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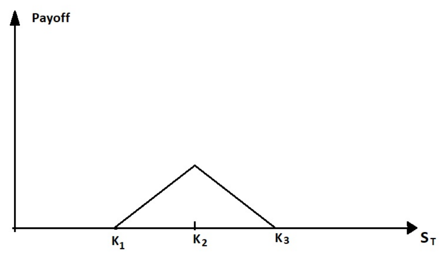
Financial Mathematics -DAV Supplement

Exercise Sheet

Exercise 1
An investor sells a European call option with strike price of K and maturity T and buys a European put option with the same strike price and maturity.
Describe the investor's position.

Exercise 2
Suppose that European call options on a stock with strike prices \$ 30 and \$ 35 cost \$ 3 and \$ 1, respectively.
(a) How can these options be used to create a bull spread?
(b) Construct a table that shows the profit and payoff this strategy.

Exercise 3
Let us consider the following payoff diagram:



Find the formal representation of the payoff using calls and puts. Here $K_2 = \frac{1}{2}(K_1+K_3)$.

Exercise 4
A trader buys two July futures contracts on orange juice. Each contract is for the delivery of 15,000 pounds. The current futures price is 160 cents per pound, the initial margin is \$ 6,000 per contract, and the maintenance margin is \$ 4,500 per contract.
(a) What price change would lead to a margin call?
(b) Under what circumstances could \$ 2,000 be withdrawn from the margin account?

Exercise 5 Let us consider a forward contract to buy one unit of a stock S with expiration date $T = 180$ days (1 year $\widehat{=}$ 360 days), and delivery price $K = 28$ EUR. Assume that until the expiration date no dividend is paid on this stock. Moreover, let $S(0) = 25$ EUR and assume that the continuous rate of a bank account is $r = 10\%$ per year.

a) Show that there exists an arbitrage opportunity in this model.

b) Find an arbitrage strategy.

c) For which K is there no possibility to achieve a riskless profit?

Exercise 6

Let us consider a long position in a forward contract to buy one unit of a stock S with expiration date $T = 90$ days (1 year $\widehat{=}$ 360 days). Assume that until the expiration date no dividend is paid on this stock. Assume that the current value of the stock is $S(0) = 40$ EUR, and the continuous rate of a bank account is $r = 5\%$ per year.

a) Find a T-Forward-price at time $t = 0$. What is the value of the forward at $t = 0$?

b) Assume that the delivery price of this forward contract is $K = 43$ EUR. How much should we (long position) pay for this contract?

Exercise 7

A company entered into a forward contract 3 months ago in order to buy a stock S. The remaining term of this forward contract is 100 days and the delivery price of the contract is 50.25 EUR. For some reasons the company does not need this stock any more and enters into a new forward contract in order to sell this stock in 100 days. At this moment the value of the stock is $S(0) = 45$ EUR and a riskless continuous interest rate $r = 4.75\%$ per year (1 year $\widehat{=}$ 360 days).

a) Calculate the T-Forward price (delivery price) of the new contract.

b) Calculate the overall position at maturity date of both contacts.

c) What is the value of the overall position today?

Exercise 8

Assume that a company wants to hedge its portfolio at time $t = 1$ by selling a futures contract. The portfolio consists of 100 units of a stock S. Today's price of the stock is $S(0) = 100$ EUR. The variance of the stock price at $t = 1$ equals 100. At any time the money can be put on or withdrawn from a bank account with a riskless (continuous) interest rate $r = 4\%$. All futures contracts considered below have maturity 1 and consist of one unit of the suitable underlying. All marking-to-market effects are not taken into consideration.

a) Assume first that the stock S is also the underlying of the futures contract. Calculate the T-Futures price at time $t = 0$.

b) Assume that on the market exist only futures on the underlying S_2. The current value of the underlying S_2 is 200 EUR, the variance at time $t = 1$ equals 400 (i.e. $\mathrm{Var}(S_2(1)) = 400$) and $\mathrm{Cov}(S(1), S_2(1)) = 180$. Give the number of futures in the minimal variance hedge.

Exercise 9

Assume that a company wants to hedge its portfolio at time $t = 1$ by selling a futures contract. The portfolio consists of 100 units of a stock S. The current price of the stock is $S(0) = 150$ EUR. The variance of the stock price at $t = 1$ equals 60. On the market exist only futures on the underlying S_2. The current value of the underlying S_2 is 230 EUR, the variance at time $t = 1$ equals 500 and $\mathrm{Cov}(S(1), S_2(1)) = 220$. At any time the money can be put on or withdrawn from a bank account with a riskless interest rate $r = 4\%$. All futures contracts considered below have maturity 1 and consist of one unit of the suitable underlying. All marking-to-market effects are not taken into consideration.

a) How many futures should one sell in the minimal variance hedging?

b) How many futures should one sell in the minimal variance hedging in case that the maturity is $T = 4$?

Exercise 10

Companies A and B have been offered the following rates per annum on a \$20 million 5-year loan:

	Fixed rate	Floating rate
Company A:	5.0%	LIBOR+0.1%
Company B:	6.4%	LIBOR+0.6%

Company A requires a floating-rate loan; company B requires a fixed-rate loan. Design a swap that will net a bank, acting as intermediary, 0.1% per annum and that will appear equally attractive to both companies.

Exercise 11

Consider a receiver forward swap with fixed interest rate $K = 5\%$, notional principal $N = 10$M, maturity 3 years and annual payment. The continuous spot rate curve today at time $t = 0$ is given by

T	1	2	3	4	5
R(0,T)	4,62%	4,65%	4,70%	4,75%	4,79%

a) Calculate the value of the swap at time $t = 0$ and the forward swap rate $S_{0,3}(0)$.

b) Calculate the value of a 3x2-swap, i.e. a receiver forward swap with $t_0 = 3$, $t_1 = 4$ and $t_2 = 5$, at time $t = 0$ with $K = 5\%$, $N = 10$M. How should we choose K in order to the forward swap has value 0 at time $t = 0$.

Exercise 12

Consider a receiver forward swap with fixed interest rate $K = 5\%$, notional principal $N = 10$M EUR, maturity 5 years and annual payment. The continuous spot rate curve today at time $t = 0$ is given by

T	1	2	3	4	5	6	7	8
R(0,T)	4,62%	4,65%	4,70%	4,75%	4,79%	4,82 %	4,82%	4,82%

a) Calculate the forward interest rates $F(0, t_i, t_{i+1})$.

b) Calculate the simple spot rates $L(0, t_i)$.

c) Calculate the value of the swap at time $t = 0$ and the forward swap rate $S_{0,5}(0)$.

d) Calculate the value of a 6x2-swap, i.e. a receiver forward swap with $T_0 = 6$, $T_1 = 7$ and $T_2 = 8$, at time $t = 0$ with $K = 5\%$, $N = 10$M EUR. How should we choose K in order to the forward swap has value 0 at time $t = 0$.