Trends and Volatility in Mortality and Longevity Risk: Insights from Econometric and Actuarial Modeling

Presentation at
Institut Fuer Finanz-Und Aktuarwissenschaften
Universitaet Ulm

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Research Aims

- Research program: Australian Research Council Linkage Grant (with PwC, APRA, World Bank)
- Longevity Trends by Country and major Cause of Death
- Stochastic Trends for Country Level and by Age
- Principal Components and Number of Factors
- Cointegration and Common Stochastic Trends
- Australian data by Age group - Smooth by Age and Cointegration
- Cause of Death data: Trends by Age, Cohort and Time
- Actuarial Loss Reserving Models - Trends by Age (Development period), Cohort (Accident period) and Time (Calendar period)

Background

- Australia has a compulsory 9 per cent Superannuation Guarantee contribution (introduced 1992)
- Significant accumulation of assets for retirement
- Only one life insurer offers life annuities and longevity insurance variable annuity (Lifestream Guaranteed Long Term Income)
- Government means tested pension
- Henry Tax Review - lack of longevity insurance market
- High current interest in longevity risk from government and industry

Longevity Trends by Country

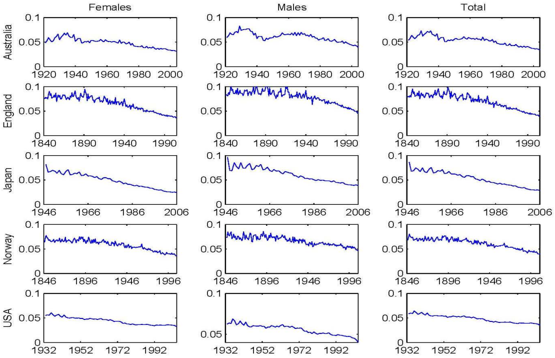
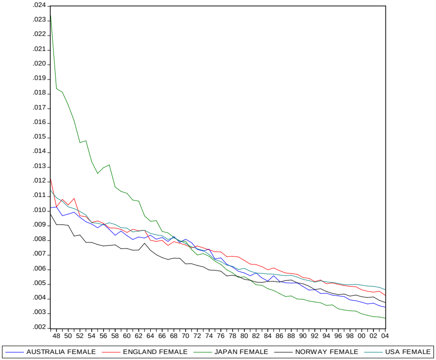


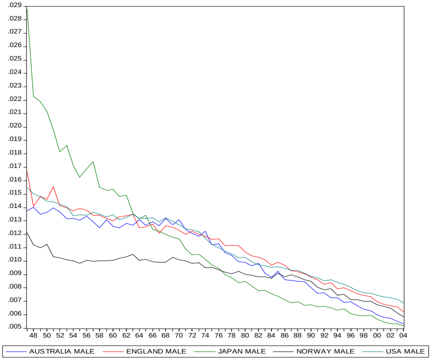
Figure: Average Death Rates for Developed Countries

Longevity Trends by Country - Standardised Rates

Females



Males



Longevity Trends for Australia by Age

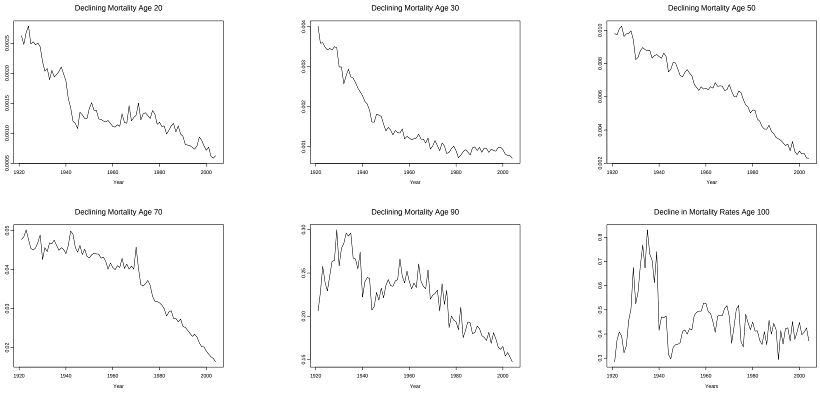


Figure: Age Specific Death Rates at Different Ages

Unit roots

Trend-stationary time series

$$y_t = \mu + \phi y_{t-1} + u_t \tag{1}$$

with $|\phi| < 1$

The random walk with drift

$$y_t = \mu + y_{t-1} + u_t \tag{2}$$

If $\phi = 1$ then the series has a unit root and as $T \to \infty$ the effect of the shocks persist and accumulate as stochastic trends in the series:

