Investment Decisions with Loss Aversion over Relative Consumption

Georg Gebhardt
University of Munich *

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Abstract

We study an exchange economy in which investors are loss averse over relative consumption, that is, they suffer a utility loss if they consume less than members of their reference group. As a consequence there is an incentive to hold the same portfolio of risky assets as the reference group. Thus, risk premia can be supported in equilibrium that diverge from the risk premia obtained without loss aversion over relative consumption. This effect may be used to explain time-varying risk premia that are empirically observed for many assets.

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*Department of Economics, University of Munich, Ludwigstr. 28 (Rgb.), D-80539 Munich, Tel.: +49 89 2180 2876, e-mail: Georg.Gebhardt@LRZ.uni-muenchen.de. Financial support by Deutsche Forschungsgemeinschaft through grant SFB TR 15 is gratefully acknowledged.

1 Introduction

In this paper we study the investment decisions of fully rational investors who exhibit loss aversion over relative consumption. We show in a simple exchange economy that such preferences lead to multiple equilibria, which differ with respect to equilibrium asset prices. This effect may be used to explain time-varying risk premia that are empirically observed for many assets.

We consider investors who are risk averse about consumption. In addition they suffer a utility loss if they consume less than members of their reference group. In the following we will use the term "utility from consumption" to refer to the utility an investor would derive from an allocation if he did not care about the consumption of his reference group. The additional utility reduction brought about by falling behind his peers will be termed "psychological loss".

The main intuition for our result is as follows: Without the loss component there is a unique set of asset prices in our exchange economy so that all investors are exactly willing to hold on to their endowment. We will call these prices the benchmark prices in the following. If prices were to deviate from this benchmark, all investors would want to simultaneously buy or sell and the asset markets would not clear anymore.

With loss aversion over relative consumption, however, the investors might not want to adjust the portfolio they are holding. Suppose, all peers of an investor keep their endowment of a risky asset although asset prices diverge from the benchmark. Then the investor faces the following trade off: Either he replicates his peers' portfolio and faces reduced utility from consumption. Or he maximizes utility from consumption but risks a psychological loss because he consumes less than his reference group if his portfolio performs badly.

The latter effect will dominate if asset prices are not too far away from the benchmark. This holds because a small deviation from the portfolio that maximizes utility from consumption results only in a second order loss, while the psychological loss is of first order. The latter is an implication of the kink at the reference point and, for that matter, of loss aversion. This implies that equilibrium asset prices are no longer unique. Instead, we can support a range of asset prices around the benchmark level in equilibrium.

If the asset is risk free, trading in it only changes the intertemporal distribution of consumption but does not expose the investor to the risk that his lifetime consumption is lowered relative to the consumption of his reference group. Therefore, the pricing of a risk free asset is not affected by the introduction of loss aversion over relative consumption. Hence, deviations from the benchmark price are deviations of the risk premium from the benchmark risk premium.

When describing investor preferences we depart from the strict von Neumann-Morgenstern paradigm in two ways, both of which are well established in the literature. First, the investors in our model do not derive utility exclusively from the absolute amount of their own consumption but compare the amount they consume to the consumption of a reference group. This idea dates back at least to Veblen (1922) and has recently been applied to a number of research fields. In macroeconomics and consumption based asset pricing so called "catching up with the Joneses" specifications of the utility function (Abel 1990, Campbell and Cochrane 1999) have been used. In microeconomics utility functions that include equity concerns have proved useful to organize the data obtained from many experiments (Fehr and Schmidt 1999, Bolton and Ockenfels 2000).

The second component is loss aversion, that is, investors suffer a psychological loss the more their consumption drops below a reference level and this loss is concave in the distance from the reference point. Loss aversion has been introduced by Kahnemann and Tversky (1979) as a building block of their prospect theory to explain a broad array of so called anomalies that had been found in experimental studies. Recently, it has been applied to financial markets by Benartzi and Thaler (1995) and Barberis, Santos, and Huang (2001), who introduce investors that suffer a loss, whenever the value of their portfolio falls below the value it had in the previous period. In contrast to Benartzi and Thaler (1995) and Barberis, Santos, and Huang (2001), investors in our model suffer a loss if members of their reference group consume more than they do. This implies that if everybody gets rich in a bull market an investor who gets only a small positive return suffers a utility loss.

