2010 FMA European Conference: New Perspectives on Skewed Distributions in Finance and Actuarial Science

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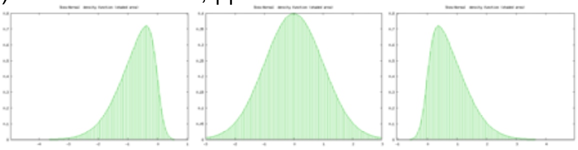
I. Motivation

- Financial data and actuarial claims data are not normally distributed.
- When skewness is present the skew-normal (SN) and its extensions (e.g., skew-student) are promising candidates to consider for both theoretical and empirical work in finance and actuarial science.

II. Skewed Distributions and Their Properties

- The distribution of a random vector **x** is multivariate skew-normal with location parameter ξ, scale parameter Ω and shape parameter a, that is **x** $\sim SN_p(\xi,\Omega,\alpha)$, if its pdf is: $f(\mathbf{x};\xi,\Omega,\alpha) = 2\phi_p(\mathbf{x};\xi,\Omega)\Phi[\alpha^T(\mathbf{x}-\xi)]$, with $\varphi_p(\cdot;\xi,\Omega)$ = pdf of $N_p(\xi,\Omega)$, $\varphi(\cdot)$ = pdf of N (0,1), $\Phi(\cdot)$ = cdf of N (0,1).
- Alternative representation for use in finance: A continuous random variable Y is said to have a skew-normal distribution if and only if the following representation holds: $Y = \xi + \omega X = \xi + \omega\left(\delta\left|Z_1\right| + \sqrt{1-\delta^2}Z_2\right)$ with: $\delta = \alpha\omega / \sqrt{1+\omega^2\alpha^2} \in (-1,1)$; Z_1 and Z_2 are independent N(0;1) random variables, | | stands for Half-Gaussian => Interpretation as <u>skewness shock</u>
- All moments of the SN distribution exist and are finite; the SN distribution extends the normal one in several ways:
  - Inclusion: The normal distribution is a skew-normal distribution with shape parameter equal to zero: $x \sim SN_p(\xi,\Omega,0) \leftrightarrow x \sim N_p(\xi,\Omega)$.
  - Affinity: Any affine transformation of a skew-normal random vector is skew-normal, too.
  - Invariance: The distribution of any norm of x-ξ does not depend on the shape parameter.



$\delta < 0$ $\quad$ $\delta = 0$ $\quad$ $\delta > 0$

Field of Research	Authors
Portfolio Selection/ Capital Asset Pricing Model (CAPM)	Adcock/Shutes (2001) Adcock (in: Genton 2004) Meucci (2006) Adcock (CS 2007) Adcock (Working Paper 2010) Adcock (AOR 2010)
Modelling of Volatility (ARCH/GARCH…)	Bartolucci/De Luca/Loperfido (2000) De Luca/Loperfido (in: Genton 2004; 2007) Cappuccio/Lubian/Raggi (2004) Bauwens/Laurent (JBES 2005) Liseo/Loperfido (JSPI 2006) Franceschini/Loperfido (2009)
Application to Hedge Funds	Adcock (EJoF 2005) Eling/Farinelli/Rossello/Tibiletti (IJMF 2010)
Application to Option Returns	Adcock/Eling (WP 2010)

III. Use of Skewed Distributions in Finance

- Returns on financial assets are not well described by the multivariate normal distr. For portf. selection and asset pricing models it is desirable to use a coherent multivariate probability distr.
- SN and skew-student distributions are natural candidates for both theoretical and empirical work in finance:
  - Parameterisation of the SN and skew-student is parsimonious.
  - Interpretable in terms of the efficient markets hypothesis (unobserved shock as departure from efficiency).
  - SN and skew-student lead to theoretical results which are useful for portfolio selection and asset pricing.
- For example, portfolio selection: For the normal, Student and indeed any elliptically symmetric distributions, portfolio selection is invariant to the choice of utility function; we end up on the efficient frontier; for the skew-normal and skew-Student distributions, there is an equivalent result: a single mean variance efficient surface (Adcock (2007, 2010); uses extended SN) Practical implications are obvious: don't waste time looking at different utility functions!
- Challenges: Time series effects (e.g. De Luca et al. (2006) for a GARCH model; estimation, particularly for the multivariate case, is not easy; for work in international finance, it may be necessary to have more than one skewness shock, ie more than one truncated variable; the portfolio selection invariance property may work for all skew-elliptical distributions, but this is work in progress.

IV. Use of Skewed Distributions in Actuarial Science

- SN has been used in studies on risk measurement and capital allocation, which are two of the most popular research fields in actuarial science
- For example, TCE and Capital Alloc. for the SN (Vernic, 2006):

$$TCE_\alpha(X) = E[X|X \geq q_{1-\alpha}(X)] = \sum_{i=1}^{n} E[X_i|X \geq q_{1-\alpha}(X)]$$

$$TCE_\alpha(X) = \mu + \frac{2}{1-\alpha}\left[\sigma\varphi\left(VaR_\alpha(Y)\right)\Phi\left(\frac{\delta VaR_\alpha(Y)}{\sqrt{\sigma^2-\delta^2}}\right) + \frac{\delta}{\sqrt{2\pi}}\bar{\Phi}\left(\frac{\sigma VaR_\alpha(Y)}{\sqrt{\sigma^2-\delta^2}}\right)\right] \Rightarrow \gamma_i = \frac{E[X_i|X \geq q_{1-\alpha}(X)]}{E[X|X \geq q_{1-\alpha}(X)]}$$

Field of Research	Authors
Risk Measurement	Vernic (IME 2006) Arellano-Valle and Genton (Metron 2010)
Capital Allocation	Vernic (IME 2006)
Goodness of Fit	Bolance et al. (IME 2008) Eling and Hess (2010) Zappa (2010)

- Some empirical studies consider the SN distribution (goodness of fit context), but research is in early stages

V. Conclusion and Future Research Perspectives

- Conclusion: 1. Skewed distributions are very flexible and easy to interpret 2. They have theoretical advantages for use in finance (useful for portfolio selection and asset pricing) 3. Wide range of potential applications in finance and actuarial science (portfolio selection, modelling of volatility, option pricing, risk measurement, capital allocation,…)
- Future Research: 1. More applications needed to increase awareness of the advantages of skewed distributions. 2. For example modeling of operational risk, modeling of option returns 3. Several quite specific research challenges, some financial and some statistical.

<u>**All references and further information is available from the presenters upon request (just write an e-mail or talk to us right after the session)**</u>