$$y_t = \mu + y_0 + \sum_{t=0}^{\infty} u_t \tag{3}$$

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Unit Root Tests - by Country

	Constant	Lags	Constant, Trend	Lags
Australia	0.9809	1	0.2383	1
Δ Australia	0	0	0	0
England	0.998	2	0.0345	0
Δ England	0	1	0	1
Japan	0.0129	1	0.9374[1]	1
Δ Japan	0	0	0	0
Norway	0.9999	1	0.9275[1]	1
Δ Norway	0	0	0	0
USA	0.8311	0	0.8487[1]	0
Δ USA	0	0	0	0

Table: ADF Tests on Male Standardised Mortality Rates

[1] Indicates significant trend

Principal Components

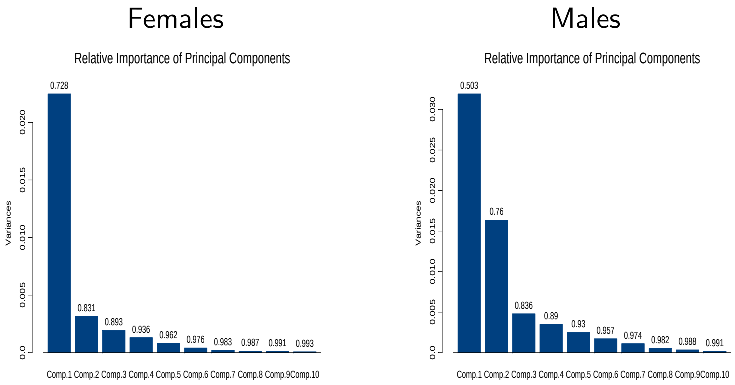

Figure: Principal Components for Australia - Mortality Rate Levels

Principal Components by Country - Time and Cohort Trends

Country	Difference	Number of Factors	Percentage Variation
Australia	Diagonal	8	98.7
	Horizontal	8	98.6
England	Diagonal	7	98.6
	Horizontal	7	98.9
Japan	Diagonal	6	98.8
	Horizontal	5	98.1
Norway	Diagonal	9	98.4
	Horizontal	8	98.1
USA	Diagonal	10	97.4
	Horizontal	10	97.7

Table: PCA Factors using for differences in rates for countries in study

Principal Components

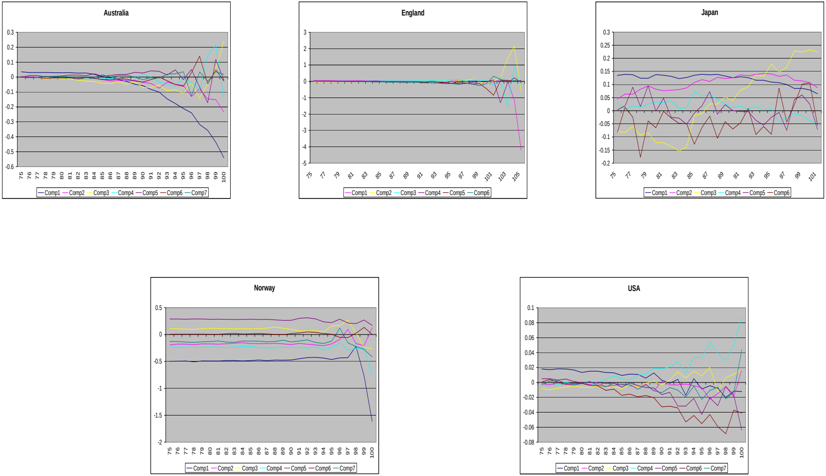

Figure: Multiple PCA Factors for Mortality Differences

VAR and VECM

VAR(p) for p lags:

$$m_t = A_0 + A_1 m_{t-1} + A_2 m_{t-2} + \cdots + A_p m_{t-p} + e_t$$

where $\mathbf{m}_t = (m_{1t}, \ldots, m_{kt}, \ldots, m_{Kt})$ for $k = 1, \ldots, K$ time series.

