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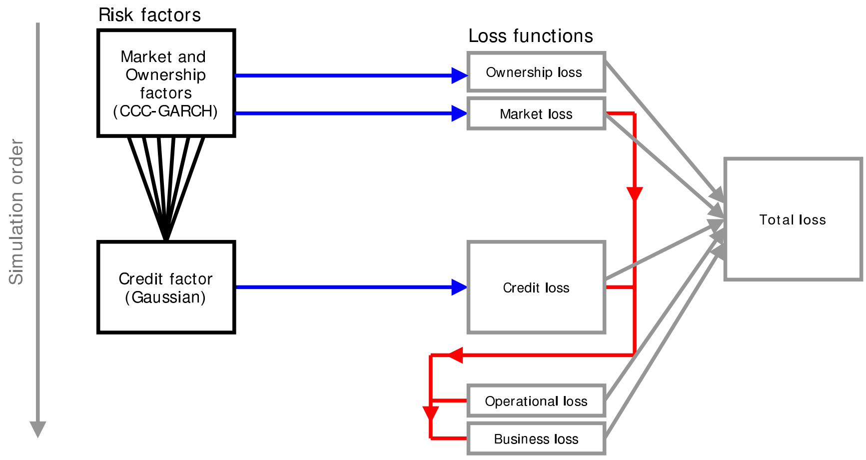
Note

Risk Capital Aggregation



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Abstract

This article presents a new approach to determine the total risk of a financial institution. The proposed model includes components for credit, market, operational and business risk. Moreover it includes a component for the ownership risk that stems from holding a life insurance company. The approach may be characterised as a base-level aggregation method. However, due to lack of appropriate data, some of the aggregation steps are done on the top level instead. The economic risk factors used in the base-level aggregation are described by a multivariate GARCH model with Student's t-distributed innovations. The loss distributions for the different risk types are determined by non-linear functions of the fluctuations in the risk factors. Hence, these marginal loss distributions are indirectly correlated through the relationship between the risk factors. The model was originally developed for DnB NOR, the largest financial institution in Norway, and one of the largest ones in the Nordic region. Being adapted to the requirements in the Basel II regulations, it will play an important part in measuring and assessing the risk level of the institution.

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1 Introduction

Risk aggregation refers to the task of incorporating multiple types or sources of risk into a single metric (Basel Committee on Banking Supervision, 2003). Most financial institutions are exposed to credit, market and operational risk. Moreover, business risk, see e.g. Saita (2004), has grown as the structure of financial institutions continuous to change. For marginal evaluation of credit and market risk, most financial institutions are equipped with advanced risk assessment software. For operational risk, loss databases and measurement methodologies are currently under development. Business risk, however, has so far received less attention, probably due to the fact that there is no minimum capital linked to it. Finally, up to now, there exists no state-of-the-art approach for aggregating the marginal risk types to the total risk. Risk managers struggle with a number of important issues, including weakly founded correlation assumptions, inconsistent risk metrics and differing time horizons for the different risk types.

In this paper, we present a model that aggregates the different risk types of a financial institution to assess the total risk. The model was originally developed for the Norwegian financial group DnB NOR. Being adapted to the requirements in the Basel II regulations, we believe it to be applicable in a broader context. The new model, which has been implemented in the group's system for risk management, is the second generation of the total risk model used in DnB NOR. The first model was developed in 2000, and is described in Dimakos and Aas (2004).

Like other financial institutions, DnB NOR is exposed to credit, market, operational and business risk. In addition, DnB NOR faces the risk that stems from its ownership in the life insurance company Vital. This is due to external parameters for life insurance operations in Norway, i.e. regulations of risk and profit sharing between policyholders and owner. Even though this ownership risk is somewhat special to DnB NOR, we include it as an illustration of how a variety of risk types can be incorporated into a single metric. Market risk is the dominant risk factor of the ownership risk, and our model for this risk is an example of how to incorporate active portfolio management by including rebalancing strategies. Finally, it should be noted that liquidity risk is another important risk. We have not included this risk type in our total risk model because it is difficult to model it in a manner that is consistent with the other risk types. Instead, liquidity risk is managed and controlled through limits and stress testing.

A first challenge in risk aggregation is specifying a common time horizon for all the risk types. Market risk is typically measured on a daily basis. Credit, operational and business risk are typically calibrated to a one-year horizon. We use the convention for modelling risk and assessing capital in banks, which is to adopt to a one-year horizon, see Kuritzkes et al. (2002). A one-year time horizon is reasonable as it corresponds to the internal capital allocation and budgeting cycle, it is a period in which an institution can access the markets for additional capital, and it is also the horizon used in the New Basel Accord.

Another main challenge is how to obtain the simultaneous distribution of all the risk types. Typically, the risk manager has some knowledge of the marginal distribution of each risk type. However, since the underlying distribution of each risk type does not have the same distributional form, it is necessary to do a numerical integration or simulation to aggregate them. The approaches proposed in the literature for combining marginal risk distributions into a total risk distribution can be divided into two main categories; base-level and top-level aggregation methods. In the base-level aggregation approach, the idea is to identify the economic risk factors that have most influence on the different risk types and develop a simultaneous model for these risk factors. The simultaneous model includes a description of the dependency structure of the risk factors, either through a correlation matrix or a copula. The losses related to the different risk types are determined by non-linear functions of the fluctuations in the risk factors. The marginal loss distributions are indirectly correlated through the relationship between the risk factors. The principles of the base-level aggregation approach is shown in Figure 1.1. As far as we know, Alexander and Pezier (2003) are the only ones to use this type of approach for risk aggregation. However, they focus on credit and market risk only.

In the top-level aggregation approaches, one develops marginal models for the yearly loss distribution of each risk type independently. These marginal distributions are then merged to a joint distribution using a correlation structure or a copula function. This kind of approach is used by Kuritzkes et al. (2002), Ward and Lee (2002), Rosenberg and Schuermann (2004) and Dimakos and Aas (2004). The top-level aggregation approach is illustrated in Figure 1.2. In this paper, we use the base-level aggregation method for combining credit, market and ownership risk. For operational and business risk, there are no obvious economic risk factors. Hence, these risk types are linked to the other ones at the loss distribution, i.e. top, level.

A third challenge is to obtain reliable parameter estimates. While there for credit, market and ownership risk exists historical data for estimating the model parameters, this is rarely the case for operational and business risk. In our opinion, subjective expert opinions can improve assessments that are based solely on internal historical experiences. Hence, expert evaluations are an important ingredient in our model.