Economists have studied the implications of relative consumption concerns for asset prices at least since Abel (1990), who introduced consumers who compare their own consumption to past aggregate consumption. More closely related to our approach are Gali (1994) and Gomez (2007) who, like us, consider investors who evaluate their consumption relative to the contemporaneous consumption of their peers. They find, at least for certain specifications, that relative income concerns can contribute to the explanation of asset pricing anomalies such as the equity premium puzzle. These models, however, have unique equilibria and cannot serve as a tool to study coordination problems in financial markets. Multiple equilibria do not arise because relative consumption concerns in this literature do not take the form of loss aversion. It is the combination of loss aversion and relative income concerns that gives rise to the multiplicity of equilibria.

Most closely related our approach is DeMarzo, Kaniel, and Kremer (2007). Like us, they obtain multiple equilibria, some of which involve time-varying returns. They use a model in which relative income concerns arise endogenously because investors anticipate that they will want to buy a scarce asset in the future; this asset becomes more expensive for an investor whenever the other investors in his cohort have earned high returns. An investor can insure against the price changes in the scarce asset if he replicates the portfolio his cohort is buying. A core assumption is that the asset cannot be traded ex-ante, that is, markets are incomplete. With our preference formulation, multiple equilibria can arise even if markets are complete.

In section 2 we lay out the basic model, section 3 characterizes the equilibria and section 5 concludes.

2 The Model

The model we use is a three period ($t \in \{1, 2, 3\}$) version of the Lucas (1978) exchange economy. The investors consume a perishable consumption good, which is exclusively produced as a dividend by a three-period-lived asset, called the tree asset. It is in fixed supply of one and in period 1 all investors are endowed with an equal amount. In every period t, the investors first derive a dividend of d_t units of the consumption good from each unit of the asset they hold; the random dividend payment is i.i.d. distributed with the pdf $f(d)$. Then the investors must decide how much to trade in the asset at the ex dividend price of p_t, before they consume their end of period holdings of the consumption good.

In contrast to Lucas (1978) we do not employ a representative investor but a continuum of identical investors of size one, who compare themselves to all other investors[1]. Each investor i evaluates his own consumption stream (c_{i1}, c_{i2}, c_{i3}) relative to the consumption of his peers (c_{j1}, c_{j2}, c_{i3}) according to

$$EU_i = E\left[U^C(c_{i1}, c_{i2}, c_{i3}) + U^L(c_{i1}, c_{i2}, c_{i3}, c_{j1}, c_{j2}, c_{j3})\right].$$

Let β be a discount factor, then utility from consumption (U^C) is given by

$$U^C(c_{i1}, c_{i2}, c_{i3}) = \sum_{t=1}^{3} \beta^{t-1} u(c_{it}).$$

The investor values consumption ($u'(\cdot) > 0$) and is risk averse ($u''(\cdot) < 0$). We assume Inada conditions $\lim_{c \to 0} u'(c) = \infty$ and $\lim_{c \to \infty} u'(c) = 0$ to guarantee an interior solution.

The loss component of utility (U^L) is a function of the difference in consumption between investor i and the investors in his reference group, indexed by j. To compare losses that occur at different points in time, we have to introduce a discount factor. We follow Barberis, Santos, and Huang (2001) in assuming

[1]Because all agents will hold the same portfolio in equilibrium, any equilibrium can also be supported with any other reference group specification.

that investors discount losses with a rate that is equal to the risk free rate. To clarify that this discount factor is exogenous, we write it directly as function of the utility function (which is exogenous) and the aggregate consumption, which is equal to the realization of the dividend payment and, thus, exogenous as well:

$$\rho_t = \begin{cases} 1, & \text{if } t = 0 \ , \\ E_t \ \beta \frac{u'(d_{t+1})}{u'(d_t)}, & \text{if } t > 0 \end{cases}$$

