1. Suppose that $n$ oligopolists operate in a market with inverse demand given by $P(Q) = a - Q$, where $Q = q_1 + \ldots + q_n$ and $q_i$ is the quantity produced by firm $i$. Each firm has a constant marginal cost of production, $c$, and no fixed cost. The firms choose their quantities in two periods as follows:

(a) (1) firm 1 chooses $q_1 \geq 0$, (2) firms $i, 2 \leq i \leq n$ observe the chosen value and then simultaneously choose $q_i$. [4 Points]

(b) (1) firms $j, 1 \leq j < z \leq n$, choose $q_j \geq 0$, (2) firms $i, z \leq i \leq n$, observe the chosen values and then simultaneously choose $q_i$. [4 Points]

What is a subgame-perfect Nash Equilibrium for game (a)? What about for game (b)? [8 Points]

2. Two investors have each deposited $D$ with a bank. The bank has invested these deposits in a long-term project. If the bank is forced to liquidate its investment before the project matures, a total of $2r$ can be recovered, where $D > r > D/2$. If the bank allows the investment to reach maturity, however, the project will pay out a total of $2R$, where $R > D$.

There are two dates at which the investors can make withdrawals from the bank: date 1 is before the bank’s investment matures; date 2 is after. For simplicity, assume that there is no discounting. If both investors make withdrawals at date 1 then each receives $r$ and the game ends. If only one investor makes a withdrawal at date 1 then that investor receives $D$, the other receives $2r - D$, and the game ends.

Finally, if neither investor makes a withdrawal at date 1 then the project matures and the investors make withdrawal decisions at date 2. If both investors make withdrawals at date 2 then each receives $R$ and the game ends. If only one investor makes a withdrawal at date 2 then that investor receives $2R - D$, the other receives $D$, and the game ends. If neither investor makes a withdrawal at date 2 then the bank returns $R$ to each investor and the game ends.

Give all subgame-perfect Nash Equilibria for this 2-period bank-run dynamic game. [6 Points]
3. Consider the “battle of the sexes” game, as shown in the figure below.

\[
\begin{array}{c|cc}
 & B & F \\
\hline
F & 0,0 & 2,1 \\
B & 1,2 & 0,0 \\
\end{array}
\]

On a Friday evening, a couple prefers going out together (+1) than alone (0). Linda would prefer going to a Fight game (+1, but only if together with Peter), while Peter would prefer going to a Ball game (+1, but only if together with Linda). Let \( \delta = \frac{9}{10} \), and consider the infinitely repeated game based on the stage game, with discount factor \( \delta \). Find a pure-strategy subgame-perfect Nash equilibrium \( s \) of the infinitely repeated game, for which the corresponding average discounted payoffs

\[
(1 - \delta) \cdot \sum_{t=0}^{\infty} \delta^t g_i(s^t(h^t))
\]

for both players are both higher than \( \frac{4}{3} \), double the highest symmetric payoff \( \left( \frac{2}{3}, \frac{2}{3} \right) \) of a NE for the stage game.

[6 Points]