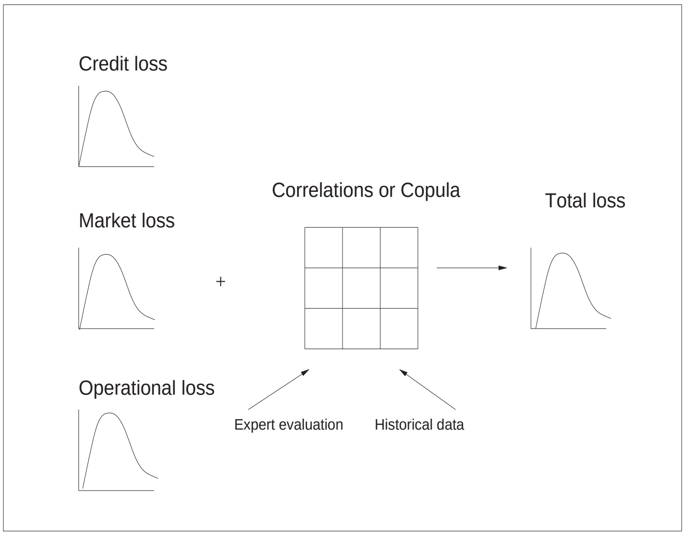

Figure 1.2. Top-level aggregation using three risk components; credit, market and operational risk. The simultaneous distribution of the risk types is defined by the marginal loss distributions and a correlation or copula structure.

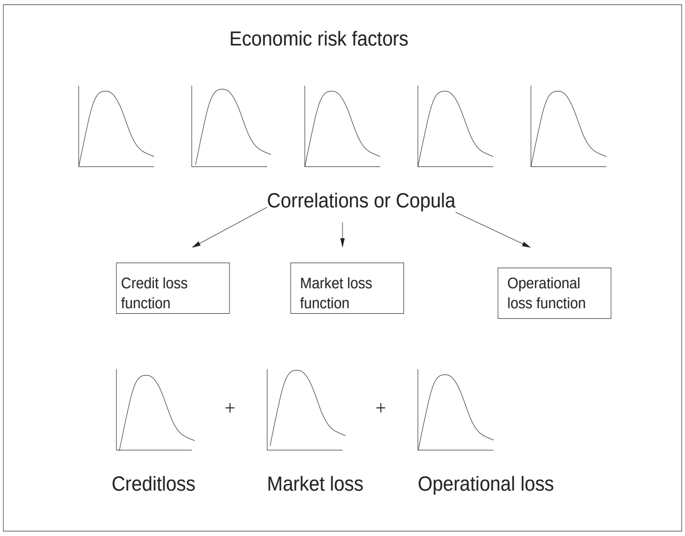

Figure 1.1. Base-level aggregation using 5 economic risk factors and three risk types; credit, market and operational risk. Marginal models for the economic risk factors are related by defining a common correlation matrix or a copula. The movements in the risk factors are transformed to losses by defining a non-linear loss function for each risk type. The resulting marginal loss distributions are then correlated indirectly through the correlation or copula of the economic risk factors.

The rest of this paper is organized as follows. In Section 2 we review existing literature on risk capital aggregation. Section 3 gives an overview of the new approach to risk aggregation proposed in this paper. Section 4 describes the models for the economic risk factors used in the base-level aggregation method for combining credit, market and ownership risk. In Section 5, the functions that transform the fluctuations in the risk factors into different risk type losses are given. Section 6 summarizes the dependency structures in the model. Section 7 gives the procedure for estimating the parameters of our system, and Section 8 contains some experimental results from our risk aggregation method. Finally, Section 9 contains some concluding remarks.

2 Previous work

Only a few approaches for aggregating risk types have been proposed in the literature. Most of them belong to the class of top-level aggregation methods. Ward and Lee (2002) and Dimakos and Aas (2004) approach the problem of risk aggregation by considering risk types pairwise. Ward and Lee (2002) use a normal copula to aggregate risk types. Some of the marginal risk distributions are computed analytically (e.g. credit risk that is assumed to follow a beta distribution) and some by simulation (e.g. mortality risk for life insurance). Dimakos and Aas (2004) decompose the joint risk distribution into a set of conditional probabilities, and impose conditional independence so that only pairwise dependence remains. The total risk is then the sum of the conditional marginals and the unconditional credit risk, which serves as their anchor. Kuritzkes et al. (2002) make a simplifying assumption of joint normality, allowing for a closed-form solution. Rosenberg and Schuermann (2004) estimate market (Student's t), credit (Weibull) and operational (empirical) risk distributions using a combination of data from regulatory reports, market data and vendor data, and combining these marginals in an internally consistent manner with a normal copula. They do not estimate the correlation matrix of the copula, but use an average of what has been reported in other studies.

Alexander and Pezier (2003) are the only ones to use a top-level aggregation approach for risk aggregation. They have proposed a multifactor approach for aggregating credit and market risk, in which the profit and loss of different business units are linked to changes in 6 risk factors through a linear regression model. The risk factors are modelled by normal mixture distributions, and a normal copula is used to link them together. The authors advocate the choice of tail correlations rather than usual correlations to model the dependence between risk factors.

Recently, Schlottmann et al. (2005) have proposed a completely different approach to risk aggregation by defining a multi-objective problem. Saita (2004) and Alexander (2005) discuss alternative risk aggregation techniques and some of the problems that arise. Saita (2004) states that parameter estimation appears to be a major concern when deciding which aggregation technique to adopt, while Alexander (2005) argues that there is considerable model risk arising from crude aggregation rules and inadequate data.

This paper builds on and extends the framework presented in Dimakos and Aas (2004). There are three main differences. First, two new risk types are included; ownership and business risk. Second, a GARCH-based risk factor approach is introduced to model and aggregating market and ownership risk. Finally, both the credit risk model and the correlation structure between credit risk and the other risk types are substantially improved. Of the other approaches reviewed in this section, our method bears most resemblance with the one of Alexander and Pezier (2003). There are however three main differences. We introduce three additional risk components; business, operational and ownership risk. We use significantly more risk factors, and we allow for nonlinear relationships between losses and risk factors.

3 Overview

Our model for total risk is defined by the models for the separate risk components and the relationship between these. Risk is in our setting defined as losses, and the total loss is given by the sum of the marginal losses. Economic capital should cover unexpected losses, while average losses, measured over a normal business cycle, represent expected costs which should be primarily covered through correct pricing. Since there exist no explicit, analytical formulae for the total loss distribution, we obtain it through Monte Carlo simulation of the risk components. By generating a sufficient number of scenarios of the different risk types, we obtain their distribution and also the distribution of the sum, i.e. the total risk.

Relating our approach to the two main categories of risk aggregation approaches, we use the base-level aggregation method for combining market, ownership and credit risk. For operational and business risk there are no obvious economic risk factors. Hence, these risk types are linked to the other ones at the loss distribution (i.e. top) level.

As stated in Section 1, we use a one-year time horizon, which is the convention for assessing total economic capital in banks. Hence, our final aim is to obtain a distribution for yearly total losses. However, to get there, we have found it appropriate to start with models on a finer resolution for two of the risk types; market and ownership risk (see Sections 5.2 and 5.3). Hence, we need two models to aggregate market, ownership and credit risk. The first model correlates the risk factors that influence market and ownership risk on a daily resolution, while the second model describes the correlation structure between these risk factors, aggregated to a yearly resolution, and a yearly risk factor for credit risk. We will return to these models in Section 4. In what follows, we give an overview of the full procedure for determining the total loss distribution in our risk aggregation approach. See also Figure 3.1.

1. Simulate correlated realisations of N market and M ownership risk factors on a daily resolution.

2. Simulate annual realisations of the credit risk factor, conditional on the realisations of the market and ownership risk factors aggregated to a yearly resolution.