The parameter $\alpha \geq 0$ measures the importance of the psychological loss relative to utility from consumption. α can be interpreted as the amount of period 1 Euros the investor is willing to pay to avoid consuming one period 1 Euro less than his reference group. The loss component is then given by

$$U^L(c_{i1}, c_{i2}, c_{i3}, c_{j1}, c_{j2}, c_{j3}) = \alpha \int_0^1 \min \left\{ \sum_{t=1}^{3} \prod_{s=0}^{t-1} \rho_s (c_{it} - c_{jt}), 0 \right\} dj.$$

In this formulation, the investor suffers a loss if he consumes less than his peers. If he consumes more, there is no psychological effect. We have chosen this formulation because it is so simple. For our results to go through, it is only necessary that an agent may suffer a loss if he invests in other assets than his peers. For example, if he suffers also a loss if he consumes more than his peers (as suggested by Fehr and Schmidt (1999) and Bolton and Ockenfels (2000)) our results are qualitatively unaffected. Even if the investor has a psychological gain if he consumes more, our results will still hold as long as this gain is weaker than the loss he suffers when consuming less.

3 Equilibrium

Let $1 + \omega_{it}$ be the amount of the tree asset investor i holds at the end of period t, that is, if ω_{it} is zero, he exactly holds on to his endowment. An equilibrium consists of a sequence of asset prices $\{p_t^*\}_{t=1}^3$ together with a sequence of investment strategies $\{\omega_{it}^*\}_{t=1}^3$ so that, in every period t, given $\{p_t^*\}_{t=0}^3$ and the strategies of the other investors all investors maximize their expected utility, and all markets clear.

We start by considering the benchmark case where all investors only care about their own consumption, that is, $\alpha = 0$. In this case the following proposition holds:

Proposition 1 *If $\alpha = 0$, the unique equilibrium prices of one unit of the tree asset are $p_3^0 = 0$ in period 3 and*

$$p_t^0 = E_t\left(\beta\frac{u'(d_{t+1})}{u'(d_t)}(d_{t+1} + p_{t+1})\right) \tag{1}$$

in periods $t \in \{1, 2\}$

Proof.

The proof is a special case of the proof of Proposition 3 if we set $\alpha = 0$ □

Now suppose $\alpha > 0$. It is instructive to consider first the special case of a deterministic dividend equal to $\hat{d}$: In equilibrium the agents in our economy must find it optimal to keep their endowment, one unit of the tree asset, so that they consume exactly the dividends of one unit of the tree asset in each period. It turns out that the price at which a risk free tree asset trades in the model in periods 1 and 2

$$p_t^{rf} = \beta(\hat{d} + p_{t+1}^{rf})$$

is independent of α. p^{rf} is an equilibrium price by the following argument: From Proposition 1 follows that $\omega_i = 0$ maximizes utility from consumption if $p = p^{rf}$. Furthermore, the psychological loss

$$U^L(\omega_{i1}, \omega_{i2}) = \alpha \min\left\{-p_1\omega_{i1} + \rho_1((d_2 + p_2)\omega_{i1} - p_2\omega_{i2}) + \rho_1\rho_2(d_3 + p_3)\omega_{i2}, 0\right\}$$

is zero for any portfolio ω_{11}, ω_{12} if we plug in $\rho_1 = \rho_2 = \beta$ and $p_t^{rf} = \beta(\hat{d} + p_{t+1}^{rf}$. Hence, to hold on to his endowment is optimal for investor i and p^{rf} is an equilibrium.

