This Appendix provides the basic derivations of the multi-country trade model of heterogeneous firms presented in Hesse (2014). The ensuing presentation borrows from Dixit & Stiglitz (1977), Melitz (2003) as well as Helpman et al. (2010a) and its technical appendix, Helpman et al. (2010b).

1. A FIRM’S REVENUE AND EXPORT DECISION

a. Domestic Demand

The preferences of a representative consumer are given by a C.E.S. utility function over a continuum of varieties indexed by $\omega$:

$$U = \left[ \int_{\omega \in \Omega} y(\omega)^{\rho} d\omega \right]^{\frac{1}{\rho}},$$

where $y(\omega)$ indexes the amount of variety $\omega$ and $\Omega$ represents the set of available varieties within the sector. These varieties are substitutes, implying $0 < \rho < 1$ and an elasticity of substitution between any two varieties of

$$\sigma = \frac{1}{1-\rho} > 1 \iff \rho = 1 - \frac{1}{\sigma} = \frac{\sigma - 1}{\sigma}. \quad (A.1)$$

The consumer’s constrained maximization problem may be solved by the Lagrangian

$$\mathcal{L} = U^\rho - \lambda \left( \int_{\omega \in \Omega} p(\omega)y(\omega)d\omega - I \right),$$

where $U^\rho$ is a strictly increasing transformation of $U$, $p(\omega)$ the price of variety $\omega$, and $I$ the consumer’s income. The maximization problem yields the following first-order condition

$$\frac{\partial \mathcal{L}}{\partial y(\omega)} = \rho y(\omega)^{\rho-1} - \lambda p(\omega) = 0.$$
By dividing the first-order condition of one variety $\omega_1$ by the first-order condition of another variety $\omega_2$, we obtain the relative demand

$$\frac{y(\omega_1)}{y(\omega_2)} = \left(\frac{p(\omega_1)}{p(\omega_2)}\right)^{\frac{\sigma}{1-\sigma}}.$$ 

Multiplying both sides with $y(\omega_2)$ and using (A.1) yields

$$y(\omega_1) = y(\omega_2) \left(\frac{p(\omega_1)}{p(\omega_2)}\right)^{-\sigma}.$$ 

When multiplying both sides with $p(\omega_1)$ and taking the integral with respect to $\omega_1$, we get

$$\int_{\omega \in \Omega} p(\omega_1) y(\omega_1) d\omega_1 = \int_{\omega \in \Omega} y(\omega_2) p(\omega_1)^{1-\sigma} p(\omega_2)^{\sigma} d\omega_1.$$ 

On the left-hand side we now have the consumer’s total expenditure on all varieties, $R$, which is assumed to be equal to his income $I$, i.e.,

$$R = I = y(\omega_2) p(\omega_2)^{\sigma} \int_{\omega \in \Omega} p(\omega_1)^{1-\sigma} d\omega_1.$$ 

Solving for $y(\omega_2)$ yields the Marshallian demand for $\omega_2$

$$y(\omega_2) = \frac{Ip(\omega_2)^{-\sigma}}{\int_{\omega \in \Omega} p(\omega_1)^{1-\sigma} d\omega_1}.$$ 

By defining an index of the overall price level

$$P = \left[\int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega\right]^\frac{1}{\sigma},$$

Marshallian demand for a variety $\omega$ simplifies to

$$y(\omega) = p(\omega)^{-\sigma} P^{\sigma-1} I = \left(\frac{p(\omega)}{P}\right)^{-\sigma} I \frac{P}{P}.$$ 

Domestic demand, denoted by $y_d(\omega)$, can accordingly be written as

$$y_d(\omega) = p_d(\omega)^{-\sigma} P_d^{\sigma-1} I_d = \left(\frac{p_d(\omega)}{P_d}\right)^{-\sigma} I_d \frac{P_d}{P_d},$$

where $p_d(\omega)$ denotes the price of the good in the domestic market while $P_d$ and $I_d$ indicate the domestic aggregate price and domestic income, respectively.

**b. Domestic Revenue**

With a firm’s domestic output being equal to domestic demand, domestic firm revenue can be written as

$$r_d(\omega) = y_d(\omega) p_d(\omega) = I_d \left(\frac{p_d(\omega)}{P_d}\right)^{1-\sigma}.$$ 

