DISCRETELY ADJUSTED OPTION HEDGES*

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This paper analyses the distribution of returns on a hedged portfolio, consisting of a European call option and its associated stock, when the portfolio is rebalanced at discrete time intervals. Under the assumptions of the Black-Scholes model this distribution is particularly skew. In tests of the average return on a hedged portfolio this skewness leads to biased *t*-statistics. The paper explores the nature and extent of this bias and suggests procedures for overcoming it. Other aspects of discrete hedging are also discussed.

1. Introduction

The creation and maintenance of a riskless hedge plays an essential role in the derivation of the Black-Scholes option pricing formula. In the case of a European call option, a hedge portfolio is constructed by establishing a long position in the option and a short position in the underlying stock on which the option is written. The relative position in the two securities in the hedge portfolio is determined by the first partial derivative of the option pricing formula with respect to the stock price. [For a more complete description, see either Black and Scholes (1973) or Smith (1976).] Given their assumptions, the effect of diffusion in the stock price is thus eliminated and with continual adjustment of the hedge composition the value of the hedge at maturity becomes riskless. Exploitation of this observation leads to the derivation of the option pricing formulae.

In practice it is clearly not possible to rebalance a portfolio continuously. In their empirical tests, Black and Scholes (1972) assumed that the hedge

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portfolio was adjusted on a daily basis. When this is done, the hedge portfolio return becomes risky. Black and Scholes argue that this return will be uncorrelated with the market and that it will have an expected value of zero when allowance is made for interest on the net equity position in the hedge. In addition, Black and Scholes suggest that the uncertainty arising from discrete rebalancing can be reduced by holding a portfolio of hedges comprised of different options.

Discretely rebalanced option hedges are thus of some importance in constructing empirical tests of the Option Pricing Model and its descendants. Also, market makers on the traded options exchanges generally employ some form of discontinuously adjusted hedging to reduce their risk exposure. Furthermore, in a wider context, there is now considerable interest in risk management and hedging against risk (for example, in hedging against exposure to foreign exchange rate fluctuations). Therefore, just as mean-variance measures were developed to test security pricing models and to evaluate portfolio management performance, it would seem to be quite important to develop similar criteria for measuring the performance of hedge portfolios.

It is the aim of this paper to analyze the distribution of the returns on the hedge portfolios when rebalancing takes place at discrete points in time. This distribution is particularly skew and some of the consequences of this skewness are explored. Initially, these consequences are examined in a world where the assumptions of the Black-Scholes model hold with certainty.

The layout of the paper is as follows. Section 2 derives an approximate expression for the return on the hedge when rebalancing occurs at discrete intervals. This result has been derived earlier by Thorpe (1976). The return on the hedge portfolio can be expressed as the product of three components. These are:

- (i) a deterministic function of the underlying variables evaluated at the time when the hedge is constructed;
- (ii) a random variable drawn from a chi-squared distribution with one degree of freedom and translated to have a mean of zero;
- (iii) the time interval between adjustments to the hedge.

In section 3 the deterministic component of the hedge return is analyzed. Some comparative statistics are presented.

In section 4 we consider the stochastic component of hedge return. The return distribution is strongly skewed and leptokurtic (Skewness measure 2.8; kurtosis 15). Roughly two-thirds of the drawings from this distribution are small and negative. The other third are positive and can be quite large. In consequence, standard *t*-tests used to compute the significance of mean hedge return are biased in favor of producing significant negative *t*-statistics. We explore the nature of this bias and methods of reducing or eliminating it.

In section 5 we present the results of a Monte Carlo simulation of the distribution of hedge returns and examine the efficacy of various methods for reducing bias.

The relationship between the return on the hedge and the return on the market is examined in section 6. Hedge return is not independent of market return and the implications of forming a portfolio of hedges are examined.

In section 7 we consider the potential impact of relaxing some of the assumptions of Black and Scholes. The implications of these relaxations for our analysis and for empirical testing are discussed.

Finally, in section 8, there is an assessment of the significance of the findings in the paper. It is emphasized that the characteristics of discrete hedging obtained in sections 2–6 of the paper are based on the assumption that the Black–Scholes assumptions hold with certainty. However, it is contended that features of discrete hedging obtained under these ideal circumstances can provide both a valuable benchmark and a useful device for analyzing what actually happens with real-world data. The final section also comments on some work by Galai (1975) in this area where he examined a different aspect of the consequences of discrete hedging. The paper concludes with some suggestions for future research in this area.¹

2. The return on a hedge portfolio

It is convenient to use the notation proposed by Smith (1976):

- t = current calendar time,
- t^* = option expiration date,
- T = time to expiration ($=t^* - t$),
- c = price of a European call option at time t ,
- r = riskless interest rate continuously compounded,
- S = stock price at time t ,
- X = the exercise price of the option,
- Δt = rebalancing period; a hedge constructed at time t is rebalanced at time $t + \Delta t$,
- $S + \Delta S$ = stock price at time $t + \Delta t$,
- $c + \Delta c$ = option price at time $t + \Delta t$,
- σ^2 = variance rate of return on stock,
- d_1 = $[\log(S/X) + (r + \frac{1}{2}\sigma^2)T]/\sigma\sqrt{T}$,
- d_2 = $d_1 - \sigma\sqrt{T}$,
- $N(\cdot)$ = the cumulative normal distribution function,
- $Z(\cdot)$ = the unit normal density function.

¹When this paper was completed we learned that researchers at the University of Berkeley (Rubinstein, (1978) Rudd, (1978) and Bhattacharya, (1979) had obtained results similar to some of those derived in the earlier part of the present paper.

