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Mathematics of Games

Exercise session 10

14.07.2014, 12pm-14pm, N24-H14

Hand-in IN PAIRS!

1. Consider the following 2-period bargaining game between a firm and a union bargaining over wages. The firm's profit, denoted by π, is uniformly distributed in $[0, \pi^*]$, but the true value of π is privately known by the firm. If not employed, union members earn nothing. In the first period, the union makes a wage offer, w_1. If the firm accepts it, then the game ends: the union's payoff is w_1 and the firm's is $\pi - w_1$. If the firm rejects this offer then the game proceeds to the second period. The union makes a second wage offer, w_2. If the firm accepts it, then the present values of the payoffs are δw_2 for the union and $\delta(\pi - w_2)$ for the firm, where δ is the discount factor. If the firm rejects the union's second offer then the game ends and payoffs are zero for both. Give a Perfect Bayesian Equilibrium.

[10 points]

2. Consider the following two-players signaling game, where the Sender's type $\theta \in [0, 1]$ is picked according to an uniform distribution, and the receiver action space as well as the sender message space are also both equal to the interval $[0, 1]$. The receiver's payoff function is $u_r(t, m) = -(m - t)^2$ and the sender's is $u_s(t, a) = -(a - (t + b))^2$, so when the sender's type is t, the receiver's optimal action is $a = t$ but the sender's optimal message is $m = t + b$. Suppose all types in the interval $[0, x_1)$ send one message while those in $[x_1, 1]$ send another. For an equilibrium to exist, which range should $b \in [0, 1]$ have? Give lower and upper bounds for b within $[0, 1]$. Given b, which value should x_1 have?

[5 points]

3. Consider the same signaling game but now with $n \geq 2$ intervals: $[0, x_1), \ldots, [x_k, x_k + 1], \ldots, [x_{n-1}, 1]$, where $0 \leq k \leq n - 1$, $x_0 = 0$ and $x_n = 1$. Given b, how much larger than its precedent should an interval be for an equilibrium to exist? Then, given b, for an equilibrium to exist, which upper bound should n have ?

[5 points]