Note that with $p_d(\omega) = y_d(\omega)^{1-\frac{1}{\rho}} P_d^{-\frac{1}{\rho}} I_d^{\frac{1}{\rho}}$ and (A.1) domestic revenue can also be written as in HIR, i.e.,

$$r_d(\omega) = y_d(\omega)^{1-\frac{1}{\rho}} P_d^{-\frac{1}{\rho}} I_d^{\frac{1}{\rho}} = y_d(\omega)^{\rho} P_d^{1-\rho} = y_d(\omega)^{\rho} A_d,$$
where $A_d$ is called the domestic demand shifter, with $A_d = P_d^I I_d^{-\rho}$. As with increasing productivity a firm’s output and thereby its domestic revenue will increase continuously, we can write domestic revenue — and revenue in general — as

$$r_d(\varphi) = y_d(\varphi)^\varphi A_d.$$  

**c. Revenue from Exporting**

By assuming country specific iceberg trading costs, $\tau_c$, such that $\tau_c > 1$ units of a variety must be exported for a unit to arrive in country $c$, we can write the revenue from exporting to country $c$ as

$$r_{x,c}(\varphi) = \frac{y_{x,c}(\varphi)}{\tau_c} P_{x,c}(\varphi) = \left(\frac{y_{x,c}(\varphi)}{\tau_c}\right)^\rho P_{x,c}^\rho I_{x,c}^{1-\rho} = \left(\frac{y_{x,c}(\varphi)}{\tau_c}\right)^\rho A_{x,c},$$

where $A_{x,c} = I_{x,c}^{1-\rho} P_{x,c}^\rho$ is the demand shifter of country $c$.

**d. \( \Upsilon \) and a Derivation of $y_d(\varphi) = y(\varphi)/\Upsilon$**

Using the first-order conditions (5), we can write a firm’s total output,

$$y(\varphi) = y_d(\varphi) + \sum_{c=1}^c y_{x,c}(\varphi),$$

as

$$y(\varphi) = y_d(\varphi) + \sum_{c=1}^c \tau_c y_d(\varphi) \left(\frac{A_{x,c}}{A_d}\right)^{\frac{\varphi}{\rho}} = y_d(\varphi) \left(1 + \sum_{c=1}^c \tau_c \frac{A_{x,c}}{A_d} \right)^{\frac{\varphi}{\rho}},$$

where $\tau_c$ equals 1 if the firm exports to country $c$ and 0 otherwise. By defining $\Upsilon \equiv 1 + \sum_{c=1}^c \tau_c \left(\frac{A_{x,c}}{A_d}\right)^{\frac{\varphi}{\rho}}$, we obtain

$$y_d(\varphi) = y(\varphi)/\Upsilon.$$  

**e. Total Revenue**

A firm’s total revenue is given by

$$r(\varphi) \equiv r_d(\varphi) + \sum_{c=1}^c r_{x,c}(\varphi) = y_d(\varphi)^\varphi A_d + \sum_{c=1}^c \tau_c^{-\rho} y_{x,c}(\varphi)^\varphi A_{x,c}.$$  

Using again the first-order conditions (5), this can be written as

$$r(\varphi) = y_d(\varphi)^\varphi A_d + \sum_{c=1}^c \frac{\tau_c}{\tau_c} y_d(\varphi)^\varphi A_{x,c} \left(\frac{A_{x,c}}{A_d}\right)^{\frac{\varphi}{\rho}}$$

$$= y_d(\varphi)^\varphi A_d \left(1 + \sum_{c=1}^c \frac{\tau_c}{\tau_c} \left(\frac{A_{x,c}}{A_d}\right)^{\frac{\varphi}{\rho}}\right) = y_d(\varphi)^\varphi A_d \Upsilon.$$  

With $y_d(\varphi) = y(\varphi)/\Upsilon$ we obtain

$$r(\varphi) = y(\varphi)^\varphi A_d \Upsilon^{1-\varphi}.$$  

(A.2)
Using the earlier definition of \( r(\varphi) \) in (A.2), the production function (2), and the first-order conditions (8) and (9), we are now able to express revenue as

\[
 r(\varphi) = \left( \frac{\zeta_d}{\zeta_d - 1} \right)^{\frac{\rho \gamma}{1 - \rho \gamma}} \left( \frac{\rho(1 - \gamma \zeta_d)}{\epsilon(1 + \rho \gamma)} \right)^{\frac{1 - \gamma_d}{\epsilon}} A_d \Gamma^{\frac{\varphi}{\epsilon}}. \tag{A.3}
\]

where \( \Gamma \equiv 1 - \rho \gamma - \rho(1 - \gamma \zeta_d)/\delta \). In a next step, we compute the firm’s profits by making once more use of the first-order conditions

\[
 \pi(\varphi) = \frac{\Gamma}{1 + \rho \gamma} r(\varphi) - f_d - \sum_{c=1}^{\ell_c} I_c f_{c,c}.
\]

Furthermore, we know that the firm with the lowest productivity, \( \varphi_d \), makes zero profit and is not exporting, hence no productivity gains from exporting are possible, i.e., \( \varphi_d \equiv \varphi'_d \). It follows

\[
 \frac{\Gamma}{1 + \rho \gamma} r(\varphi_d) = f_d \quad \Rightarrow \quad r(\varphi_d) = \frac{1 + \rho \gamma}{\Gamma} f_d. \tag{A.4}
\]