It is emphasized that for most of this paper the Black-Scholes assumptions hold with certainty. The formula for the call option is therefore

$$c(S, T) = SN(d_1) - X \exp(-rT)N(d_2), \quad (1)$$

and $c + \Delta c$ has a similar functional form. At time t a hedge portfolio is constructed by purchasing one call option at price c and selling $\partial c / \partial S$ units of stock short. From (1), $\partial c / \partial S = N(d_1)$ so that the initial investment in the hedge is $c - SN(d_1)$. At time $t + \Delta t$, the value of the hedge portfolio is

$$c + \Delta c - (S + \Delta S)N(d_1). \quad (2)$$

Note that in expression (2) d_1 is still a function of the original variables S and T (because the hedge was not adjusted since time t), even though the option price and stock price will surely have changed. The return on the hedge portfolio during the time interval Δt is therefore

$$\Delta c - \Delta SN(d_1). \quad (3)$$

If the initial investment of $c - SN(d_1)$ was borrowed at the riskless rate r , then the net investment is zero. The return on the hedge allowing for the opportunity cost of its equity is thus

$$\Delta c - \Delta SN(d_1) - r\Delta t[c - SN(d_1)] + 0(\Delta t^2). \quad (4)$$

Henceforth, we shall refer to expression (4) as HR . The purpose of the present section is to analyze the structure of HR . From (1) and (4) we see that

$$HR = \Delta c - \Delta SN(d_1) + r\Delta rX \exp(-rT)N(d_2) + 0(\Delta t^2). \quad (5)$$

As Thorpe (1976) shows, the change in the option value Δc during a small time interval Δt is

$$\Delta c = c_1 \Delta S + c_2 \Delta t + \frac{1}{2} c_{11} \sigma^2 S^2 u^2 \Delta t + 0(\Delta t^{3/2}), \quad (6)$$

where the subscripts 1 and 2 denote partial derivatives with respect to S and t , respectively, and u is a random variable drawn from a normal distribution with zero mean and unit variance. The variable u generates the stochastic component of the stock price change. Thus

$$HR = [c_2 + \frac{1}{2} c_{11} \sigma^2 S^2 u^2 + rX \exp(-rT)N(d_2)] \Delta t + 0(\Delta t^{3/2}). \quad (7)$$

From differentiating (1) we find that

$$c_2 = -X \exp(-rT) \left[\frac{Z(d_2)\sigma}{2\sqrt{T}} + rN(d_2) \right], \quad (8)$$

$$c_{11} = \frac{Z(d_1)}{S\sigma\sqrt{T}}.$$

Substituting these relationships into (7) we obtain

$$\begin{aligned} HR &= \frac{\sigma Su^2 Z(d_1) \Delta t}{2\sqrt{T}} - \frac{\sigma X \exp(-rT) Z(d_2) \Delta t}{2\sqrt{T}} + O(\Delta t^{3/2}) \\ &= \frac{\sigma S}{2\sqrt{T}} Z(d_1) (u^2 - 1) \Delta t + O(\Delta t^{3/2}), \end{aligned}$$

since $SZ(d_1) = X \exp[(-rT)Z(d_2)]$.

Therefore, neglecting terms involving higher powers of Δt ,

$$HR = \frac{\sigma S}{2\sqrt{T}} Z(d_1) (u^2 - 1) \Delta t. \quad (9)$$

If we define

$$\lambda = \frac{\sigma S}{2\sqrt{T}} Z(d_1), \quad y = u^2 - 1,$$

then HR becomes²

$$HR = \lambda y \Delta t. \quad (10)$$

Notice that λ is a deterministic function of the variables S , T , X , r and σ^2 evaluated at time t . The stochastic component of HR is embodied in y . Since $y + 1$ is distributed as chi-squared with one degree of freedom, we have

$$\begin{aligned} E(y) &= 0, \\ E(y^2) &= 2. \end{aligned} \quad (11)$$

²Note that expression (10) for HR is an approximate one since terms of order $(\Delta t)^{3/2}$ and higher were neglected in its derivation. The numerical analyses presented in sections 3 and 4 are therefore also approximate since they are based on eq. (10). However, this approximation serves to illustrate the main features of the distribution of hedge returns. When eq. (10) is used, it is more correct to say that the leading component of HR is distributed as chi-squared with one degree of freedom (since the higher order terms in Δt have not been included in the analysis).

OA' . Suppose there is a large stock price jump so that $\Delta S = MN$. The new stock price is given by ON . The corresponding option price $c + \Delta c$ is located at B and so Δc is represented by the line EB . Since $\Delta S = MN = PE$, the term $\Delta SN(d_1)$ is given by the line EC . Hence the first component of the hedge return

$$\Delta c - \Delta SN(d_1)$$

corresponds to the line BC . As ΔS gets smaller, this component of the hedge return is reduced. When $\Delta S = ML$ this component is exactly zero. If $\Delta S < ML$ but greater than minus MK , the term is negative. If ΔS is less than minus MK the value of $\Delta c - \Delta SN(d_1)$ again becomes positive.

This analysis has not considered the contribution of the last term in (5),

$$rX \exp(-rT)N(d_2)\Delta t.$$

This deterministic term is hard to represent diagrammatically. However, incorporation of it does not change the qualitative interpretation of fig. 1.

3. Analysis of deterministic component of hedge return

It was shown in the previous section that the deterministic component of the hedge return is given by

$$\lambda = \frac{\sigma S}{2\sqrt{T}} Z(d_1) = \frac{1}{2}\sigma^2 S^2 c_{11}.$$

We see that λ depends on the second partial derivative of the option price with respect to the stock price. This dependence of λ on the curvature of the option curve conforms with the geometrical representation of the hedge return in the previous section.

Clearly, since S and T will change over time, the return on a hedge in which the number of options held is kept constant will be heteroscedastic. If there considerable variation, there will be problems in making statistical inferences about hedge return.

Table 1 shows how λ depends on S and T . Whilst $\lambda\Delta t$ is clearly quite a smooth function of S and T , as might have been expected from examination of its functional form above, there is nevertheless considerable variation over the domain considered. In particular, option hedges that expire 'close to the money' will exhibit a considerable amount of heteroscedasticity. This heteroscedasticity could be removed, at least in principle, either by adjusting the trading interval Δt inversely with λ or by adjusting the number of options held inversely with λ (whilst, of course, still holding an appropriate short position in the underlying stock). Table 1 indicates that unless this is done

there can be quite extreme heteroscedasticity in the hedge returns, particularly in the last unit of time prior to expiration. (This example was constructed with reasonable parameter values and with the time unit corresponding to one quarter of a year and Δt corresponding to one trading day.)

Table 1

Variation of deterministic component of hedge return with stock price S and time to maturity T .

