Exercise 1  (theory)  (2 points)

Suppose you work at a call center and calls arrive according to a Poisson process, \( \{N_t\}_{t \geq 0} \) with rate \( \lambda = 10 \) (\( t \) in hours). Calculate the expectation and the variance of the number of calls which arrive during the 8 hours you work there on a typical day.

Exercise 2  (theory)  (2 + 2 points)

Consider a homogeneous Poisson process \( \{N_t\}_{t \geq 0} \) with rate \( \lambda = 2 \).

a) Calculate the probability \( P(N_2 \geq 5) \) and find \( P(N_5 = 4 \mid N_3 = 2) \).

b) Find \( P(N_4 - N_2 = 2 \mid N_3 = 4) \).

Exercise 3  (theory)  (2 points)

Let \( n \in \mathbb{N} \) and let \( X_1, \ldots, X_n \) be iid. random variables with cumulative distribution function \( F \). Express the cumulative distribution function of \( X_{(1)} \) in terms of \( F \).

Exercise 4  (theory)  (5 points)

Let \( \{N_t^{(1)}\}_{t \geq 0} \) and \( \{N_t^{(2)}\}_{t \geq 0} \) be two independent homogeneous Poisson processes with positive intensities \( \lambda_1 \) and \( \lambda_2 \), respectively. Show that the sum of those processes, i.e.,

\[
\{N_t^{(3)}\}_{t \geq 0} \quad \text{with} \quad N_t^{(3)} = N_t^{(1)} + N_t^{(2)} \quad \forall t \geq 0
\]

is again a homogeneous Poisson process and determine its intensity.

Exercise 5  (programming)  (4 + 2 points)

a) Write a Matlab program to simulate a homogeneous Poisson process with intensity \( \lambda = 5 \) in the interval \([0, 10] \)

1. by generating its interarrival times
2. by using a grid with a mesh size of \( h = 0.05 \)
and plot one of its realizations (for each method).

b) Run your simulation programs from a) repeatedly (at least \( 10^4 \) times) and plot a
histogram of the values of \( N_1 \) you obtain (relative frequencies) for each method and
the pdf of its theoretical distribution.

Exercise 6 (programming) (3+1 points)

Consider the function

\[
f(x) = \frac{1}{2}x^2 - 10 \exp \left( -\frac{x^2}{20} \right) \cos(3x).
\]

a) Write a Matlab program to find the minimum of \( f \) using simulated annealing with
geometric cooling. Start from \( X_0 = 10 \) and \( T_0 = 10 \) and use geometric cooling with
\( \beta = 0.99 \). Draw proposals using a random walk sampler with normally distributed step
sizes.

b) Run your program from a) several times and change the standard deviations of the
step sizes (try \( \sigma = 0.1, 0.5 \) and 1). Use a sample size of at least \( N = 10^3 \). What do you
observe?