Exercise sheet 5 (total – 16 points) till November 23, 2016

Exercise 5-1 (3 points)

Let $\{N(t), t \in \mathbb{R}^+\}$ be the Poisson process with intensity $\lambda$. Compute

1. $P(N(1) = 1, N(2) = 2, N(3) = 4)$,
2. $P(N(1) \leq 1, N(2) = 2, N(3) \geq 4)$,
3. $P(N(t) = 2k + 1), k \in \mathbb{N}$.

Exercise 5-2 (3 points)

Let $\{N(t), t \in \mathbb{R}^+\}$ be the Poisson process with intensity $\lambda$. Compute

1. $P(N(3) \geq 4, N(2) = 2 | N(1) = 1)$,
2. $P(N(t) = i | N(s) = j), t > s$.
3. $E_{\frac{1}{N(t)+1}}$.

Exercise 5-3 (3 points)

Let $\tau_n$ be the time moment of the $n$th jump for the Poisson process. Prove that the distribution density of $\tau_n$ equals

$$\frac{\lambda^n x^{n-1}}{(n-1)!} e^{-\lambda x}, x \geq 0,$$

i.e., $\tau_n \sim Erlang(\lambda, n)$.

Exercise 5-4 (5 points)

Let $N^{(1)} = \{N^{(1)}(t), t \in \mathbb{R}^+\}$ and $N^{(2)} = \{N^{(2)}(t), t \in \mathbb{R}^+\}$ be independent Poisson processes with intensities $\lambda_1$ and $\lambda_2$ built on the independent sequences $T^{(1)}_1, T^{(1)}_2, \ldots$ and $T^{(2)}_1, T^{(2)}_2, \ldots$. Show that $N = \{N(t) := N^{(1)}(t) + N^{(2)}(t), t \in [0, \infty)\}$ is a Poisson process with intensity $\lambda_1 + \lambda_2$.

Exercise 5-5 (2 points)

A battery has a lifetime distributed uniformly over the interval (30, 60) (in units of hours). Let $N(t)$ be the number of batteries that have failed after $t$ hours. What is $\lim_{t \to \infty} \frac{N(t)}{t}$?