Values of underlying parameters

Exercise price: $X = 50$

Riskless rate: $r = 0.015$ per time period (quarter of year)

Variance rate: $\sigma^2 = 0.025$ per time period

Rebalancing interval: $\Delta t = \frac{1}{60}$

Stock price S	Time to maturity T								
	0.05	0.20	0.40	0.60	0.80	1.00	2.00	3.00	4.00
42	0.00000	0.00285	0.00919	0.01242	0.01382	0.01434	0.01354	0.01205	0.01081
44	0.00017	0.01158	0.01848	0.01975	0.01954	0.01891	0.01551	0.01317	0.01155
46	0.00734	0.02948	0.02942	0.02689	0.02460	0.02272	0.02696	0.01396	0.01205
48	0.06055	0.04980	0.03816	0.03194	0.02796	0.02514	0.01781	0.01440	0.01232
49	0.09996	0.05636	0.04055	0.03324	0.02880	0.02573	0.01799	0.01449	0.01237
50	0.11745	0.05859	0.04131	0.03362	0.02903	0.02588	0.01803	0.01450	0.01237
51	0.10020	0.05623	0.04042	0.03312	0.02869	0.02564	0.01793	0.01444	0.01232
52	0.06323	0.05004	0.03810	0.03182	0.02783	0.02501	0.01769	0.01430	0.01223
54	0.01090	0.03216	0.03048	0.02739	0.02484	0.02282	0.01686	0.01383	0.01192
56	0.00068	0.01604	0.02149	0.02168	0.02082	0.01980	0.01568	0.01316	0.01147
58	0.00002	0.00638	0.01353	0.01590	0.01649	0.01642	0.01425	0.01234	0.01092
60	0.00000	0.00207	0.00770	0.01090	0.01242	0.01308	0.01270	0.01141	0.01029

For the record, the values of S , T , σ^2 , r and X which individually maximize λ are given by the following relationships:

$$S = X \exp [(-r - \sigma^2/2)T],$$

$$T = -\frac{\sigma^2 + \sqrt{[\sigma^4 + ((\log S/X)(2r + \sigma^2))^2]}}{2(r + \frac{1}{2}\sigma^2)^2},$$

$$\sigma^2 = (2 + 2\sqrt{[1 + (\log(S/X) + rT)^2]})/T,$$

$$r = -\frac{1}{T}(\log(S/X) + \frac{1}{2}\sigma^2 T),$$

$$X = S \exp ((r + \frac{1}{2}\sigma^2)T).$$

4. Analysis of the stochastic component of hedge return

We saw in section 2 that the stochastic component of the return on the hedge portfolio, y , corresponds to a displaced chi-squared distribution with one degree of freedom. Although the return on the hedge portfolio will vary both on account of changes in λ and the stochastic nature of y , to focus attention on the changes induced by the stochastic component we assume λ is held constant.³ As mentioned before, assuming homoscedasticity is not valid, but homoscedasticity can, in principle, be achieved by varying the number of options held inversely with λ . (Assuming, of course, that the higher-order terms in Δt remain negligible.)

In this section we analyse the distribution of hedge portfolio returns on this basis. In particular, we examine the biases which can arise from the skewness of the y -distribution in the calculation of t -statistics for confidence interval construction. The results presented in this section are illustrative rather than definitive since formula (10) for HR neglects higher powers of t .

With λ assumed constant, the estimation bias will exactly correspond to that obtained in the following experiment. A random sample of size n is taken from a chi-squared distribution with one degree of freedom and t -statistics for confidence limits on the sample mean are computed. What is the resulting distribution of the t -statistics like? How does the bias change as n changes? We are assured that for large samples, the t -statistics will be 'correct' by an appeal to the law of large numbers. However, it is of interest to examine the behavior of the t -statistics for small values of n such as 10, 20 and 30 since it has been suggested that weekly rebalancing of hedge portfolios may be adequate and also since the 60 trading days of a three-month option is not necessarily large enough for the law of large numbers to apply. For small sample sizes, the t -statistics will be biased downwards. This is because a negative mean return is caused by a predominance of negative daily hedge returns. These negative returns are closely bunched together and the variance estimate tends therefore to be low. This leads to a large negative t -statistic. Conversely a positive mean return is caused by a predominance of positive widely dispersed daily returns. The t -statistic tends to appear insignificant because of the resulting high variance estimate.

Table 2 shows the result of a simulation experiment. The approximate frequency distribution of t is displayed for the case when the parent population is chi-squared and when it is normal.

As can be seen, there are substantial differences in the distribution of the estimated t -statistic when the sample is taken from the chi-squared distri-

³In the next section we analyse the structure of hedge returns when the option is held to maturity, and we thus account for the combined effect of variations in λ and in the stochastic component. The distribution of hedge returns in this case will be examined using Monte Carlo simulation and so the higher order (Δt) terms, ignored in sections 3 and 4 will be automatically included in the analysis.

bution instead of the normal. These differences will be exacerbated if λ varies over time.

How do we cope with statistical problems caused by this bias effect? There are several approaches: First, we could construct frequency tables of the estimated t -statistics. These tables could be used for confidence interval estimation, just as regular t -tables are used to construct confidence intervals for regression estimates when residuals are homoscedastic and normal.

Table 2
Distribution of estimated t -statistic for the mean hedge return.^a

Range of t	Sample size = 10		Sample size = 20	
	Chi-Sq.	Normal	Chi-Sq.	Normal
< -3	0.093	0.007	0.051	0.004
-3-- -2	0.070	0.030	0.060	0.025
-2-- -1	0.137	0.134	0.141	0.135
-1-- 0	0.262	0.329	0.290	0.336
0-- 1	0.338	0.329	0.348	0.336
1-- 2	0.096	0.134	0.105	0.135
2-- 3	0.004	0.030	0.005	0.025
> 3	0.000	0.007	0.000	0.004

^aThis table illustrates the differences in the distribution of t -statistics when the parent population is chi-squared instead of normal for sample sizes of 10 and 20. In the chi-squared case there is an increased probability of getting a significant, negative t value, which leads to a downward bias.

Second, we could attempt to adjust our estimated t -statistic so that it corresponds more closely to a drawing from a true t -distribution. Since the estimation bias is caused primarily by skewness, the adjustment should naturally be a function of the estimated skewness. A procedure to do this is given by Johnson (1978),

$$\text{Adj. } t = \frac{(\bar{x} - \mu) + \mu_3/6\sigma^2 N + (\mu_3/3\sigma^4)(\bar{x} - \mu)^2}{\sqrt{(s^2/N)}}. \quad (12)$$

In this expression, $\bar{x}$ is the sample mean and μ is the hypothesized mean, N is the sample size, μ_3 is the third central moment, σ^2 is the population variance and s^2 is the sample variance. If μ_3 and σ^2 are unknown, then sample estimates can be used. Johnson suggests that the third term in the numerator can be discarded. Johnson reports that the adjusted t -estimate is tolerably close to the true t -distribution for sample sizes as small as thirteen drawn from an exponential distribution (chi-squared with two degrees of

freedom). Since the residual returns on hedges are more highly skewed than the exponential distribution, the efficacy of this adjustment for our purpose is not yet established.