3. Compute

   (i) yearly credit losses, based on the realisations of the credit risk factor and portfolio characteristics,

   (ii) yearly market losses, based on the realisations of the daily market risk factors, position limits and stop-loss strategies,

   and

   (iii) yearly ownership losses, based on the realisations of the daily ownership risk factors, buffer capital and rebalancing strategies.

4. Simulate yearly operational losses conditional on the other yearly loss distributions.

5. Simulate yearly business losses conditional on the other yearly loss distributions.

6. Compute the total yearly losses as the sum of the credit, marked, ownership, business, and operational losses.

Steps (a) and (b) are described in Sections 4.1 and 4.2, respectively. Sections 5.1-5.3 treat the approaches used in step (c), while Sections 5.4 and 5.5 describes the marginal models used in steps (d) and (e). Finally, Section 6 describes how the operational and business losses are linked to the other risk types.

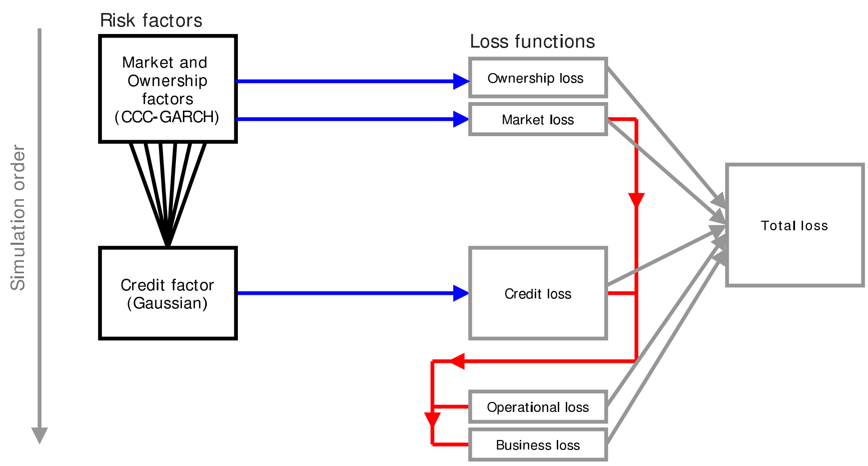

Figure 3.1. Overview of the full procedure for determining the total loss distribution.

4 Modelling economic risk factors

In this section we describe the models for the underlying economic risk factors that influence credit, market and ownership risk. As indicated in Section 3, we have developed two models. The first model describes the risk factors that influence market and ownership risk on a daily resolution. The second model describes the dependence structure between the credit risk factor and the market and ownership risk factors on a yearly resolution.

4.1 Model for the market and ownership risk factors

We have identified a set of risk factors that influence the market and ownership risk. These can be categorised into Norwegian and international stock indices, Norwegian and international bond indices, Norwegian and international interest rates, exchange rates and hedge fund and real estate indices. Returns from financial market variables measured over daily time intervals are characterized by two stylized facts, volatility clustering and non-normality. Volatility clustering means that small changes in the price tend to be followed by small changes, and large changes by large ones. The empirical distribution of returns is also more peaked and has fatter tails than the normal distribution.

We use the multivariate constant conditional correlation (CCC) GARCH(1,1) model (Bollerslev, 1990) with the Student's t-distribution as a conditional distribution for the market and ownership risk factors. The success of the GARCH class of models at capturing volatility clustering in financial markets is extensively documented, surveys are given in Ghysels et al. (1996) and Shepard (1996). We choose the Student's t-distribution as the conditional distribution because it is well recognized that GARCH models, coupled with the assumption of conditionally normal distributed innovations, are unable to fully account for the tails of the daily return distributions. There are alternative multivariate GARCH models, see e.g. Bauwens et al. (2003). However, the literature includes several studies showing that that the CCC-GARCH model often provides better fit than multivariate GARCH models with larger flexibility. Moreover, most multivariate GARCH models have a large number of parameters and are computationally too demanding for high dimensional problems.

4.2 Model for the credit risk factor conditional on the market- and ownership risk factors

Our model includes one risk factor for credit risk, which is assumed to represent the common source driving the entire credit portfolio. This risk factor has yearly notations, which distinguishes it from the risk factors for the market and ownership risk. Therefore, the simultaneous model for the market, ownership and credit risk factors must be on a yearly resolution. While daily data are characterized by volatility clustering and non-normality, distributions of yearly log-returns are much closer to the normal distribution. Hence, we use a multivariate normal distribution with constant volatility to model the credit, market and ownership risk factors simultaneously.

In practice, the credit risk factor is linked to the other risk factors through conditional simulation. First, the yearly log-increments of the market and ownership risk factors are formed as the sum of the simulated daily log-increments. Then, these are transformed to standard normal variables, and a new standard normal variable, i.e. a realisation of the credit risk factor, is generated conditionally on these. See Appendix A for how to determine the conditional distribution of the credit risk factor.

5 Marginal yearly loss distributions

5.1 Credit loss function

The credit risk faced by a financial group is defined as the risk of losses resulting from failure of its financial counterparties to meet their obligations. We divide the entire credit portfolio into K industrial sectors. The model for each sector is presented in Section 5.1.1. Section 5.1.2 describes how we combine the sector models to create a model for the full portfolio. Our model bears some resemblance with the Basel II IRB model. There are however three main differences. While the different sectors are perfectly correlated in the Basel model, we allow for diversification effects between sectors. Next, in our model, the correlation used in Basel II is multiplied by a sector dependent factor that is larger than one if the average default rate of a specific sector historically has been characterised by large fluctuations, and smaller than one otherwise. Finally, while the IRB model is strictly valid only for a portfolio having an infinitesimally small weight on its largest exposure, we take into account undiversified unsystematic risk due to large exposures.

5.1.1 Sector model

We start by assuming that the standardised yearly asset return R_{ik} of firm i in subportfolio k is driven by a single common factor Y_k and an unsystematic noise component ϵ_{ik}

$$R_{ik} = \sqrt{\rho_k}\, Y_k + \sqrt{1 - \rho_k}\, \epsilon_{ik}, \tag{5.1}$$

where Y_k and ϵ_{ik} are i.i.d. $N(0, 1)$. The component ϵ_{ik} represents risk specific to firm i in subportfolio k and Y_k is the risk common to all firms in the subportfolio. Using this approach, the asset returns of two firms are correlated with linear correlation coefficient $\mathrm{Corr}[R_{ik}, R_{jk}] = \mathrm{E}[R_{ik} \cdot R_{jk}] = \rho_k$. Moreover, the asset returns of all firms follow the multivariate normal distribution.