The intuition for this result is that a risk free asset just shifts lifetime consumption over time and does not expose the investor to any risk. The way we have chosen to discount the loss component does imply that each investor does not mind consuming less than his reference group today if he can make up for it tomorrow by consuming just the risk free rate times the amount more. The following proposition sums up the result:

Proposition 2 *If var$(d) = 0$ and $E(d) = \hat{d}$ in all three periods, the unique equilibrium prices of one unit of the tree asset are $p_3^{rf} = 0$ in period 3 and*

$$p_t^{rf} = \beta(\hat{d} + p_{t+1}^{rf}) \tag{2}$$

in periods $t \in \{1, 2\}$

Proof.

Plugging in for ρ_1, ρ_2 and p_t^{rf} we get $U^L = 0$. By the arguments used for the proof of Proposition 3, p_t^{rf} is the unique equilibrium price. □

If the tree asset is risky, however, buying more or less of the tree asset does not only shift consumption over time. Additionally, it exposes the investor to the risk that the present value of his lifetime consumption is lower than the one of his reference group. This expected loss makes the investor reluctant to deviate from the portfolio his reference group is holding even if that portfolio does not optimize his expected utility from consumption.

Given that all other investors do hold the endowment, the period t expectation of U^L as a function of ω_{i1} and ω_{i2} is

$$E_t U^L(\omega_{i1}, \omega_{i2}) = \alpha E_t \min\{-p_1\omega_{i1} + \rho_1((d_2 + p_2)\omega_{i1} - p_2\omega_{i2}) + \rho_1\rho_2(d_3 + p_3)\omega_{i2}, 0\}$$

As an example, consider an investor, who has bought the same portfolio as his reference group in period 1 but buys marginally more of the tree asset in period 2. Then he suffers a loss whenever the realization of d_3 is such that

$$\frac{d_3}{p_2} < \frac{1}{\rho_2} = E_2\left(\frac{1}{\beta}\frac{u'(d_2)}{u'(d_3)}\right) = \hat{r}.$$

In other words, he suffers a loss if the realized return is below $\hat{r}$. The threshold return $\hat{r}$ is the return a risk free asset would pay in our exchange economy. Similarly, if he buys a little less of the asset than his reference group, he suffers a loss in every state of the world, in which

$$\frac{d_3}{p_2} > \hat{r}.$$

We can conclude that on expectation there is always a loss, if investor i deviates from the portfolio of his reference group in period 2. If he buys more of the asset, he loses if the dividend is low. If he buys less than his reference group, he suffers if the dividend is high. This loss is always of first order and prevents the investor from deviating from $\omega_{i2} = 0$ even if p_2 is such, that without the psychological loss component, the investor would want to deviate. The same argument can be made for period 1.

From the first order conditions of the utility maximization problem, we can see that an investor does not have an incentive to sell part of his endowment in period t as long as

$$p_t \leq \frac{\beta E_t\, u'(d_{t+1})(d_{t+1} + p_{t+1})}{u'(d_t) + \alpha \int_0^{\frac{p_t}{\rho_t} - p_{t+1}} \left(\rho_t \frac{d_{t+1}+p_{t+1}}{p_t} - 1\right) df(d_{t+1})}. \tag{3}$$

He does not have an incentive to buy more of the tree asset, if

$$p_t \geq \frac{\beta E_t\, u'(d_{t+1})(d_{t+1} + p_{t+1})}{u'(d_t) + \alpha \int_{\frac{p_t}{\rho_t} - p_{t+1}}^{\infty} \left(\rho_t \frac{d_{t+1}+p_{t+1}}{p_t} - 1\right) df(d_{t+1})}. \tag{4}$$

If $\alpha = 0$, that is, for agents who do not suffer from inequity aversion the two conditions coincide and the unique equilibrium price is given by (1). For positive values of α, however, inequality (3) defines a price $\bar{p}$ above which the equilibrium price must not rise and inequality (4) defines a price $\underline{p}$ below which the equilibrium price cannot fall. Because $\bar{p} > \underline{p}$ there is a range of prices and, for that matter, returns that can be supported as an equilibrium.