Third, we could exploit the fact that, assuming constant λ , the aggregate hedge return over n trading periods should be distributed as chi-squared with n degrees of freedom. This is a tabulated distribution. Therefore if we could obtain an accurate variance estimate, we could use the tabulated distribution for construction of confidence intervals. Now, as mentioned above, direct estimation of hedge return variance is not particularly accurate. However, it should be recalled that the basic process driving the hedge return is the random diffusion of the stock price. If the variance rate of that underlying process is known exactly (or at least reasonably accurately) then an accurate estimate of the hedge return variance can be made.

Before we leave this section, one final philosophical point should be made. It is necessary to distinguish between testing mean hedge return for significant departures from zero and testing the assumptions of the Black–Scholes option pricing model. All of the tests discussed here are designed to test mean hedge return, which for many purposes is the ‘bottom line’. In such a test, one wants to develop procedures that are fairly robust and which are not particularly sensitive to changes in the underlying model. If, instead, one’s sole preoccupation is with testing the Black–Scholes assumptions, then one should ignore hedge returns and directly examine security returns. Also, if one accepts the Black–Scholes assumptions and wishes to test the ability of a trader to select overvalued and undervalued options, then one should recognise that setting up each hedge position is equivalent to making a forecast of the stock return variance. Therefore, the appropriate test in such circumstances is to construct a confidence interval for observed stock return variance.

5. Simulation investigation of the empirical bias effect

In order to illustrate the feasibility of these various approaches, we suggest reference to table 3. This table presents the results of extensive simulations of the returns on a hedge portfolio. The Black–Scholes assumptions are assumed to hold perfectly in these simulations. We performed ten thousand simulations of hedge performance over ten, twenty, forty and eighty trading intervals.

That is to say, we simulated how a correctly valued and balanced hedge would perform if hedge proportion were adjusted at the start of regular intervals (of length Δt — e.g. daily adjustment). Such readjustment would occur at the start of each of the ten (or twenty, forty or eighty) intervals between the date of initial purchase and the option expiration date.

Stock returns were randomly generated according to a lognormal distribution and all simulated trading in the option and the underlying stock took place at the theoretically correct price. Any net equity in the hedge was invested in risk-free bonds. The terminal value of such a hedge should be zero on average. Any deviation from zero must be a consequence of risk and bias induced by the discrete hedge adjustment process.

The specific values of parameters used in the simulation are relatively unimportant. Merton (1973), for example, has shown that the Black-Scholes option pricing formula can be transformed into a function involving only two parameters rather than the original five untransformed parameters. However, the actual values used were:

$$X = 50,$$

$$r = 0.008/\text{month},$$

$$\sigma^2 = 0.01/\text{month},$$

$$\Delta t = 0.05 \text{ months} = (\text{roughly speaking}) \text{ one trading day}.$$

These values were chosen to be representative of a 'typical' situation.

We consider options that were initially at the money, out of the money and in the money. For each scenario, a frequency table is presented for each of five statistics. These statistics are:

- *Raw t-statistic* — This is the t -statistic that one would normally calculate to test the hypothesis that the mean hedge return is zero
- *Raw α -statistic* — This is the observed return on the hedge over its lifetime. This statistic has been rescaled and its actual tabulated values are meaningless. However, examination of the distribution of α is indicative of how well the 'law of large numbers' applies (i.e., how close the observed distribution of α is to the normal distribution). This 'law' appears to hold quite well for α , but at the same time, it fails miserably for the observed t -statistic.
- *Johnson adjusted t* — This is simply the statistic defined in eq. (12) with sample moments used instead of population moments.
- *'Homoscedastic' t* — This is the t -statistic that would be calculated if, instead of holding one option in the hedge, $1/\lambda$ options are held in the hedge. In other words, the portfolio composition is being adjusted in an attempt to remove heteroscedasticity. Since λ can become extremely small for deep in the money and out of the money options, an arbitrary bound was placed on the amount of this adjustment to prevent our computer program 'blowing up'.
- *Doubly corrected t* — This is the adjusted t -statistic as defined in eq. (12), calculated for a 'homoscedastic' hedge.

Examination of table 3 indicates many more apparently significant raw t -

statistics than the t -distribution would indicate. Virtually all of these significant observations lie in the negative tail. The problem is clearly greatest when the trading intervals are fewest. On the other hand, the problem persists even with as many as eighty trading intervals. Extremely negative t -statistics (i.e., less than -4) are really quite common.

By itself, the 'Johnson adjusted t ' does not seem to be particularly useful. This is not particularly surprising when one considers that heteroscedasticity is also present.

The 'Homoscedastic t ' is not helpful at all. However, it may be of interest to a hedger who manipulates the composition of his hedge to control heteroscedasticity. (Aside from the empirical problems caused by heteroscedasticity, its elimination would tend to minimize aggregate hedge risk.) Clearly the statistic does not appear to be distributed anything like the $\chi^2(n)$ distribution. This is to be expected, since there is sampling error in calculating the hedge return variance.

The 'Doubly corrected t ' statistic appears to be a distinct improvement over 'Raw t ' statistic. The lower tail is appreciably shortened. For forty and eighty trading intervals, this statistic is quite close to the true t -distribution. Generally, being in, at or out of the money does not appear to have much relevance.

However, extreme outliers are most frequent when there are few trading intervals and the option is initially at the money. In such cases, the correction is helpful but not as effective as in other circumstances. This is because the higher-order terms omitted in eq. (6) start to become of some importance in such circumstances.

It is of some interest to note the first and ninety-ninth percentiles of our simulated statistics. One tends to think of observations outside the range between these percentiles as being 'statistically significant' and tending to favor the alternative hypothesis. The first percentiles for the 'Raw t ' statistic are -5.6 , -4.4 , -3.7 and -3.5 , for ten, twenty, forty and eighty trading intervals respectively. Likewise, the ninety-ninth percentiles are 1.7 , 1.7 , 1.8 and 1.9 , respectively. For the 'Doubly corrected t ' statistic, the first percentiles are -4.8 , -3.2 , -2.8 and -2.5 , and the ninety-ninth percentiles are 2.1 , 2.2 , 2.3 and 2.3 , respectively. Naturally, these percentiles are estimates based on our simulations and are subject to sampling error. Moreover, as we shall see later, the percentiles in the extreme tails are quite sensitive to changes in the assumptions concerning the nature of the underlying distribution generating stock returns.