We define a binary random variable Z_{ik} for each firm in the subportfolio, taking value 1 (defaulted) with probability p_k and value 0 with probability $1 - p_k$. From the theory of Merton (1974), we have

$$Z_{ik} = 1 \text{ if } R_{ik} \leq \Phi^{-1}(p_k) \quad \text{and} \quad Z_{ik} = 0 \text{ if } R_{ik} > \Phi^{-1}(p_k),$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution. It can easily be shown that the probability of firm i defaulting a specific year, given that the systematic factor Y_k has the value y_k, is

$$P(R_{ik} \leq \Phi^{-1}(p_k)|Y_k = y_k) = \Phi\left(\frac{\Phi^{-1}(p_k) - \sqrt{\rho_k}\, y_k}{\sqrt{1-\rho_k}}\right). \tag{5.2}$$

The yearly credit loss L_k of the subportfolio is given by

$$L_k = E_k\, s_k \sum_{i=1}^{n_k} Z_{ik},$$

where E_k and s_k are the total exposure and the average loss given default rate for the subportfolio, respectively. If n_k is very large, and the subportfolio is infinitely granular (i.e. the weight of its largest exposure is infinitesimally small), then conditional on a realisation y_k of the common factor Y_k, the individual defaults are independent. In such a portfolio, it can be shown (Schönbucher, 2002), that the fraction of clients that defaults, i.e. $\sum_{i=1}^{n_k} Z_{ik}$, is equal to the individual conditional default probability in (5.2). That is, the loss in the subportfolio, conditional on $Y_k = y_k$ is

$$(L_k|Y_k = y_k) = E_k\, s_k\, \Phi\left(\frac{\Phi^{-1}(p_k) - \sqrt{\rho_k}\, y_k}{\sqrt{1-\rho_k}}\right). \tag{5.3}$$

As stated above, Equation (5.3) is only valid if the subportfolio is infinitely granular. This is a reasonable assumption, except possibly for the largest exposures in the group. In our model the undiversified risk resulting from large companies is taken into account by treating all commitments in a sector with exposures larger than a certain limit separately. Hence, Equation (5.3) is altered to

$$(L_k|Y_k = y_k) = E_k\, s_k\, \Phi\left(\frac{\Phi^{-1}(p_k) - \sqrt{\rho_k}\, y_k}{\sqrt{1-\rho_k}}\right) + \sum_{j=1}^{m_k} E_{kj}\, s_{kj}\, I_{kj}, \tag{5.4}$$

where the last sum is over the m_k largest commitments in sector k, and E_{kj} and s_{kj} are the exposure and loss given default rate for the jth of these commitments, respectively. Further, $I_{kj} = 1$ with probability $\Phi\left(\frac{\Phi^{-1}(p_k)-\sqrt{\rho_k}\, y_k}{\sqrt{1-\rho_k}}\right)$ and $I_{kj} = 0$ otherwise.

5.1.2 Portfolio model

Our approach presumes that the credit portfolio of a financial institution can be split into K subportfolios. Moreover, we assume that the single common factor Y_k of each subportfolio depends on the yearly log-increments X^{credit} of the credit risk factor described in Section 4 as follows

$$Y_k = \sqrt{\beta_k}\, X^{credit} + \sqrt{1-\beta_k}\, \eta_k. \tag{5.5}$$

Here X^{credit} and η_k are i.i.d. $N(0, 1)$. The component η_k represents risk specific to sector k, and β_k represents the correlation between Y_k and the credit risk factor. It should be noted that if β_k equals 1 for all sectors and the ρ_k-values are chosen in a specific way, our model is reduced to the Basel II model (Gordy, 2003).

Let x^{credit} be the log-increment of the credit risk factor a specific year. Combining Equations (5.4) and (5.5), the total yearly credit loss is given by

$$L^{credit} = \sum_{k=1}^{K} \left[E_k\, s_k\, \Phi \left(\frac{\Phi^{-1}(p_k) - \sqrt{\rho_k}\, \left(\beta_k\, x^{credit} + \sqrt{1-\beta_k^2}\, \eta_k \right)}{\sqrt{1-\rho_k}} \right) + \sum_{j=1}^{m_k} E_{kj}\, s_{kj}\, I_{kj} \right]. \tag{5.6}$$

5.2 Market loss function

Market risk is a consequence of the open positions of the financial institution in capital, interest rate and foreign exchange markets. It is typically measured by VaR on a short time horizon such as 10 days (Basel Committee on Banking Supervision, 1995), assuming that market liquidity will always be sufficient to allow positions to be closed out at minimal losses.

We need to scale the market risk to a one-year time horizon, to be consistent with the other risk types captured by the economic capital framework of our model. A simple way of scaling is to multiply the 10-day VaR by the square root of 25 (assuming 250 days in a trading year). This approach assumes, however, that the daily returns of all the risk factors are normal and serially independent, neither of which is true for our multivariate GARCH model with Student's t-distributed innovations. Moreover, the scaling approach assumes that the average size of positions taken remains relatively constant throughout the year, and ignores how management is likely to react in the event of a series of losses. Management intervention policies such as stop loss limits can substantially limit the cumulative effect of losses in a severe downside scenario (Hickman et al., 2002).

Hence, we propose another approach that includes the influence of active management. We incorporate the fact that an intermediate loss is likely to be realized to avoid the risk of large losses, by fixing a liquidation period for each position. Moreover, it is clear that the liquidity of investment instruments varies greatly, and this is taken care of by allowing each liquidation period to have different length. In what follows, we describe how the market losses are derived.

The total market risk of the financial institution is assumed to be composed of risk associated with K main asset classes, each corresponding to one of the risk factors described in Section 4. We include both market risks stemming from trading activities in the interest rate, currency and equity markets, and from banking activities, where the investments have a longer-term perspective. Specifically, it should be noted that interest rate risk arising from non-trading activities is included. Hence, our model extends Basel II even in this context.

In DnB NOR, like in many other financial institutions, the market risk is managed by applying risk limits to traders' or portfolio managers' activities. When modelling the market risk, we take as a starting point these limits rather than the actual exposures. This is due to the use of the model for capitalisation purposes. It is the potential market risk measured on a one-year time horizon that is of interest, not the risk associated with current short term positions.

We assume that each of the K main asset classes is an aggregate of a large number of instruments and positions depending on fluctuations in one specific risk factor, e.g. an interest rate. Connected to each class is an anticipated maximum exposure (defined as the expected utilisation of the limit), E^k, and a liquidation period Δ_k. Note that the limits themselves are aggregates of limits at lower levels. Some of the asset classes are composed of both long and short positions, meaning that the sign of the net position will vary over time. Hence, to be on the conservative side for these classes, we assume that the financial institution is always positioned the wrong way.

Let $x_t^{market,k}$ be the log-increment of market risk factor k on day t. Define r_t^k to be the change in asset k associated with a specific day t. We then have that

$$r_t^k = \begin{cases} 1 - \prod_{s=t+1}^{t+\Delta_k} \exp\left(x_s^{market,k}\right) & \text{if class } k \text{ consists of only long positions} \\ \\ \left|1 - \prod_{s=t+1}^{t+\Delta_k} \exp\left(x_s^{market,k}\right)\right| & \text{if class } k \text{ consists of both long and short positions} \end{cases}$$

The change in the market portfolio associated with a specific day t is defined as the sum of the changes in all asset classes, i.e.

$$L_t^{market} = \sum_{k=1}^{K} L_t^k = \sum_k E^k \, r_t^k,$$

and the market loss over one year is defined as the worst of these changes, i.e.

$$L^{market} = \max\left(\max_t L_t^{market}, 0\right).$$

5.3 Ownership loss function

In addition to the traditional credit, market and operational risk, our model includes risk associated with the ownership in a life insurance company. This risk, which we refer to as ownership risk, arises when the financial institution, as the owner of the insurance company, has to report a net loss for these operations and possibly provide the insurance company with new equity. We here assume that the ownership risk is associated only with negative movements in the financial assets of the life insurance company. In the model implemented for DnB NOR, we also have included the risk of loss due to unforeseen increases in life claims (e.g. caused by changes in death probabilities and disability rates). However, this insurance risk is very small compared to the market risk of the life insurance company. Hence, for the sake of simplicity we ignore it here.