In the following, we use the expression for \( r(\varphi) \) from (A.3) and determine the relative revenue of a firm in comparison to the firm with the lowest productivity. We obtain

\[
 \frac{r(\varphi)}{r'_d} = \left( \frac{\varphi}{\varphi_d} \right)^{\frac{\varphi}{\epsilon}} \Rightarrow \quad r(\varphi) = r'_d \left( \frac{\varphi}{\varphi_d} \right)^{\frac{\varphi}{\epsilon}} \Gamma^{\frac{\varphi}{\epsilon}}. \tag{A.5}
\]

Since we can decompose a firm’s productivity into its initial productivity, \( \varphi' \), and the possible productivity gain from exporting, \( e^{\varphi(\varphi')} \), we can write revenue as

\[
 r(\varphi') = r'_d \left( \frac{\varphi'}{\varphi_d} \right)^{\frac{\varphi'}{\epsilon}} \Gamma^{\frac{\varphi'}{\epsilon}} e^{\varphi(\varphi')}. \]

2. A FIRM’S AVERAGE WAGE

By the same token, we are able to compute \( a_c(\varphi) \). We employ the first-order condition (9) and get

\[
 a_c(\varphi) a_c(\varphi_d)^{\beta} = \Gamma^{\frac{\varphi}{\epsilon}} \left( \frac{\varphi}{\varphi_d} \right)^{\frac{\varphi}{\epsilon}} \Rightarrow \quad a_c(\varphi) = a_c(\varphi_d) \left( \frac{\varphi}{\varphi_d} \right)^{\frac{\varphi}{\epsilon}} \Gamma^{\frac{\varphi}{\epsilon}}. \tag{A.6}
\]

Using (A.4) together with (9), we can compute

\[
 a_c(\varphi_d) = \left( \frac{\rho(1 - \gamma k)}{1 + \rho \gamma} \right)^{\frac{1}{\epsilon}} f_d = \left( \frac{\rho(1 - \gamma \zeta_d)}{\epsilon \Gamma} \right)^{\frac{1}{\epsilon}} f_d.
\]

With the wage condition from (10), the lowest wage paid by a domestic firm is then

\[
 w(\varphi_d) \equiv w'_d = h \left( \frac{a_c(\varphi_d)}{a_{\min}} \right)^{\gamma_d} = \left( \frac{\rho(1 - \gamma \zeta_d)}{\epsilon \Gamma \gamma_{\min}} f_d \right)^{\frac{\gamma_d}{\epsilon}}.
\]

This yields a wage relation that is solely dependent on \( \varphi', \Gamma(\varphi), \varphi_d, \) and parameters, namely

\[
 \frac{w(\varphi)}{w'_d} = \left( \frac{a_c(\varphi)}{a_c(\varphi_d)} \right)^{\gamma_d} = \left( \frac{\varphi}{\varphi_d} \right)^{\frac{\gamma_d}{\epsilon}} \Gamma^{\frac{\varphi(\varphi')}{\epsilon}} \Rightarrow \quad w(\varphi) = w'_d \left( \frac{\varphi}{\varphi_d} \right)^{\frac{\gamma_d}{\epsilon}} \Gamma^{\frac{\varphi(\varphi')}{\epsilon}}.
\]
As can be seen from this last equation, wages increase with firm productivity and are always higher for exporting firms than for non-exporting firms. Ultimately, we decompose productivity into its components and obtain

\[ w(\varphi') = w_d \left( \frac{\varphi}{\varphi_d} \right)^{\varphi_d} \int \frac{\varphi^{\rho \varphi_d}}{\varphi_d} e^{-\frac{\varphi^{\rho \varphi_d}}{\varphi_d}}. \]

3. A FIRM’S MEASURE OF WORKERS HIRED

In a similar manner, we can derive the lowest measure of workers hired

\[ h(\varphi_d) \equiv h_d = m(\varphi_d) \left( \frac{a_{\min,d}}{a(\varphi_d)} \right)^{\varphi_d} = \rho \gamma \frac{r_d^d}{1 + \rho \gamma} \left( \frac{a_{\min,d}}{a(\varphi_d)} \right)^{\varphi_d} \]

Using (A.5) and (A.6), the relation to \( h(\varphi) \) is then given by

\[ h(\varphi) = \frac{h_d}{h_d} = \left( \frac{a_{\min,d}}{a(\varphi_d)} \right)^{\varphi_d} \int \frac{\varphi^{\rho \varphi_d}}{\varphi_d} e^{-\frac{\varphi^{\rho \varphi_d}}{\varphi_d}} \]

which ultimately leads with (1) to

\[ h(\varphi') = h_d \left( \frac{\varphi'}{\varphi_d} \right)^{(1-\varphi)} \int \frac{(1-\varphi)^{\rho \varphi_d}}{1-\varphi} e^{(1-\varphi)[\varphi']} \]

REFERENCES