6. The relationship between hedge return, market return and return on other hedges

In this section we examine some aspects of the relationship between the return on the hedge and the return on the market. We also analyse some

Table 3a
 Simulated hedge returns (10000 simulations, 10 trading intervals): observed cumulative probability distribution.

Ratio of initial share price to exercise price = 1.00 (at the money)										
Range of statistic										
	< -4	< -3	< -2	< -1	< 0	< 1	< 2	< 3	< 4	
Raw t -statistic	0.0292	0.0611	0.1269	0.2614	0.5072	0.8901	0.9964	0.9999	1.0000	
Raw z -statistic	0.1145	0.1792	0.2745	0.3842	0.5072	0.6302	0.7457	0.8329	0.8892	
Johnson adjusted t	0.0240	0.1024	0.1024	0.2327	0.4879	0.8310	0.9874	0.9995	0.9999	
'Homoscedastic' t	0.0416	0.0803	0.1457	0.2836	0.5561	0.9062	0.9961	0.9999	1.0000	
Doubly corrected t	0.0151	0.0317	0.0802	0.2401	0.5355	0.8675	0.9884	0.9985	0.9992	
Ratio of initial share price to exercise price = 0.9 (out of the money)										
Range of statistic										
	< -4	< -3	< -2	< -1	< 0	< 1	< 2	< 3	< 4	
Raw t -statistic	0.0193	0.0459	0.1289	0.2989	0.5568	0.8948	0.9982	1.0000	1.0000	
Raw z -statistic	0.0167	0.0327	0.0621	0.1492	0.5568	0.8828	0.9365	0.9575	0.9718	
Johnson adjusted t	0.0191	0.0552	0.1396	0.2915	0.5422	0.8258	0.9888	0.9998	1.0000	
'Homoscedastic' t	0.0355	0.0716	0.1489	0.3073	0.5805	0.9036	0.9975	1.0000	1.0000	
Doubly corrected t	0.0118	0.0340	0.1001	0.2688	0.5618	0.8633	0.9870	0.9991	0.9950	
Ratio of initial share price to exercise price = 1.1 (in the money)										
Range of statistic										
	< -4	< -3	< -2	< -1	< 0	< 1	< 2	< 3	< 4	
Raw t -statistic	0.0191	0.0526	0.1325	0.2997	0.5488	0.8901	0.9978	1.0000	1.0000	
Raw z -statistic	0.0213	0.0394	0.0779	0.1829	0.5488	0.8427	0.9076	0.9391	0.9574	
Johnson adjusted t	0.0191	0.0574	0.1408	0.2934	0.5360	0.8251	0.9873	0.9998	1.0000	
'Homoscedastic' t	0.0380	0.0760	0.1487	0.2979	0.5622	0.9011	0.9958	1.0000	1.0000	
Doubly corrected t	0.0124	0.0352	0.0991	0.2615	0.5439	0.8533	0.9880	0.9991	0.9996	

Table 3b
Simulated hedge returns (10000 simulations, 20 trading intervals); observed cumulative probability distribution.

<i>Ratio of initial share price to exercise price = 1.00 (at the money)</i>												
Range of statistic												
	< -4	< -3	< -2	< -1	< 0	< 1	< 2	< 3	< 4			
Raw <i>t</i> statistic	0.0143	0.0366	0.0928	0.2289	0.5028	0.8772	0.9965	1.0000	1.0000			
Raw <i>z</i> statistic	0.0509	0.0983	0.1834	0.3243	0.5028	0.6817	0.8132	0.8925	0.9395			
Johnson adjusted <i>t</i>	0.0108	0.0248	0.0658	0.1963	0.4821	0.8229	0.9809	0.9996	1.0000			
'Homoscedastic' <i>t</i>	0.0221	0.0472	0.1046	0.2488	0.5446	0.8905	0.9963	1.0000	1.0000			
Doubly corrected <i>t</i>	0.0030	0.0123	0.0489	0.2027	0.5206	0.8468	0.9983	0.9995	0.9999			
<i>Ratio on initial share price to exercise price = 0.9 (out of the money)</i>												
Range of statistic												
	< -4	< -3	< -2	< -1	< 0	< 1	< 2	< 3	< 4			
Raw <i>t</i> statistic	0.0136	0.0385	0.1065	0.2574	0.5190	0.8836	0.9970	1.0000	1.0000			
Raw <i>z</i> statistic	0.0201	0.0438	0.0910	0.2065	0.5190	0.8189	0.9115	0.9509	0.9719			
Johnson adjusted <i>t</i>	0.0133	0.0362	0.0927	0.2349	0.4985	0.8239	0.9848	0.9995	1.0000			
'Homoscedastic' <i>t</i>	0.0229	0.0522	0.1184	0.2657	0.5572	0.8964	0.9969	1.0000	1.0000			
Doubly corrected <i>t</i>	0.0044	0.0166	0.0631	0.2224	0.5351	0.8524	0.9842	0.9995	0.9999			
<i>Ratio of initial share price to exercise price = 1.1 (in the money)</i>												
Range of statistic												
	< -4	< -3	< -2	< -1	< 0	< 1	< 2	< 3	< 4			
Raw <i>t</i> statistic	0.0118	0.0387	0.1044	0.2502	0.5311	0.8846	0.9968	1.0000	1.0000			
Raw <i>z</i> statistic	0.0201	0.0449	0.0920	0.2171	0.5311	0.7947	0.8914	0.9380	0.9643			
Johnson adjusted <i>t</i>	0.0111	0.0362	0.0888	0.2274	0.5105	0.8267	0.9833	0.9993	1.0000			
'Homoscedastic' <i>t</i>	0.0200	0.0496	0.1115	0.2531	0.5507	0.8915	0.9960	1.0000	1.0000			
Doubly corrected <i>t</i>	0.0034	0.0145	0.0593	0.2069	0.5282	0.8447	0.9862	0.9993	0.9999			

Table 3c
Simulated hedge returns (10000 simulations, 40 trading intervals); observed cumulative probability distribution.