Long-run specification of VECM:

$$\Delta \mathbf{m}_t = \Gamma_1 \Delta \mathbf{m}_{t-1} + \cdots + \Gamma_{p-1} \Delta \mathbf{m}_{t-p+1} + \Pi \mathbf{m}_{t-p} + A_0 + e_t \quad (4)$$

where

$$\Gamma_i = -(I - A_1 - \cdots - A_i), \quad i = 1, \ldots, p-1 \qquad \Pi = -(I - A_1 - \cdots - A_p)$$

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VAR and Cointegration

- Lee-Carter model special case of VAR model
- VAR Models developed for standardised mortality rate at country level
- Autoregression in country mortality rates: Norway AR(2); other countries AR(1); Cross country lags significant
- Countries in the study have stochastic trends with drift; no evidence of common stochastic trends (cointegration)
- Cross country diversification.

Dynamic Heligman-Pollard (H-P) Model

- Younger and older ages stationary
- Mid range of ages evidence of unit roots - non stationary
- Large number of parameters in VAR model
- Significant number of stochastic factors (up to 8)
- Use Heligman-Pollard to smooth Australian data by age group

Heligman-Pollard model with stochastic parameters (8 parameters as VECM)

$$q_x = A^{(x+B)^C} + D\exp[-E(\log\{\frac{x}{F}\})^2] + \frac{GH^x}{1+GH^x} \quad (5)$$

