Online and Distributed Algorithms

Exercise Session 4

1. Maximum Flow: Consider an orientation (with directed edges) of a weighted graph $G$ such as the one in the minimum weighted tree problem, where now the weights represent flow maximum capacities $c$ of those oriented edges of the new oriented digraph $D$ obtained by orienting $G$. Suppose that there is exactly one node $n_1$ (called source) with no incoming edges in $D$ as well as exactly one node $n_n$ (called sink) with no outgoing edges in $D$.

A flow in $D$ is a function $f : V \times V \rightarrow \mathbb{R}$ such that:

\[ f(n_i, n_j) \leq c(n_i, n_j) \forall n_i, n_j \in V, \]

\[ f(n_j, n_i) = -f(n_j, n_i) \forall n_i, n_j \in V, \]

\[ \sum_{n_j \in V} f(n_i, n_j) = 0 \forall n_i \in V - \{n_1, n_n\}. \]

- Give a synchronous algorithm for the maximum flow of the network $D$.
- Is there a way to modify the algorithm to turn it into asynchronous?

2. How many agents are needed in a tree to identify the “bad node” called blackhole (which kills agents) if agents walk in a synchronous manner on the edges? What about if agents walk asynchronously?