Ratio of initial share price to exercise price = 1.00 (at the money)										
Range of statistic										
	< -4	< -3	< -2	< -1	< 0	< 1	< 2	< 3	< 4	
Raw <i>t</i> -statistic	0.0062	0.0238	0.0751	0.2195	0.5045	0.8733	0.9944	1.0000	1.0000	
Raw α statistic	0.0152	0.0445	0.1189	0.2633	0.5045	0.7463	0.8875	0.9507	0.9779	
Johnson adjusted <i>t</i>	0.0036	0.0132	0.0500	0.1866	0.4820	0.8231	0.9787	0.9992	1.0000	
'Homoscedastic' <i>t</i>	0.0089	0.0272	0.0824	0.2325	0.5361	0.8849	0.9936	1.0000	1.0000	
Doubly corrected <i>t</i>	0.0008	0.0056	0.0361	0.1889	0.5161	0.8458	0.9819	0.9994	1.0000	
Ratio of initial share price to exercise price = 0.9 (out of the money)										
Range of statistic										
	< -4	< -3	< -2	< -1	< 0	< 1	< 2	< 3	< 4	
Raw <i>t</i> statistic	0.0078	0.0261	0.0786	0.2164	0.5066	0.8771	0.9960	0.9998	1.0000	
Raw α statistic	0.0113	0.0322	0.0791	0.1975	0.5066	0.8146	0.9251	0.9656	0.9849	
Johnson adjusted <i>t</i>	0.0050	0.0180	0.0568	0.1879	0.4846	0.8266	0.9794	0.9993	1.0000	
'Homoscedastic' <i>t</i>	0.0111	0.0259	0.0843	0.2297	0.5387	0.8903	0.9952	0.9998	1.0000	
Doubly corrected <i>t</i>	0.0009	0.0067	0.0390	0.1876	0.5192	0.8505	0.9822	0.9990	0.9999	
Ratio of initial share price to exercise price = 1.1 (in the money)										
Range of statistic										
	< -4	< -3	< -2	< -1	< 0	< 1	< 2	< 3	< 4	
Raw <i>t</i> -statistic	0.0067	0.0226	0.0775	0.2164	0.5080	0.8732	0.9962	1.0000	1.0000	
Raw α -statistic	0.0112	0.0295	0.0774	0.1962	0.5080	0.7974	0.9097	0.9589	0.9803	
Johnson adjusted <i>t</i>	0.0051	0.0142	0.0552	0.1902	0.4861	0.8201	0.9786	0.9990	1.0000	
'Homoscedastic' <i>t</i>	0.0095	0.0243	0.0765	0.2189	0.5225	0.8801	0.9948	1.0000	1.0000	
Doubly corrected <i>t</i>	0.0005	0.0063	0.0367	0.1781	0.5005	0.8397	0.9802	0.9990	1.0000	

Table 3d

Simulated hedge returns (10000 simulations, 80 trading intervals); observed cumulative probability distribution.

<i>Ratio of initial share price to exercise price = 1.00 (at the money)</i>										
Range of statistic										
	< -4	< -3	< -2	< -1	< 0	< 1	< 2	< 3	< 4	
Raw <i>t</i> -statistic	0.0031	0.0146	0.0594	0.2059	0.5135	0.8678	0.9929	0.9999	1.0000	
Raw α -statistic	0.0026	0.0152	0.0573	0.1949	0.5135	0.8181	0.9387	0.9799	0.9932	
Johnson adjusted <i>t</i>	0.0015	0.0059	0.0376	0.1769	0.4949	0.8281	0.9759	0.9995	0.9999	
'Homoscedastic' <i>t</i>	0.0039	0.0164	0.0636	0.2131	0.5302	0.8723	0.9931	0.9999	1.0000	
Doubly corrected <i>t</i>	0.0002	0.0025	0.0290	0.1776	0.5117	0.8392	0.9802	0.9992	0.9999	
<i>Ratio of initial share price to exercise price = 0.9 (out of the money)</i>										
Range of statistic										
	< -4	< -3	< -2	< -1	< 0	< 1	< 2	< 3	< 4	
Raw <i>t</i> -statistic	0.0039	0.0159	0.0628	0.2043	0.5066	0.8788	0.9943	1.0000	1.0000	
Raw α statistic	0.0017	0.0097	0.0435	0.1668	0.5066	0.8525	0.9595	0.9871	0.9953	
Johnson adjusted <i>t</i>	0.0041	0.0082	0.0400	0.1739	0.4869	0.8344	0.9806	0.9988	1.0000	
'Homoscedastic' <i>t</i>	0.0037	0.0155	0.0630	0.2128	0.5332	0.8821	0.9926	1.0000	1.0000	
Doubly corrected <i>t</i>	0.0001	0.0033	0.0297	0.1772	0.5139	0.8492	0.9799	0.9988	1.0000	
<i>Ratio of initial share price to exercise price = 1.1 (in the money)</i>										
Range of statistic										
	< -4	< -3	< -2	< -1	< 0	< 1	< 2	< 3	< 4	
Raw <i>t</i> -statistic	0.0047	0.0147	0.0605	0.2049	0.4952	0.8630	0.9943	1.0000	1.0000	
Raw α -statistic	0.0027	0.0095	0.0446	0.1588	0.4952	0.8337	0.9497	0.9844	0.9943	
Johnson adjusted <i>t</i>	0.0024	0.0087	0.0380	0.1744	0.4767	0.8192	0.9755	0.9994	1.0000	
'Homoscedastic' <i>t</i>	0.0046	0.0168	0.0621	0.2062	0.5102	0.8631	0.9934	0.9999	1.0000	
Doubly corrected <i>t</i>	0.0004	0.0049	0.0300	0.1745	0.4927	0.8304	0.9783	0.9992	1.0000	

consequences of the formation of portfolios of hedges and comment on whether this procedure is effective in the reduction of risks associated with discrete hedging.