When computing the ownership risk, we take into account that the financial institution does not experience a loss before the loss of the life insurance company is greater than its buffer capital. Hence, the simulation of ownership loss consists of two main steps. First the daily changes in the value of the financial assets of the insurance company is simulated for the whole year. Then, this time series is compared to the insurers buffer capital to determine the loss. The first issue is treated in Section 5.3.1, the other in Section 5.3.2.

5.3.1 Computing the value of the insurance company

The value of the life insurance company is composed of the values of K different financial assets, each being represented by one of the risk factors from Section 4. Let $x_t^{owner,k}$ be the log-increment of ownership risk factor k on day t, and v_{k0} the value of asset k at time 0. Then, the value of the insurance company on day t is given by

$$V_t = \sum_k v_{k0} \cdot \prod_{s=1}^{t} \exp\left(x_s^{owner,k}\right). \tag{5.7}$$

For some of the assets, we include currency risk. The corresponding exchange rates are then among the risk factors, modelled as described in Section 4.

The computation of the value of the insurance company given by Equation (5.7) ignores how management is likely to react in the event of a series of losses. Most life insurance companies have strategies for rebalancing the portfolio that can substantially limit the cumulative effect of losses in a severe downside scenario. Hence, we have incorporated such strategies in our model. The ratio of the buffer capital to the equity proportion in the portfolio is monitored every day. If the ratio is below a certain predefined limit, the equity proportion is reduced. In the opposite situation, the equity proportion is increased.

5.3.2 Determining the ownership loss

The next step is to describe the yearly ownership loss as a function of the value of the life insurance company. The financial institution loses money whenever this value falls below certain time-varying limits that are affected by the securities adjustment reserve, interim profits, additional allocations and the guaranteed rate of return. Due to the profit- and risk-sharing between policy holders and owners of Norwegian life insurance companies, imposed by Norwegian laws, the specification of these limits is complicated and outside the scope of this paper. Here it is sufficient to understand that the yearly ownership loss is taken as the maximum daily difference between the smallest of these limits, B_t, and the value of the insurance company, i.e.

$$L^{owner} = \max_t \left(B_t - V_t \right).$$

5.4 Operational loss function

The Basel Committee defines operational risk as the risk of loss resulting from inadequate or ineffective internal processes, people or systems, and from external events such as natural disasters or criminal acts. Some of these losses occur frequently, but are of moderate size, whereas others are rare, but very large. This suggests heavy-tailed models, and sophisticated approaches like extreme value theory (EVT) have been proposed in Cruz et al. (1998), Medova (2000), Medova and Kyriacou (2000) and de Fontnouvelle et al. (2005).

In real-world applications, however, many sophisticated approaches typically fail due to the poor quality and low quantity of data internally available in the banks. To address the problem of data sparseness, the current Basel proposal requires banks to supplement internal information with external data. There are at least two commercial databases that catalogue publicly disclosed operational losses across the entire financial services sector, SAS's OpRisk Global Data, and Algo OpVantage FIRST Database. Both vendors gather information on operational losses exceeding 1 million USD from public sources, such as news reports and court filings. There is however some problems associated with the use of these databases. The data might not be relevant to the institution or they might be biased, see de Fontnouvelle et al. (2005) and Chernobai et al. (2005), in the sense that not all losses are publicly reported. Moreover, since the publicly available databases only cover losses exceeding 1 million USD, one probably underestimates the capital if one does not combine the external data with internal information on smaller losses. Results from "The 2002 Loss Data Collection Exercise" (Risk Management Group, 2003) suggest that for most banks, the amount of additional capital due to smaller losses is not negligible. This means that it is crucial to include internal data as a supplement to the commercial databases when using advanced measurement approaches for operational risk.

Today, many financial institutions have started collecting data on their own operational loss experience, but it will take some time before the size and quality of most institutions' databases allow reliable estimation of the parameters in an EVT-model. Hence, we have decided to use the Basel II standardised approach for determining economic capital for operational risk (Basel Committee on Banking Supervision, 2004b) in our total risk framework.

In the standardised approach, banks' activities are divided into 8 business lines: corporate finance, trading and sales, retail banking, commercial banking, payment and settlement, agency services, asset management and retail brokerage. The capital charge for each business line is calculated by multiplying gross income by a factor assigned to that business line. The total capital charge is computed as the three-year average of the simple summation of regulatory capital charges across each of the business lines each year.

To be able to incorporate the operational risk into the same framework as the other risk types, we need to assume a distribution for the operational losses. One of the approaches suggested by Basel Committee on Banking Supervision (2001) is to simulate the number of operational loss events for the financial institution one year from a Poisson distribution, and the severity of these events from a lognormal distribution, and compute the total operational loss as the sum of the individual events. Based on simulation studies, we found it appropriate to approximate the resulting total operational loss distribution by another lognormal distribution, i.e.

$$L^{oper} \sim \text{lognormal}(\mu^{oper}, \sigma^{oper}).$$

The 99.9% percentile in this distribution should be equal to the Basel II capital, computed as described above. In Section 7.5 we describe how the parameters μ^{oper} and σ^{oper} are estimated using this information and expert opinions.

The current choice of the lognormal distribution, and the method for estimating its parameters must be considered as preliminary. As soon as the database on internal losses is considered to be sufficiently large, we will replace the lognormal distribution with the one that best fits the data. It should be noticed that the rest of the model will not be influenced by such a replacement.

5.5 Business loss function

We define business risk as the risk of losses due to external factors, such as competitive forces (e.g. reduction in loan volume as new entrants hit the core market), the market situation (e.g. reduction in volume of assets under management as the market falls), government regulations (e.g. new legislation due to consumer pressure resulting in changes of business practice), and reputational risk (e.g. actual or perceived failure to fulfil commitments to stakeholders). This risk category has so far received less attention, probably due to the fact that there is no minimum capital linked to it. While other authors like Saita (2004), have a definition similar to ours, it should be mentioned that there are other ways of defining business risk. Alexander (2005) for instance, defines it as the risk of insolvency due to inappropriate management decisions.

