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Summer Term 2015

Stochastic Simulation

Problem Sheet 9

Deadline: July 2, 2015 at noon before the exercises

Please email your code to lisa.handl@uni-ulm.de AND hand in a printed copy of the code!

Exercise 1 (theory) (3 points)

Suppose you want to sample from some density f using the independence sampler with proposal density g and there exists a $C > 0$ so that $f(x) \leq Cg(x)\ \forall x \in \mathbb{R}$.

Show that the Metropolis-Hastings acceptance rate in this setting is at least $1/C$, i.e., show that $P(U \leq \alpha(X, Y)) \geq 1/C$ holds for independent random variables $X \sim f$, $Y \sim g$ and $U \sim \mathcal{U}(0, 1)$.

Exercise 2 (theory) (4 points)

Let (X, U) be a random point uniformly distributed on the area between some density $f(x)$ and the x-axis.

a) Find the joint density $g(x, u)$ of that point.

b) Show that the marginal density of X is $f(x)$.

c) In class you were told that the conditional densities, in this case $f_{X|U}$ and $f_{U|X}$, are proportional to the joint density $g(x, u)$, where the variable you condition on is treated as a constant. Determine the conditional distributions of X and U given that the respective other random variable is known.

d) Explain why the slice sampler is a special case of the Gibbs sampler.

Exercise 3 (programming) (4 points)

Implement the slice sampler in Matlab to draw from

$$f(x) = \frac{1}{2}\frac{1}{\sqrt{2\pi}} \exp\left\{\frac{-(x-2)^2}{2}\right\} + \frac{1}{2}\frac{1}{\sqrt{2\pi}} \exp\left\{\frac{-(x+2)^2}{2}\right\}.$$

Start from $x = 0$, run the chain for at least $N = 10^4$ steps and give a histogram of the results.

Hint: You have to compute the sets $\{x \in \mathbb{R} : f(x) \geq y\}$ numerically. For example, you can use the Matlab function `fzero(f, x0)` to find the zero of a function `f` which is closest to `x0`.

Exercise 4 (programming) (4 points)

Write a Matlab program to sample from the density

$$f(x) = Cx^2 \exp\left(-\frac{x}{2}\right) \mathbb{I}\{x \geq 0\} \tag{1}$$

with unknown normalizing constant $C > 0$ using the random walk sampler with standard normally distributed proposals. Run your sampler for at least $N = 10^5$ steps and plot a histogram of the values you obtained.