Suppose that the return on the stock is given by

$$\Delta S/S = m\Delta t + \sigma u\sqrt{\Delta t}, \quad (13)$$

(once again, this is an approximation to a log-normal diffusion with higher-order terms in Δt neglected; the expression becomes exact as Δt tends to zero), and that the return on the market is given by

$$\Delta M/M = \alpha_M \Delta t + \sigma_M w\sqrt{\Delta t}, \quad (14)$$

where u and w are jointly normally distributed unit normal variates. If the correlation coefficient between u and w is ρ , then we may rewrite u as

$$u = \rho w + v\sqrt{(1 - \rho^2)}, \quad (15)$$

where v is yet another unit normal variate independent of w . Then the hedge return $\lambda y \Delta t$ or $\lambda(u^2 - 1)\Delta t$ may be rewritten as

$$HR = \lambda \Delta t (\rho^2 w^2 + 2\rho(\sqrt{(1 - \rho^2)})wv + (1 - \rho^2)v^2 - 1) \Delta t. \quad (16)$$

Clearly this is not independent of what happens to the market as it depends directly on w . Nowever, the covariance with the market return is

$$\begin{aligned} \text{cov}(HR, \Delta M/M) &= [E((\rho^2 w^2 + 2\rho(1 - \rho^2)wu \\ &\quad + (1 - \rho^2)v^2 - 1)w)] \Delta t^{3/2} \\ &= [\rho^2 E(w^3) + 2\rho(\sqrt{(1 - \rho^2)})E(w^2 v) \\ &\quad + (1 - \rho^2)E(v^2 w) - E(w)] \Delta t^{3/2} \\ &= 0 \quad (\text{since all terms are zero}). \end{aligned}$$

Thus, when we ignore higher-order terms of Δt , we see that the hedge return is uncorrelated with the market return.

Using the same type of approach it can be easily shown that

$$\text{cov}[(u^2 - 1), (w^2 - 1)] = 2\rho^2.$$

Consequently when hedges are constructed on two stocks whose returns are jointly distributed with correlation coefficient ρ , the hedge returns have correlation ρ^2 .

Now, if correlation between stock returns arises through their common dependence on a single market index, it is well known that asymptotically as the number of securities tends to infinity the return on an equal-weighted portfolio becomes perfectly correlated with the return on the index. Empirically, one finds that the standard deviation of return on a randomly selected equal-weighted portfolio of common stocks rapidly approaches about one-half the standard deviation of a single stock's return as the number of stocks held increases. Because of the above result, the return on a similarly constructed portfolio of option hedges will, in the limit, be perfectly correlated with the square of the index return. With no pretence either of rigor or of strict arithmetic correctness, we assert that diversification among option hedges will reduce the standard deviation of return on the portfolio of option hedges to about one-quarter of the standard deviation of return on a single hedge.

Thus the conclusions of this section are as follows. The return on an individual hedge portfolio is uncorrelated with the return on the market, but the return on the hedge is not independent of the return on the market. If a portfolio of hedges is constructed, the return on the portfolio is uncorrelated with the market. However the returns on the individual hedges in the portfolio are not uncorrelated with each other. It is sometimes claimed in empirical work that the formation of portfolio of option hedges eliminates the risks associated with discrete hedging. We would suggest that this procedure may reduce such risks but that it does not eliminate them.

7. Relaxation of Black and Scholes assumptions

The foregoing mixture of formal and informal analysis relies heavily on the assumptions that:

- (i) the variance rate of return on the stock is known;
- (ii) the daily stock returns are normally distributed (strictly speaking we mean log-normally distributed; however, practically speaking there is no perceptible difference between the two on a daily basis).

Relaxing these assumptions will change our conclusions to some extent.

For example, suppose that investors and the overall market are learning about the variance rate of return on the stock by observing realised daily returns. Then, assuming some type of Bayesian updating of variance estimates, a large rise or fall in stock price will lead to an upward revision in the variance estimate and hence to an upward revision of the option pricing function, as this function increases with variance. (Note, of course, that in this alternative world the option pricing function need not be the Black-Scholes function.) Then the hedge value, in addition to rising because of the large value of y , will also rise because of the revision of variance estimates.

Table 4
Simulated hedge returns, $t(5)$ distributed returns (10,000 simulations); observed cumulative probability distribution

Ratio of initial share price to exercise price = 1.00 (at the money)												
10 trading intervals												
	Range of statistic											
	< -4	< -3	< -2	< -1	< 0	< 1	< 2	< 3	< 4			
Raw t -statistic	0.0664	0.1226	0.2120	0.3641	0.5841	0.9277	0.9996	1.0000	1.0000			
Raw α -statistic	0.1988	0.2779	0.3798	0.4799	0.5841	0.6782	0.7550	0.8164	0.8620			
Johnson adjusted t	0.0573	0.0952	0.1663	0.3288	0.5646	0.8561	0.9665	0.9999	0.9999			
'Homoscedastic' t	0.0933	0.1496	0.2383	0.3935	0.6345	0.9364	0.9992	1.0000	1.0000			
Doubly corrected t	0.0325	0.0583	0.1294	0.3276	0.6063	0.8803	0.9933	0.9975	0.9981			
20 trading intervals												
	Range of statistic											
	< -4	< -3	< -2	< -1	< 0	< 1	< 2	< 3	< 4			
Raw t -statistic	0.0415	0.0884	0.1789	0.3351	0.5772	0.9240	0.9994	1.0000	1.0000			
Raw α -statistic	0.1111	0.1863	0.2953	0.4375	0.5772	0.7051	0.7939	0.8549	0.8951			
Johnson adjusted t	0.0281	0.0559	0.1223	0.2852	0.5547	0.8458	0.9953	1.0000	1.0000			
'Homoscedastic' t	0.0582	0.1096	0.2015	0.3566	0.6155	0.9315	0.9995	1.0000	1.0000			
Doubly corrected t	0.0088	0.0254	0.0867	0.2837	0.5846	0.8680	0.9936	0.9991	0.9994			

40 trading intervals

	Range of statistic								
	< -4	< -3	< -2	< -1	< 0	< 1	< 2	< 3	< 4
Raw t -statistic	0.0318	0.0675	0.1507	0.3045	0.5655	0.9159	0.9993	1.0000	1.0000
Raw z -statistic	0.0518	0.1104	0.2168	0.3770	0.5655	0.7259	0.8287	0.8904	0.9282
Johnson adjusted t	0.0145	0.0353	0.0935	0.2570	0.5397	0.8391	0.9900	0.9999	1.0000
'Homoscedastic' t	0.0380	0.0805	0.1595	0.3260	0.5884	0.9214	0.9997	1.0000	1.0000
Doubly corrected t	0.0028	0.0143	0.0734	0.2615	0.5643	0.8534	0.9901	1.0000	1.0000

80 trading intervals

	Range of statistic								
	< -4	< -3	< -2	< -1	< 0	< 1	< 2	< 3	< 4
Raw t -statistic	0.0205	0.0526	0.1286	0.2855	0.5619	0.9087	0.9988	1.0000	1.0000
Raw z -statistic	0.0157	0.0534	0.1359	0.3104	0.5619	0.7719	0.8788	0.9335	0.9603
Johnson adjusted t	0.0069	0.0220	0.0724	0.2426	0.5360	0.8381	0.9891	0.9999	1.0000
'Homoscedastic' t	0.0236	0.0573	0.1335	0.2943	0.5743	0.9116	0.9995	1.0000	1.0000
Doubly corrected t	0.0011	0.0096	0.0593	0.2394	0.5494	0.8451	0.9896	0.9999	1.0000

Conversely, a small change in stock price will cause variance estimates to fall and hedge return will also fall. It can thus be seen that this type of variance learning mechanism will tend to exacerbate the skewed nature of hedge returns.