As for operational risk, we use a top-down approach to compute business risk. We start by determining the economic capital, defined as the 99.97% quantile of the corresponding loss distribution, and then we assume a distribution for the business losses. The economic capital is measured on the basis of fluctuations in income and expenses that cannot be linked to any other risk category. It is computed according to the following formula

$$C^{business} = \sum_{k=1}^{K} (v_k\, I_k - u_k\, E_k)\,, \tag{5.8}$$

where I_k and E_k are the kth expected income and expense items the following year, respectively, and v_k and u_k are two variability factors between 0 and 1. The idea behind Equation (5.8) is that high volatility ($v_k \approx 1$) in income increases business risk, while a flexible cost structure ($u_k \approx 1$) reduces this risk. The expected income and expense items and the two variability factors are estimated from historical data (see Section 7.6).

Having determined the economic capital, the next step is to assume a distribution for the business loss. As for the operational loss, the lognormal distribution was found to be a reasonable choice, i.e.

$$L^{business} \sim \text{lognormal}(\mu^{business}, \sigma^{business}).$$

The 99.97% percentile in this distribution should be equal to the economic capital, computed as described above. In Section 7.6 we describe how the parameters $\mu^{business}$ and $\sigma^{business}$ are estimated using this information and expert opinions.

6 Modelling correlation between losses

As described in Section 1, our approach includes two types of dependency structures. The first, a multivariate GARCH model for credit, market and ownership risk at the risk factor level, was described in Section 4.

For operational and business risk there are no obvious economic risk factors. Hence, we use the top-level aggregation approach to combine the corresponding loss distributions with the ones for credit, market and ownership loss. More specifically, we use a normal copula. Embrechts et al. (1999) were among the first to describe this toolkit in the financial literature. Since then, copulae have become very popular in finance. We will not go into details on copula modelling in this paper. For the purpose of understanding our approach, it is sufficient to think of a copula as the dependency structure, that in combination with the marginal distributions, defines the joint distribution. In practice, operational losses are linked to the other ones using the following approach (the approach for business losses is identical). First, the yearly credit, market and ownership losses are transformed to standard normal variables, using the empirical cumulative distribution of the losses. Then, a new set of normal variables are generated, conditional on the transformed quantities, using a pre-defined correlation matrix. Since we have no data, these correlations are based on expert judgments. Finally, the new set of variables are transformed to operational losses with the lognormal distribution described in Section 5.4.

7 Model parameters

The proposed risk aggregation model has a large number of parameters. In this section we describe how these are determined in the case of DnB NOR. They are either estimated from historical data, or determined based on expert knowledge.

7.1 Parameters of the risk factor models

We use historical data to estimate the simultaneous stochastic model for the credit, market and ownership risk factors. In the software accompanying our model, parameter estimation is performed in a separate module, with full flexibility regarding the nature and length of the historical time series. In the case of DnB NOR, we currently have one credit risk factor, 14 market risk factors and 9 ownership risk factors. The most important ones are shown in Table 7.1 along with the historical market time series that may be used as proxies.

7.1.1 Model for the market- and ownership risk factors

The parameters of the multivariate CCC-GARCH model, described in Section 4.1, are estimated using a sequential approach. In the first step, we estimate the multivariate GARCH model for the conditional variance, using the pseudo-maximum likelihood (PML) procedure (Bollerslev and Wooldridge, 1992). In the second step, the parameter ν of the multivariate Student's t-distribution, used to model the standardised residuals of the GARCH model, is determined.

We will emphasize the importance of (i) selecting an appropriate historical time period, and (ii) manually validating the estimated parameters. As far as (i) is concerned, Mikosch and Starica (2000) show that a GARCH(1,1) model should not be used to describe return series over long time intervals. One of their findings is that what seems to be empirical evidence of long memory and strong persistence of the volatility in log-returns in fact may be caused by non-stationarity in the time series. Hence, they recommend to update the parameters of a GARCH(1,1) model periodically in any practical setting.

While historical volatility is known to be an appropriate predictor for future volatility, this is usually not the case for historical and future returns. Hence, in our framework, the expected yearly returns of the risk factors are not estimated from historical data, but determined based on expert knowledge.

The estimate of the correlation matrix depends highly on the historical time periods used in the estimation. During the last 40 years, the correlation between

Table 7.1. The most important economic risk factors for credit, market and ownership risk, and historical market time series that may be used as proxies.

Risk Factor	**Historical time series**
	Credit risk
Credit risk factor	Norwegian credit loss ratios
	Market risk
The Norwegian Financial Index	FINX
The Norwegian Stock Market Index	The Oslo Stock Exchange Benchmark Index (OSEBX)
The USD/NOK exchange rate	USD/NOK
The EUR/NOK exchange rate	EUR/NOK
The JPY/NOK exchange rate	JPY/NOK
Norwegian short-term interest rate	NIBOR 3-month
Norwegian long-term interest rate	NIBOR 5-year
European short-term interest rate	EURIBOR 3-month
European long-term interest rate	EURIBOR 5-year
US short-term interest rate	LIBOR US 3-month
US long-term interest rate	LIBOR US 5-year
	Ownership risk
Norwegian Stocks	The Oslo Stock Exchange Benchmark Index (OSEBX)
International Stocks	The MCI World Index
Norwegian Credit Bonds	The Brix index for Norwegian bonds
International Bonds	The SSBWGB hedged bond index
Norwegian Government Bonds	The ST3X index
Real estate	OSE4040 Real estate
Hedgefond	S&P Hedge Fund Index

international equities and bonds has mostly been positive. There have also been periods of negative correlation, like it currently is. In these cases, one needs to make a choice: to trust the negative correlation regime to last for one more year, or to take the conservative approach and choose the average positive correlation.

A similar dilemma arises when considering the correlation between different stock markets. There have been a number of studies, see for instance Longin and Solnik (2001), suggesting that the correlations between market returns increase in periods of global turbulence. Should one incorporate such extreme correlations instead of the average ones? Like Alexander and Pezier (2003) we think the answer to this question is yes. Using tail correlations, one might overstate the total risk. However, in many cases this is better than a potential underestimation, using average correlations.

7.1.2 Model for the credit risk factor conditional on market and ownership risk factors

The parameters to be estimated for the model described in Section 4.2, are the yearly correlations between the credit risk factor and each of the market and ownership risk factors. As shown in Table 7.1, a time series of Norwegian yearly credit loss ratios is used as a proxy for the credit risk factor. In Norway, the banks started to register such numbers about 15-20 years ago, hence the estimation of the correlations is to be based on a relatively small data set. This means that the estimated numbers should be manually verified and potentially corrected if there seems to be obvious peculiarities.

7.2 Parameters of the credit loss function

In the case of DnB NOR, we have currently divided the credit portfolio into 34 industrial sectors. The parameters to be estimated for each sector k are

- p_k, the probability of default, which is the average percentage of obligors from this sector that will default during the course of the year,
- s_k, the loss given default, which is the average percentage of exposure the bank might lose in case a borrower from this sector defaults,
- E_k, the exposure at default, which is the average amount outstanding in case a borrower defaults in this sector,
- ρ_k, the average correlation between asset returns of two obligors from this sector, and
- β_k, the correlation between the sector index and the credit risk factor.

The parameters p_k, s_k, and E_k are computed by estimating the corresponding quantities for each client in the subportfolio, and then computing a weighted average of the single-client numbers. The single-client estimates are output from DnB NOR's other credit management systems. Typically these are score card or simulation models.

