, the risk-free interest rate is 9 % per annum, and the volatility of the futures price is 25 % per annum. Determine the arbitrage-free put price.