On the other hand, if individual investors have seriously misestimated the variance rate of return that is generally expected on the stock, then it is likely that those investors' option value estimates and hedge ratios will disagree markedly with market estimates. Consequently, day-to-day hedge returns based on observed market prices will be far from riskless. In fact, in cases of serious misestimation, direct dependence on ΔS will tend to swamp any other components of hedge return. In that case, the discussion here will appear to have been somewhat irrelevant. However, if nothing else, a simple check on the skewness and homoscedasticity of hedge returns costs little if the results are negative and is quite informative if the results are positive.

Finally, there is considerable evidence that suggests that daily stock returns are too leptokurtic to be normally distributed. A possible alternative that has been suggested is that daily returns are drawn from a t -distribution with five degrees of freedom. Because of the law of large numbers, this would effectively imply a lognormal return distribution for any period longer than a few days. Therefore, in spite of the different assumed behavior of daily stock price changes, it might well be inferred that the Black-Scholes option pricing formula should still apply (with minor reservations). If this is so, then daily hedge returns will be even more highly skewed than before and will be negative roughly 75% of the time. Still greater caution in evaluating crude t -statistics would then seem appropriate and it would then seem even more imperative either to adjust the t -statistics for skewness or to use the proper distribution of the t -statistics given the assumed return generating process.

Table 4 shows the effect on our results of changing the assumed distribution generating stock returns. Instead of a lognormal distribution of stock prices, we assumed that daily returns were drawn from a t -distribution with five degrees of freedom and with equivalent mean and variance. Otherwise, everything is the same as described in section 5.

As can be seen, the reported t -statistics appear to be even more significant. Unbelievably significant t -statistics (i.e., in the range of -6 to -10 or even lower) are relatively common even with eighty trading days. Given these results, we find it hard to accept as valid (without further explanation) any empirical work that does not explicitly check for possible bias, no matter how impressive the reported t -statistics may sound.

The first percentiles for the 'Raw t ' statistic are -7.3 , -5.8 , -5.3 and -4.8 , for ten, twenty, forty and eighty trading intervals, respectively. The ninety-ninth percentiles are 1.5, 1.5, 1.6 and 1.6, respectively. For the 'Doubly corrected t ' statistic, the first percentiles are -7.3 , -3.9 , -3.2 and -3.0 , and the ninety-ninth percentiles are 1.8, 1.8, 2.0 and 2.0, respectively.

8. Discrete hedging in empirical work

In order to examine the components of the return on the hedging portfolio when empirical observations are used it is convenient to decompose the components of the hedge return. The approach adopted is similar to one suggested by Galai (1975). We assume that a hedge portfolio is constructed at time $(t-1)$ and liquidated at time t . The notation is as follows:

- c_{t-1}^A = actual (market) option price at time $t-1$,
- c_{t-1}^M = model price at time $t-1$,
- c_t^A = actual price at time t ,
- c_t^M = model price at time t ,
- S_{t-1} = stock price at time $t-1$,
- S_t = stock price at time t .

Suppose that the hedge is constructed by buying one call option (at market price) at time $t-1$ and going short in the stock. The initial investment is

$$c_{t-1}^A - N(d_1)S_{t-1}, \quad (17)$$

where d_1 is evaluated at time $(t-1)$, and the hedge return is given by

$$c_t^A - c_{t-1}^A - N(d_1)[S_t - S_{t-1}] - r\Delta t\{c_{t-1}^A - N(d_1)S_{t-1}\}, \quad (18)$$

where although $\Delta t=1$ in this notation we have retained Δt to preserve generality. For a number of reasons the hedge return given by (18) need not have the same distributional characteristics as the hedge returns analysed earlier. These differences can arise on account of departures from the Black-Scholes assumptions. It is also worth noting that the amount of the equity in the hedge position changes so that part of the variation in the dollar returns arises from this source of variation.

Eq. (18) can be rewritten in the form

$$\begin{aligned} & c_t^M - c_{t-1}^M - N(d_1)(S_t - S_{t-1}) - r\Delta t(c_{t-1}^M - N(d_1)S_{t-1}) \\ & - (F_t - F_{t-1}) + r\Delta tF_{t-1}, \end{aligned} \quad (19)$$

where $F_t = c_t^M - c_t^A$.

The first line of (19) gives the hedge returns on the assumption of model prices whilst the second line gives the correction for deviation between the model and the market prices. Notice that we could further analyse the hedge return by allowing for deviations between the stock price S_t found in the

market and the stock price that would be generated from S_{t-1} under the assumptions of lognormality. This point is not pursued here.

Galai has analysed some empirical option data by decomposing the elements of the hedge return using a form similar to eq. (19). He finds that the contribution from the $(F_t - F_{t-1})$ term swamps the distribution in terms of its contribution to the mean and variance of the hedge return. At first blush this might seem to render much of our previous analysis redundant. We suggest that this is not the case. First a number of tests of the option pricing model involve buying and selling at model prices. In this case the $(F_t - F_{t-1})$ term vanishes and so the distribution of the hedge returns can be analysed as we have suggested. In particular the pioneering tests of Black and Scholes were carried out under conditions where there was no secondary market and they used model prices for 'daily' rebalancing. Second we suggest that our procedures for removing the variation due to the changing amount of the equity in the hedged position and for adjusting for skewness should lead to a clearer picture of the distribution of hedge returns. Third we contend that the whole question of the formation of hedged portfolio has to be examined carefully. It is possible for the $(F_t - F_{t-1})$ terms to dominate the return distribution of a single hedge but if these terms are approximately uncorrelated across hedges the dominant contribution to the distribution of the return on a portfolio of hedges may come from the batch of terms in (19). It should prove interesting to analyse some of these considerations in the context of empirical data.

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