The estimation of the correlation ρ_k consists of two main steps. First, we determine the ρ_k that gives the same economic capital for sector k and confidence level 99.9% using our model and the Basel II formula, and then this ρ_k is multiplied by a volatility factor to obtain a final estimate. In what follows the two steps are described in more detail.

Using the standard Basel II IRB formula, the economic capital for sector k and confidence level 99.9%, given the single-client estimates p_{ki}, s_{ki}, and E_{ki} is,

$$C_k^{Basel}(0.999) = \left\{ \sum_{i \in k} E_{ki}\, s_{ki} \left[\Phi\left(\frac{\Phi^{-1}(p_{ki}) + \sqrt{\rho_{ki}}\Phi^{-1}(0.999)}{\sqrt{1-\rho_{ki}}} \right) - p_{k,i} \right] \right\} \cdot M_{ki}. \tag{7.1}$$

The asset correlation ρ_{ki} is given by

$$\rho_{ik} = 0.12 \times \frac{1 - e^{-50\,p_{ik}}}{1 - e^{-50}} + 0.24 \times \left(1 - \frac{1 - e^{-50\,p_{ik}}}{1 - e^{-50}}\right), \tag{7.2}$$

and the maturity adjustment is given by (Basel Committee on Banking Supervision, 2004a)

$$M_{ik} = \frac{1 + (m_{ik} - 2.5)\, b(p_i)}{1 - 1.5\, b(p_i)},$$

where

$$b(p_i) = (0.11852 - 0.05478 \times \log(p_i))^2,$$

and m_{ik} is the maturity for client i in sector k. In the current version of our model, m_{ik} is set to 2.5 years for all clients. The economic capital for sector k and confidence level 99.9% using our model is obtained by computing 99.9% percentile of the loss distribution given by Equation (5.3) and subtracting the expected loss, i.e.

$$C_k^{Model}(0.999) = E_k\, s_k \left[\Phi\left(\frac{\Phi^{-1}(p_k) + \sqrt{\rho_k}\Phi^{-1}(0.999)}{\sqrt{1-\rho_k}} \right) - p_k \right].$$

A first estimate for ρ_k is obtained by solving $C_k^{Basel}(0.999) = C_k^{Model}(0.999)$ with respect to ρ_k. This ρ_k is multiplied by a volatility factor f_k to obtain a final estimate. The factor f_k is estimated from historical data. It is large if the average annual default rate of sector k has been characterised by large fluctuations during the historical time period used for estimation (1989-2004), and small if the fluctuations have been small.

The last parameters to be estimated are the correlations β_k, between the sector indices Y_k and the credit risk factor X^{credit}. The annual historical observations for

Y_k are obtained by setting the right-hand side of Equation (5.3) equal to the observed credit loss ratios for sector k, D_k, and solving this equation with respect to Y_k (we use the estimates for ρ_k and p_k obtained as described above). Correspondingly, the annual historical observations for the credit risk factor are obtained from the observed average credit loss ratios for the whole country.

In some sectors, there have been no, or very few, defaults during the historical time period available. For these sectors, we produce fictitious credit loss ratios as follows. First, we estimate an overall model for the relationship between credit loss ratios and accounting variables, using yearly credit loss ratios from all Norwegian firms during the time period 1989–2004. Then, this model is used to predict the number of defaults in each sector and the total credit portfolio for each year in the same time period. The β_k-values are estimated based on these predicted time series, as shown above. Since the correlations are computed based on only 11 data points, they are not very robust. Hence, in a second step, we validate and potentially correct them with historical data for US credit bonds and expert knowledge.

7.3 Parameters of the market loss function

For the market loss function given in Section 5.2, there are no parameters to estimate. The exposures E^k and the liquidation periods Δ_k must however be specified. For DnB NOR, the liquidation periods vary from 250 days for equity investments (as the vast majority of the financial institution's stock investments are long-term) to two days for positions in the most commonly traded currencies.

7.4 Parameters of the ownership loss function

There are no parameters to estimate for the ownership model given in Section 5.3 either. However, the start value $v_{k,0}$ of each financial asset k is needed as input. Moreover, the parameters that determine the rebalancing strategy and those determining the buffer capital limits must be given.

7.5 Parameters of the operational loss function

As mentioned in Section 5.4, the 99.9% quantile of the lognormal distribution must correspond to the economic capital from the standardised approach. In addition, the risk managers of DnB NOR felt that they had a relatively clear opinion on the size of the most frequent aggregate yearly operational loss, i.e. the mode of the loss distribution. The parameters of the lognormal distribution described in Section 5.4 are therefore determined from the 99.9% quantile $C^{oper}(0.999)$ and the mode m^{oper} by solving the equations

$$m^{oper} = \exp(\mu^{oper} - (\sigma^{oper})^2) \quad (7.3)$$

and

$$C^{oper}(0.999) = \exp\{\mu^{oper} + \sigma^{oper}\,\Phi^{-1}(0.999)\}. \tag{7.4}$$

7.6 Parameters of the business loss function

The procedure for business risk is the same as the one for operational risk, i.e. the parameters of the lognormal distribution described in Section 5.5 are determined by specifying the 99.97% quantile and the mode of the business loss distribution. The first is set equal to the economic capital for business risk computed as described in Section 5.5. To compute the economic capital, one must first determine expected income and expense items and the parameters v_k and u_k in Equation (5.8). These are estimated from relevant historical profit and loss time series, removing volatility stemming from other risk types. It should be noted that manual validation is an important part of the estimation procedure. Finally, like for the operational loss distribution, we had to rely on expert opinions and subjective choices when specifying the mode.

7.6.1 Parameters of the copulae linking operational and business risk to the other risk types

Since we have no historical operational and business loss data, the correlations between operational losses and the other loss distributions and between business losses and the other loss distributions, needed for the conditional simulation described in Section 6, cannot be estimated. Hence, like Rosenberg and Schuermann (2004), we have to base the correlations on expert judgments and what has been reported in other studies.

8 Practical use

The suggested approach has been implemented in the statistical software package S-PLUS. To save computational time, the simulation code is written in C++. DnB NOR started using the model in the second quarter of 2005. Being adapted to the requirements in the Basel II regulations, the new model will play an important part in measuring and assessing the risk level of the financial institution. Economic capital may be calculated separately for each business area, as well as aggregated for the whole financial institution. The calculations are used in profitability measurements and as decision support within risk management. Using the proposed model and the accompanying software, one may compare risk across risk categories and business areas. For instance, Figure 8.1 shows the composition of economic capital for each business area and the whole financial group.

As it is impossible to guard against all potential losses, DnB NOR has stipulated that economic capital should cover 99.97% of potential losses within a one-year horizon. This level is in accordance with an Aa level rating for ordinary long-term debt. We found 500,000 simulations to be sufficient to estimate the 99.97% quantile with the required accuracy. The computational time is satisfactory for practical use. On a standard PC, 500,000 simulations is performed within 20-25 minutes.

