1. Consider the following 2-period bargaining game between a firm and a union bargaining over wages. The firm’s profit, denoted by \( \pi \), is uniformly distributed in \([0, \pi^*]\), but the true value of \( \pi \) is privately known by the firm. If not employed, union members earn nothing. In the first period, the union makes a wage offer, \( w_1 \). If the firm accepts it, then the game ends: the union’s payoff is \( w_1 \) and the firm’s is \( \pi - w_1 \). If the firm rejects this offer then the game proceeds to the second period. The union makes a second wage offer, \( w_2 \). If the firm accepts it, then the present values of the payoffs are \( \delta w_2 \) for the union and \( \delta (\pi - w_2) \) for the firm, where \( \delta \) is the discount factor. If the firm rejects the union’s second offer then the game ends and payoffs are zero for both. Give a Perfect Bayesian Equilibrium. \[10 \text{ points}\]

2. Consider the following two-players signaling game, where the Sender’s type \( \theta \in [0, 1] \) is picked according to an uniform distribution, and the receiver action space as well as the sender message space are also both equal to the interval \([0, 1]\). The receiver’s payoff function is \( u_r(t, m) = -(m - t)^2 \) and the sender’s is \( u_s(t, a) = -(a - (t + b))^2 \), so when the sender’s type is \( t \), the receiver’s optimal action is \( a = t \) but the sender’s optimal message is \( m = t + b \). Suppose all types in the interval \([0, x_1]\) send one message while those in \([x_1, 1]\) send another. For an equilibrium to exist, which range should \( b \in [0, 1] \) have? Give lower and upper bounds for \( b \) within \([0, 1]\). Given \( b \), which value should \( x_1 \) have? \[5 \text{ points}\]

3. Consider the same signaling game but now with \( n \geq 2 \) intervals: \([0, x_1\), \ldots, \([x_k, x_k + 1]\), \ldots, \([x_{n-1}, 1]\), where \( 0 \leq k \leq n - 1 \), \( x_0 = 0 \) and \( x_n = 1 \). Given \( b \), how much larger than its precedent should an interval be for an equilibrium to exist? Then, given \( b \), for an equilibrium to exist, which upper bound should \( n \) have? \[5 \text{ points}\]