Prof. Dr. Karsten Urban
Kristina Steih

Institute for Numerical Mathematics
Ulm University
Summer Term 2014

Numerical Finance – Overview

IMPORTANT: The following questions are meant for you to check if you have understood the contents of the lecture. However, these are **not** exam questions. In the exam, we expect you to

- show that you have understood the different ideas/techniques (that's what you can check here),
- know the definitions and theorems (to the point of being able to write them down),
- be able to explain the different numerical techniques, their implementations and their behaviour (what we did in the exercises).

Generation of Random Numbers

There are two main steps in the generation of pseudo-random numbers with arbitrary distributions:

1. Generation of uniformly distributed numbers.
   - What methods are there?
   - Why is the choice of parameters in linear congruential generators crucial?
   - How can a generator be tested?
   - What is the general difference between statistical tests and the notion of discrepancy to ensure the uniform distribution of a sequence?
2. Transformation of uniformly distributed numbers to obtain other distributions.
   - What general methods are there?
   - How can normally distributed random variables be generated?

Numerical Cubature

1. Product Formulas
   - What is meant by the *curse of dimensionality*?
2. Monte Carlo Methods:
   - MC techniques are said to break the curse of dimensionality. Why?
   - What are advantages/disadvantages of the MC method?
   - What can be improved? How?
3. Quasi-Monte Carlo Methods:
   - What are low-discrepancy sequences? What may be their advantages in comparison with grids?
   - What is the idea behind Quasi-Monte Carlo integration? Why is the Koksma-Hlawka inequality so important in this context?

- QMC methods can accelerate the Monte Carlo convergence rate $O(\frac{1}{\sqrt{n}})$ to nearly $O(\frac{1}{n})$. Explain. What does the convergence rate depend on?
- What are (t,m,j)-nets? How do they relate to general low discrepancy sequences?
- What is the role of the parameters t and b in (t,m,j)-nets? Is it better for t and b to be large or small?

4. Sparse Grids (Smolyak Algorithm)
   - What is the difference between the Smolyak quadrature and the standard product formula?
   - What do we know about the integration error? Compare with the results for QMC integration.
   - Explain Figure 3.3.
   - Does the curse of dimensionality play a role in Smolyak methods?

Basis notions in financial mathematics

- What is a Wiener Process?
- What are SDEs? Explain the notion of strong/weak solutions. When do strong solutions exist?
- What do we need the Itô formula for?
- What is the statement of the Feynman-Kac theorem? Why is it so important?

Numerical Valuation of European Options

- Explain the Binomial Tree method.
- Discuss the assumptions of the binomial method. Where exactly do they come into play?
- What's the idea of the derivation of the BS-PDE? What's the idea behind the proof of the existence of a solution?
- The Black-Scholes formula is an easy way to calculate option prices - what do we need numerical methods for?

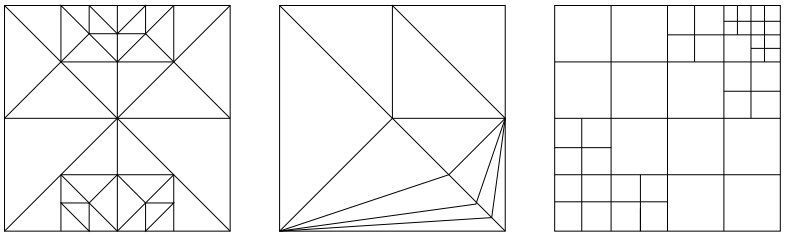
Stochastic Differential Equations

- What is the idea behind the Euler-Maruyama method? Compare with the deterministic approach.
- Explain the different approaches to measure the error of numerical solutions of SDEs.
- Explain strong convergence and consistency.
- Explain weak convergence and consistency.
- What do we know about the Euler method?
- How can higher order methods be obtained?
- How to you use SDEs to price options? What error components are there? Which ones would you focus on when improving your pricing method?

Numerical Methods for PDEs

1. Finite Difference Methods
   - Explain the idea of Finite Difference Methods.
   - What does one know about consistency, stability and convergence for FD methods? What assumptions are necessary?
   - Explain Figure 7.1.
2. Finite Element Methods

   General Questions / Elliptic Setting:
   - What is the problem with the classical formulation of PDEs, i.e., what do we need variational formulations for?
   - What is meant by the variational formulation of a PDE? Also explain the definitions of a weak derivative and the Sobolev Space.
   - What do we know about the existence of weak solutions? When do they correspond to our classical solutions?
   - What is the idea behind the Galerkin method? Explain the importance of Céa's Lemma.
   - What do we know about the approximation error? Compare with the results for FD.
   - What is a nodal basis? What is the Courant Finite Element? How are the corresponding entries in the stiffness matrix derived?
   - Why should one use reference elements? How is the assembly process implemented?
   - Are these grids admissible / regular / (quasi-) uniform ?



   Parabolic Problems:
   - What are Bochner spaces?
   - What semidiscretization methods are there? What is a general θ-scheme?
   - What do we know about stability and error convergence?

American Option Pricing

- Explain how the Binomial Tree method has to be modified to solve American option pricing problems.
- Why are American option pricing problems instances of free-boundary problems? What does this mean for finding a solution via the Black-Scholes PDE?
- Write down the American option pricing problem in the form of an obstacle problem (7.2.1). How are the explicit constraints avoided?
- How does one approach obstacle problems with Finite Differences?

- What does the variational formulation of the model obstacle problem look like (and why is this so)?
- Summarize the classical iterative methods for the solution of $Ax = b$. What is meant by a SOR-method?
- How does the SOR method have to be changed to solve complementary problems of the form $(Au - f)^T(u - g) = 0$, $Au \geq f$, $u \geq g$?
- What are the main steps in the proof of the convergence theorem?

Exotic Options

- What particular problem has to be solved to obtain prices for Asian options? Why is this difficult?
- What is the Peclét number? Can the Black-Scholes equation be convection dominated?
- What is the idea of the SUPG method?

Lévy finance

- What are Lévy processes and why are they used in financial modeling? What are the components of the characteristic triplet?
- What is the connection with (Feller) semigroups?
- What are the challenges for MC methods? What are the main ideas?
- How can the FFT be used for pricing in this setting?
- What is a PIDE?