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Mathematics of Games

Exercise session 3

13.05.2013, 12pm-2pm, N24-H15

Hand-in in PAIRS, before class starts!

1. (a) Compute the pure and mixed strategy Nash equilibria and the corresponding payoffs in the "battle of the sexes" game as shown in the figure below.

	B	F
F	0,0	2,1
B	1,2	0,0

   (b) If now partners get 1 as payoff from going to favorite program without each other, how many pure and mixed strategy Nash equilibria are there? Show all Nash Equilibria and the corresponding payoffs in the variant game as shown in the figure below.

	B	F
F	1,1	2,1
B	1,2	0,0

2. Two investors have each deposited D with a bank. The bank has invested these deposits in a long-term project. If the bank is forced to liquidate its investment before the project matures, a total of $2r$ can be recovered, where $D > r > D/2$. If the bank allows the investment to reach maturity, however, the project will pay out a total of $2R$, where $R > D$.
   There are two dates at which the investors can make withdrawals from the bank: date 1 is before the bank's investment matures; date 2 is after. For simplicity, assume that there is no discounting. If both investors make withdrawals at date 1 then each receives r and the game ends. If only one investor makes a withdrawal at date 1 then that investor receives D, the other receives $2r - D$, and the game ends.

   Finally, if neither investor makes a withdrawal at date 1 then the project matures and the investors make withdrawal decisions at date 2. If both investors make withdrawals at date 2 then each receives R and the game ends. If only one investor makes a withdrawal at date 2 then that investor receives $2R - D$, the other receives D, and the game ends. Finally, if neither investor makes a withdrawal at date 2 then the bank returns R to each investor and the game ends.

   Are there any pure-strategy subgame-perfect Nash Equilibria? If so, which?

3. Suppose that n oligopolists operate in a market with inverse demand given by $P(Q) = a - Q$, where $Q = q_1 + \ldots + q_n$ and q_i is the quantity produced by firm i. Each firm has a constant marginal cost of production, c, and no fixed cost. The firms choose their quantities as follows:

   - firm 1 chooses $q_1 \geq 0$, (2) firms i, $2 \leq i \leq n$ observe q_1 and then simultaneously choose q_i.

   Are there any pure-strategy subgame-perfect Nash Equilibria? If so, which?

4. The accompanying simultaneous-move game is played twice, with the outcome of the first stage observed before the second stage begins. There is no discounting. The final game payoff is the sum of the payofffs for both stages. The variable x is greater than 3, so that $(3, 3)$ is not an equilibrium payoff in the one-shot game. Give a value of x (with $x > 3$) and a strategy (played by both players) that constitute a pure-strategy subgame-perfect Nash equilibrium, such that the payoff of (A_1, C_2), which is (3,3), is achieved in the first stage.

	A_2	B_2	C_2	D_2
A_1	0,x	-1,0	3,3	0,0
B_1	0,0	0,1	-1,1	0,0
C_1	-1,0	0,-1	0,0	1,0
D_1	1,1	-1,1	x,0	0,0