Heligman Pollard Model Fit

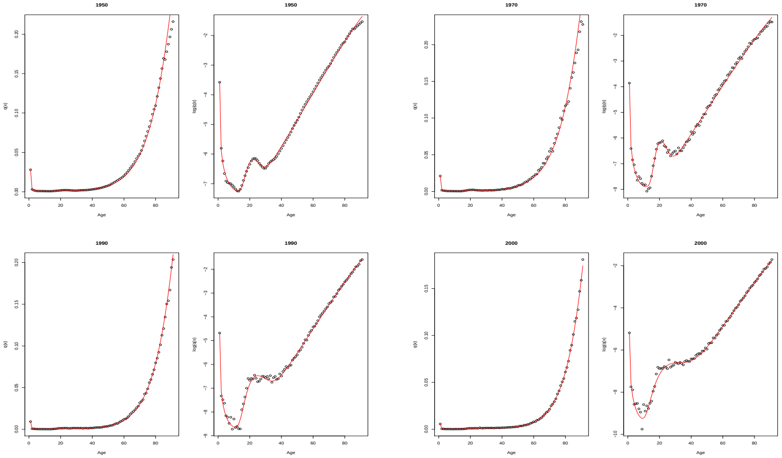

Figure: H-P Fit for Australia - Males

Heligman Pollard Model Parameters

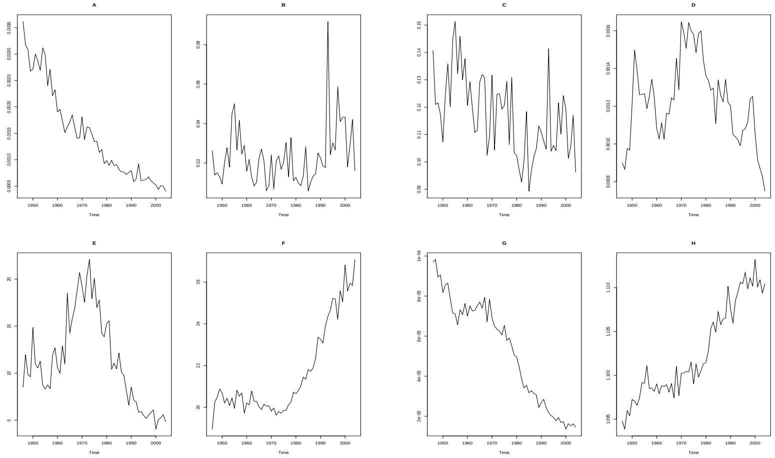

Figure: H-P Parameters for Australia - Males

Heligman Pollard as VECM in Parameters

- VECM - VAR model allows for cointegration and multiple factors for volatility
- Cross correlations included
- Improved fit compared to Lee-Carter
- Captures long run trends although issues with parsimony and forecasting

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Cause of Death Data - WHO

Cancer	Circulatory Disease	Respiratory Disease
Infectious Diseases	External causes	Others not included
Australia	Japan	Singapore
UK	Italy	Sweden
Norway	Switzerland	USA

Table: Major Causes of Death and Countries

Cause of Death Analysis

Analysis of Cause of Death data by 9 major countries and 5 major causes:

- Pattern by age similar for each cause across countries
- Time trends differ by cause, age and country (impact of standard of living, smoking, obesity, etc)
- Strong evidence of cohort trends for most causes
- Cancer and circulatory disease pattern different for males and females (impacts time trends and age pattern)

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Actuarial Models for Trends by Cause of Death

Cohort trends significant but also need to smooth age pattern and capture time trends

Actuarial loss reserving techniques can be applied:

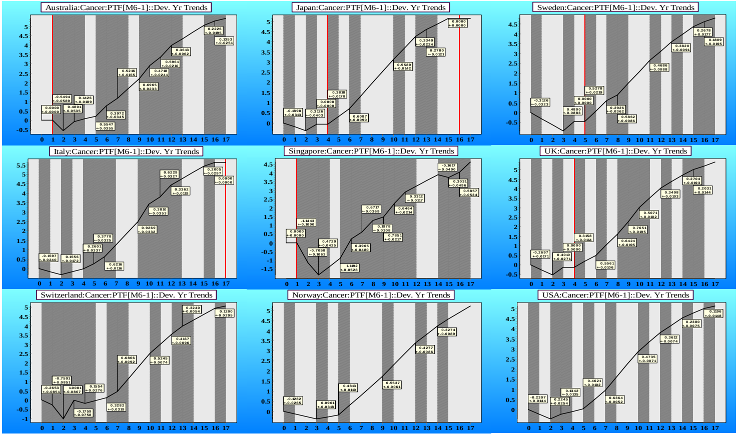
- Age trend - development period
- Cohort trend - accident period
- Time trend - calendar period

Stochastic loss reserving model with parameters for trends fitted to cause of death male data. Preliminary research.

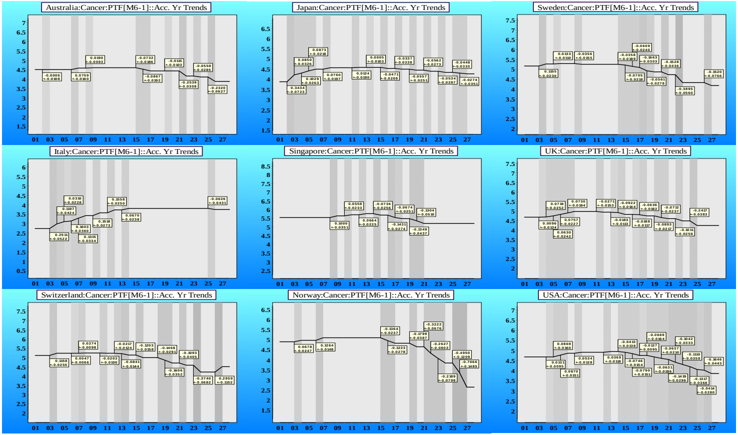
Cohorts for Cause of Death Data

Country	From	To
USA	1866-1870	2001-2005
Australia	1865-1868	1999-2003
Switzerland	1866-1870	2001-2005
Japan	1867-1871	2002-2006
Singapore	1877-1881	2002-2006
Italy	1864-1868	1998-2002
Norway	1866-1870	2001-2005
Sweden	1866-1870	2001-2005
UK	1865-1869	1995-1999