The modelling approach described in this paper reduces the economic capital substantially when compared to methods following the hypothesis of perfect correlation. Table 8.1 shows economic capital numbers (the 99.97% quantiles of the corresponding loss distributions) for the DnB NOR Group for the last 7 quarters (figures for previous periods have been restated in accordance with the new risk measurement principles). As the table shows, the net economic capital is approximately 30% smaller than the gross risk adjusted capital obtained by just adding the separate capital requirements.

In addition to being adapted to the Pillar I requirements of Basel II, the proposed model takes into account several of the risks covered by Pillar 2 of the accord. These are concentration risk connected to the credit portfolio, interest rate risk arising from non-trading activities, and business risks (earnings and costs). Hence, it represent an important tool for structuring the dialogue between the institution and its supervisor, when assessing whether the capital held by the institution covers all material risks.

Table 8.1. Economic capital (defined as the 99.97% quantiles of the corresponding loss distributions) for the DnB NOR Group. Amounts in NOK billion.

Risk Type	**2.2005**	**1.2005**	**4.2004**	**3.2004**	**2.2004**	**1.2004**	**4.2003**
Credit risk	24.8	24.2	23.6	23.7	23.9	23.4	22.0
Market risk	2.0	2.0	2.1	2.1	1.9	1.9	2.1
Ownership risk	9.0	9.0	7.2	8.2	7.4	7.0	6.6
Operational risk	4.0	4.0	4.0	3.9	3.9	3.7	3.7
Business risk	1.6	1.6	1.6	1.6	1.6	1.6	1.6
Gross economic capital	41.4	40.6	38.5	39.4	38.7	37.6	35.9
Diversification effects	(13.4)	(13.4)	(12.1)	(12.6)	(11.6)	(11.4)	(11.3)
Net economic capital	28.0	27.2	26.4	26.7	27.1	26.2	24.6

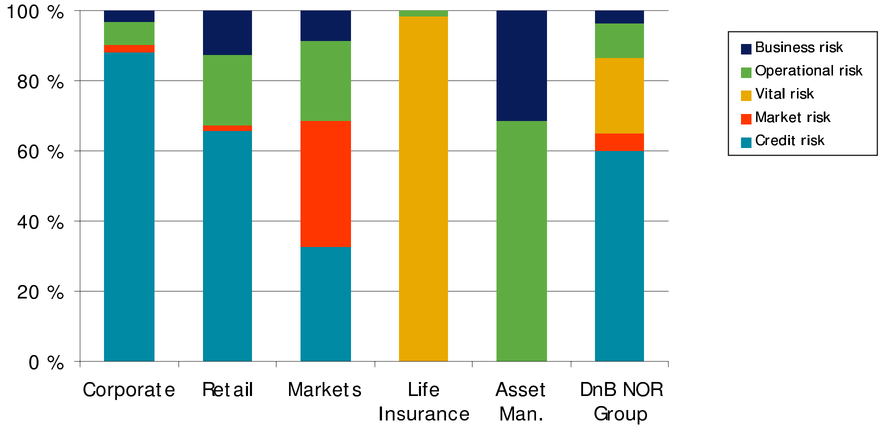

Figure 8.1. Composition of economic capital for each business area and the whole financial group.

9 Summary and discussion

In this article we have presented a new approach for determining the total risk of a financial institution. The proposed model includes components for credit, market, operational and business risk. Moreover, it includes a component for the ownership risk that stems from holding a life insurance company. The approach may be characterised as a base-level aggregation method. Due to lack of appropriate data, however, some of the aggregation steps are done at the top-level instead. The economic risk factors used in the base-level aggregation are described by a multivariate GARCH model with Student's t-distributed innovations. The loss distributions of the different risk types are determined by non-linear functions of fluctuations in the risk factors. These marginal loss distributions are indirectly correlated through the relationship between the risk factors. The model was developed for DnB NOR, the largest financial institution in Norway, and one of the largest ones in the Nordic region. Being adapted to the requirements in Pillar 1 of the Basel II regulations, it will play an important part in measuring and assessing the risk level of the institution. The proposed model also takes into account several of the risks covered by Pillar 2 of Basel II: Residual and concentration risk connected to the credit portfolio, interest rate risk arising from non-trading activities and business risks (earnings and costs).

Our motivation has been to construct a model which correlates the different risk types and that is easy to use in practice. The model has some shortcomings. In particular, the choice of the operational loss function must be considered as preliminary. The current model will be refined as soon as the database on internal losses is considered to be sufficiently large and of satisfactory quality. When more data is available, one will also be able to estimate the correlations between operational losses and the other loss distributions. In the current version of the software, these are based on expert judgments.

Liquidity risk is not a part of our total risk model. It has proved difficult to model this risk category in a manner that is consistent with the other risk categories. Liquidity risk is instead managed and controlled through limits and stress testing.

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A The conditional distribution of the credit risk factor

First, we give a general result for the multivariate normal distribution, see e.g. Johnson et al. (1995) for a proof. Let $\boldsymbol{X}$ be $N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. Partition $\boldsymbol{X}$, its mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$ as $\boldsymbol{X} = (\boldsymbol{X}_1, \boldsymbol{X}_2)^T$, $\boldsymbol{\mu} = (\boldsymbol{\mu}_1, \boldsymbol{\mu}_2)$ and

$$\boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12} \\ \boldsymbol{\Sigma}_{21} & \boldsymbol{\Sigma}_{22} \end{pmatrix}.$$

Then, the conditional distribution of $\boldsymbol{X}_1$ given $\boldsymbol{X}_2 = \boldsymbol{x}_2$ is multivariate normal with mean vector

$$\mathrm{E}[\boldsymbol{X}_1 | \boldsymbol{X}_2 = \boldsymbol{x}_2] = \boldsymbol{\mu}_1 + \boldsymbol{\Sigma}_{12}\, \boldsymbol{\Sigma}_{22}^{-1}\, (\boldsymbol{x}_2 - \boldsymbol{\mu}_2)$$

and covariance matrix

$$\mathrm{Var}(\boldsymbol{X}_1 | \boldsymbol{X}_2 = \boldsymbol{x}_2) = \boldsymbol{\Sigma}_{11} - \boldsymbol{\Sigma}_{12}\, \boldsymbol{\Sigma}_{22}^{-1}\, \boldsymbol{\Sigma}_{21}.$$

We are interested in the special case for which $X_1 \sim N(0,1)$ and $\boldsymbol{X}_2$ is multivariate normal with mean vector and covariance matrix equal to the zero-vector and the correlation matrix, respectively. We then have

$$X_1 | \boldsymbol{X}_2 = \boldsymbol{x}_2 \sim N(\boldsymbol{R}_{12}\, \boldsymbol{R}_{22}^{-1}\, \boldsymbol{x}_2, 1 - \boldsymbol{R}_{12}\, \boldsymbol{R}_{22}^{-1}\, \boldsymbol{R}_{21}),$$

where $\boldsymbol{R}_{12}$ is the vector containing the correlations between x_1 and $\boldsymbol{X}_2$, i.e. the correlations between the credit risk factor and the yearly log-increments of the market and ownership risk factors, and $\boldsymbol{R}_{22}$ is the correlation matrix of $\boldsymbol{X}_2$, i.e. of the market and ownership risk factors.