Online and Distributed Algorithms

Exercise Session 7

1. Sorting Networks: Prove or disprove the following claims.

(a) Given any correct sorting network, adding another comparator at the end destroys the sorting property.

(b) Given any correct sorting network, adding another comparator at the front does not destroy the sorting property.

(c) Every correct sorting network needs to have at least one comparator between each two consecutive wires.

(d) A network of \( n \) wires in which each wire has at least \( \frac{n}{2} \) comparators, being at least one to the previous wire (if any) and at least one to the next wire (if any), is a correct sorting network.

(e) A network which contains all \( \binom{n}{2} \) comparators between any two of the \( n \) wires, in whatever order they are placed, is a correct sorting network.

(f) Given any correct sorting network, adding another comparator anywhere does not destroy the sorting property.

(g) Given any correct sorting network, inverting it (i.e., feeding the input into the output wires and traversing the network from right to left) results in another correct sorting network.

(h) A network of binary entries and \( n \) wires which consists of two sorting networks of \( \frac{n}{2} \) wires followed by bitonic sorter of \( n \) wires (instead of a merging network) is a correct sorting network.

(i) A sorting network of \( n > 2 \) wires needs at least \( 2n \) comparators to be correct.

(j) A sorting network of \( n > 2 \) wires needs at most \( 2n \) comparators to be correct.