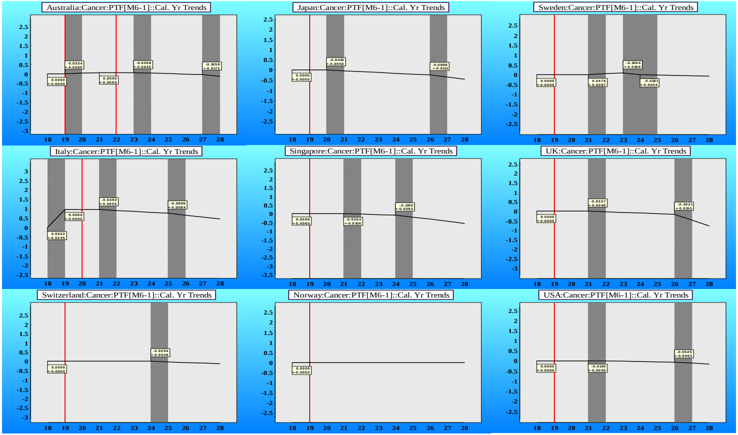
Trends by Age by Country for Cause of Death - Cancer



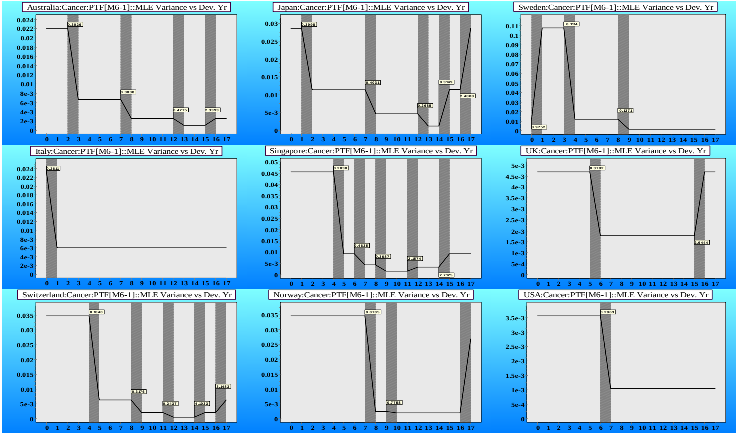
Trends by Cohort by Country for Cause of Death - Cancer



Trends by Time by Country for Cause of Death - Cancer



Volatility by Age by Country for Cause of Death - Cancer



Modeling Cause of Death with Actuarial Loss Reserve Models

Five major causes across nine countries analyzed.

Summary of key trend and volatility results:

- Age patterns differ by cause but similar across countries
- Time trends differ by country and cohort trend significant for causes
- Differing time trends and differing relative importance of causes adds complexity to modeling country age patterns, trends and volatility

Correlations of Model Residuals by Cause of Death

Final Weighted Residual Correlations Between Datasets: Japan

	Cancer	Circ	Ext	Infectious	Resp
Cancer	1	-0.062168	0.051038	-0.158574	0.37066
Circ	-0.062168	1	0.544224	0.108517	0.290305
Ext	0.051038	0.544224	1	0.273405	0.343039
Infectious	-0.158574	0.108517	0.273405	1	0.345423
Resp	0.37066	0.290305	0.343039	0.345423	1

8 iterations were executed

Final Weighted Residual Correlations Between Datasets: Australia

	Cancer	Circ	Ext	Infectious	Resp
Cancer	1	0.235318	0.301575	0.03398	0.220104
Circ	0.235318	1	0.50179	0.096593	0.274126
Ext	0.301575	0.50179	1	0.068073	0.294911
Infectious	0.03398	0.096593	0.068073	1	-0.05123
Resp	0.220104	0.274126	0.294911	-0.051229	1

