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Mathematics of Games

Exercise session 10

08.07.2013, 12pm-2pm, N24-H15

Hand-in in PAIRS, before class starts!

1. Consider the following signaling game: an entrepreneur needs outside financing of I to expanding. The entrepreneur has private information about the profitability of the existing company, either high ($\theta = H$) with probability $1 - p$ or low ($\theta = L$, where $H > L > 0$) with probability p. The financing of I would add an extra profit of R to it. The investor's alternative of return is r, and $R > I(1 + r)$. Suppose that the entrepreneur offers a potential investor an equity stake (i.e., a share) of s in the firm in exchange of the necessary financing, which the investor may reject or accept. What should be the equity stake of s? Give both a pooling as well as a separating Perfect Bayesian Nash Equilibrium.

2. In the following signaling game, the first player is a hotel guest in Bavaria, of private type t either *german* with probability 0.9 or *brazilian* with probability 0.1, known by the first player but not the second one. The german type prefers to have its national drink (*beer*) at breakfast, while the brazilian type prefers to have its national drink (*coffee*) at breakfast, both types preferring not to duel with the hotel breakfast waiter, the second player, rather than receiving its preferred breakfast drink. The hotel breakfast waiter, on the other hand, may *duel* or *not* with the hotel guest's choice, and it would prefer to duel with the skinny brazilian type but not to duel with the strong german type. These preferences may be described in the following table, the first integer representing the payoff for the first player, and the second integer representing the payoff for the second player. For instance, if the waiter decides not to duel, if the brazilian guest type had asked for coffee or if the german guest type had asked for beer, the guest will receive a payoff of 3 while the waiter will receive a payoff of 0. Which are the Perfect Bayesian Nash Equilibria?

(a_1, a_2)	$t =$ Brazilian	$t =$ German
$(Coffee, Duel)$	1,1	0,-1
$(Coffee, Not)$	3,0	2,0
$(Beer, Duel)$	0,1	1,-1
$(Beer, Not)$	2,0	3,0

3. Is there any mixed-strategy Perfect Bayesian Nash Equilibrium in Exercise 4 from Sheet 9?

4. Draw a game tree for the following extensive-form game and give all pure-strategy Perfect Bayesian Nash Equilibria. Is there any mixed strategy Perfect Bayesian Nash Equilibrium?

1 Player 1 chooses a strategy a_1 from the set $A_1 = \{L, M, R\}$ of feasible strategies, where R ends the game with payoff of 1 for player 1 and of 3 for player 2.

2 Player 2 observes if $a_1 = R$ and if not, he believes that player 1 played L with probability p and M with probability $1 - p$ and chooses a strategy a_2 from the feasible set $A_2 = \{L', R'\}$ which ends the game with payoffs given in the table below.

(a_1, a_2)	Player 1	Player 2
(L, L')	2	1
(L, R')	0	0
(M, L')	0	2
(M, R')	0	1