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Mathematics of Games

Sample Exam - Total: 60 Points

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1. Each of n players gets a unique marker color. Now each player $i \in \{1, \dots, n\}$ simultaneously chooses his position: a real number in a unitary square: $x_i \in S = [0,1] \times [0,1]$. All points in the unitary square S are colored: Each point $y \in S$ gets player i's marker color for that i with x_i closest to y, and if y has the same distance to more than one player's position x_i, its color is determined at random (uniformly). If one position x_i is chosen by more than one player, the points with minimum distance to x_i are colored randomly (uniformly) with one of those players' marker colors. Each player wants to color a largest possible part of the interval with his marker color.

   (a) If there are two players, does a pure-strategy Nash equilibrium exist? If so, give one and if not, explain why not. (5 points)

   (b) If there are three players, does a pure-strategy Nash equilibrium exist? If so, give one and if not, explain why not. (5 points)

2. Suppose there are n firms in the Cournot oligopoly model. Let q_i denote the quantity produced by firm i, and let $Q = q_1 + \ldots + q_n$ denote the aggregate quantity on the market. Let P denote the market-clearing price and assume that inverse demand is given by $P(Q) = a - Q$ (assuming $Q < a$, else $P = 0$). Assume that the total cost of firm i from producing quantity q_i is $C_i(q_i) = cq_i$. That is, there are no fixed costs and the marginal cost is constant at c, where we assume $c < a$.

   (a) Following Cournot, suppose that the firms choose their quantities simultaneously. What is the Nash equilibrium? What happens as n approaches infinity?

   (b) Following Cournot, suppose that firm 1 chooses q_1; (2) firms $2 \leq i \leq n$ observe q_1 and then simultaneously choose q_i. What is the subgame perfect Nash equilibrium?

   (10 points)

3. (a) Compute the pure and mixed strategy Nash equilibria and the corresponding payoffs in the "chicken game" (coward game) as shown in the figure below. Two cars drive into a bridge in a valley. If both dare and cross the bridge simultaneously, both crash and fall into the valley. If both chicken and do not cross, both gain nothing. However, if one crosses after the other, both win, where the first one crossing earns more than the second one. The payoffs are as in the following table. Give all Nash Equilibria, and draw a best response graphic containing all of them.

	T	S
T	-1,-1	2,1
S	1,2	0,0

(b) Let $\delta = \frac{9}{10}$, and consider the infinitely repeated game based on the stage game from (a), with discount factor δ. Find a pure-strategy subgame-perfect Nash equilibrium s of the infinitely repeated game, for which the corresponding average discounted payoffs

$$(1-\delta) \cdot \sum_{t=0}^{\infty} \delta^t g_i(s^t(h^t))$$

for both players $i = 1, 2$ are higher than 3 times the mixed strategy payoff of the stage game minus $\frac{1}{19}$.

(c) Perturb this complete information game into a incomplete information game with continuum type space, so that when the type space tends to a point, the Bayesian Nash Equilibrium tends to the mixed-strategy Nash Equilibrium.

(15 points)

4. Consider the alsacian restaurant game where a guest is a french type from Paris with probability 0.5 or a german type from Munich with probability 0.5. The german would rather have a sausage, while the french would rather have a fois gras. The french waiter loves to have a fight with other french, specially from Paris, and in particular even more if the french ask a sausage. On the other hand, the survivor instinct of the french waiter tells him it is better not to challenge the well-built german. The payoffs are as in the table below. Give all pooling and separating Perfect Nash Equilibria. Which Equilibria are eliminated by the Intuitive Criterion and which survive it?

(a_1, a_2)	French	German
$(Sausage, not)$	2,0	3,1
$(Sausage, duel)$	1,2	0,0
$(FoieGras, not)$	3,0	2,2
$(FoieGras, duel)$	0,1	1,0

(5 points)

5. Consider a first-price, sealed-bid auction in which the bidders valuations are independently and uniformly distributed on [0,1]. Which are the bidders' payoffs? Show that for two bidders, the strategy of bidding $\frac{1}{2}$ times one's valuation is a symmetric Bayesian Nash Equilibrium of this auction. (10 points)

6. Prove that the infinite-horizon Rubinstein bargaining game has a unique subgame-perfect Nash equilibrium. Use the continuation payoffs as seen in the lecture and, as in the lecture, use the following notation. Let $\underline{v}_i$ and $\overline{v}_i$ be player i's lowest and highest continuation payoffs in any perfect equilibrium of any subgame that begins with player i making an offer. Let $\underline{w}_i$ and $\overline{w}_i$ be player i's lowest and highest continuation payoffs in any perfect equilibrium of any subgame beginning with an offer by player j.
(10 points)