8 iterations were executed

Final Weighted Residual Correlations Between Datasets: Italy

	Cancer	Circ	Ext	Infectious	Resp
Cancer	1	0.057686	0.461946	0.053628	0.265988
Circ	0.057686	1	0.34739	0.306137	0.549576
Ext	0.461946	0.34739	1	0.352167	0.233657
Infectious	0.053628	0.306137	0.352167	1	0.178312
Resp	0.265988	0.549576	0.233657	0.178312	1

10 iterations were executed

Final Weighted Residual Correlations Between Datasets: Norway

	Cancer	Circ	Ext	Infectious	Resp
Cancer	1	0.142939	-0.044551	0.276948	0.232036
Circ	0.142939	1	0.130197	0.157562	0.335954
Ext	-0.044551	0.130197	1	-0.048109	0.149689
Infectious	0.276948	0.157562	-0.048109	1	0.273769
Resp	0.232036	0.335954	0.149689	0.273769	1

11 iterations were executed

Final Weighted Residual Correlations Between Datasets: Singapore

	Cancer	Circ	Ext	Infectious	Resp
Cancer	1	0.125891	0.027483	0.229997	0.151022
Circ	0.125891	1	0.185774	-0.009585	0.030465
Ext	0.027483	0.185774	1	-0.27165	0.110892
Infectious	0.229997	-0.00959	-0.27165	1	0.315497
Resp	0.151022	0.030465	0.110892	0.315497	1

7 iterations were executed

Final Weighted Residual Correlations Between Datasets: Sweden

	Cancer	Circ	Ext	Infectious	Resp
Cancer	1	0.191611	-0.07381	-0.306706	-0.03378
Circ	0.191611	1	0.116666	0.028089	0.326564
Ext	-0.07381	0.116666	1	0.068953	0.338316
Infectious	-0.306706	0.028089	0.068953	1	-0.10135
Resp	-0.033783	0.326564	0.338316	-0.101345	1

6 iterations were executed

Final Weighted Residual Correlations Between Datasets: Switzerland

	Cancer	Circ	Ext	Infectious	Resp
Cancer	1	-0.036015	0.017935	0.048508	-0.061938
Circ	-0.036015	1	0.29554	0.397873	0.078742
Ext	0.017935	0.29554	1	0.061407	0.054524
Infectious	0.048508	0.397873	0.061407	1	0.04563
Resp	-0.061938	0.078742	0.054524	0.04563	1

5 iterations were executed

Final Weighted Residual Correlations Between Datasets: UK

	Cancer	Circ	Ext	Infectious	Resp
Cancer	1	-0.11714	-0.0671	0.175351	0.217493
Circ	-0.11714	1	0.535239	-0.01761	0.227617
Ext	-0.0671	0.535239	1	0.060496	0.249007
Infectious	0.175351	-0.01761	0.060496	1	0.296123
Resp	0.217493	0.227617	0.249007	0.296123	1

8 iterations were executed

Final Weighted Residual Correlations Between Datasets: USA

	Cancer	Circ	Ext	Infectious	Resp
Cancer	1	0.455217	0.072159	0.064325	0.346568
Circ	0.455217	1	0.222332	0.132551	0.23073
Ext	0.072159	0.222332	1	0.161744	0.012277
Infectious	0.064325	0.132551	0.161744	1	0.139527
Resp	0.346568	0.23073	0.012277	0.139527	1

6 iterations were executed

Conclusions - Country and Age

- At country level: stochastic trends although not common stochastic trends
- Age groups within countries: older and younger ages close to stationary and middle ages stochastic trends
- Significant number of common factors in mortality rate levels as well as differences; multiple factor models and dimension reduction
- VAR model at country level; VECM model for age groups (stochastic parameter models that smooth across age)

Conclusions - Cause of Death

- Cause of death trends; differ by country and sex
- Similar age pattern by cause for different countries
- Significant cohort trends for most causes
- Volatilities by cause differ across age groups

Conclusions - Future challenges

- Modeling of longevity/mortality trends and volatility has many challenges
- Number of parameters to capture trends and volatility: fit to data versus forecasting
- Hierarchical and Bayesian models
- Financial applications and volatility