Proposition 3 *If* $\alpha \geq 0$*,* $p_3 = 0$ *and prices* $p_t \in [\underline{p}_t, \bar{p}_t]$ *for the asset can be supported as an equilibrium in periods* $t \in \{1, 2\}$*. Prices* $\underline{p}_t$ *and* $\bar{p}_t$ *are implicitly defined by*

$$\underline{p}_t = \frac{\beta \int_0^\infty u'(d_{t+1})(d_{t+1} + p_{t+1}) df(d_{t+1})}{u'(d_t) + \alpha \int_{\frac{p_t}{\rho_t} - p_{t+1}}^\infty \left(\rho_t \frac{d_{t+1} + p_{t+1}}{p_t} - 1\right) df(d_{t+1})} \tag{5}$$

and

$$\bar{p}_t = \frac{\beta \int_0^\infty u'(d_{t+1})(d_{t+1} + p_{t+1}) df(d_{t+1})}{u'(d_t) + \alpha \int_0^{\frac{p_t}{\rho_t} - p_{t+1}} \left(\rho_t \frac{d_{t+1} + p_{t+1}}{p_t} - 1\right) df(d_{t+1})} \tag{6}$$

Proof.

Note that for any distribution of portfolio choices each investor maximizes a strictly concave function under linear constraints so that the first order conditions are sufficient and necessary for the (unique) maximum. Because the utility function is not differentiable at $\omega_{it} = 0$, we use the left and right derivative. Because each investor is of negligible size, the maximization problem for each investor i is identical. Thus, equilibrium portfolio choices must be symmetric and market clearing requires that

$$\omega_{it} = 0.$$

Moreover $p_3 = 0$ must hold. Let us assume all other investors hold their share of the market portfolio and market prices are p_1 and p_2. Then, each investor i chooses portfolios $\omega_{i1}(d_1, p_1)$ and $\omega_{i2}(d_1, d_2, p_1, p_2)$, and a consumption plan $c_{i1}(d_1, p_1)$ and $c_{i2}(d_1, d_2, p_1, p_2)$ to maximize expected utility

$$\begin{aligned} E(U(c_i)) = & \int_0^\infty \int_0^\infty \int_0^\infty u(c_{i1}) + \beta u(c_{i2}) + \beta^2 u(c_{i3}) \\ & + \alpha \min\{c_{i1} - d_1 + \rho_1(c_{i2} - d_2) + \rho_1\rho_2(c_{i3} - d_3), 0\} \, df(d_1)df(d_2)df(d_3) \end{aligned}$$

under the constraints of

$$\begin{aligned} c_{i1} &= d_1 - p_1\omega_{i1} \\ c_{i2} &= (d_2 + p_2)(1 + \omega_{i1}) - p_2(1 + \omega_{i2}) \\ c_{i3} &= (d_3 + p_3)(1 + \omega_{i2}) \end{aligned}$$

Taking derivatives with respect to ω_{i1} and ω_{i2} we find that $\omega_{i2} = 0$ is optimal for any realization of d_2 given $\omega_{i1} = 0$ if and only if

$$-\beta u'(d_2)p_2 + \beta^2 \int_0^\infty u'(d_3)d_3 df(d_3) + \alpha \int_0^{\frac{p_2}{\rho_2}} (\rho_2 \frac{d_3}{p_2} - 1) df(d_3) p_2 \leq 0$$

and

$$-\beta u'(d_2)p_2 + \beta^2 \int_0^\infty u'(d_3)d_3 df(d_3) + \alpha \int_{\frac{p_2}{\rho_2}}^\infty (\rho_2 \frac{d_3}{p_2} - 1) df(d_3) p_2 \geq 0$$

which implies for p_2

$$p_2 \geq \frac{\beta \int_0^\infty u'(d_3)d_3 df(d_3)}{u'(d_2) + \alpha \int_{\frac{p_2}{\rho_2}}^\infty \left(\rho_2 \frac{d_3}{p_2} - 1\right) df(d_3)}$$

and

$$p_2 \leq \frac{\beta \int_0^\infty u'(d_3)d_3 df(d_3)}{u'(d_2) + \alpha \int_0^{\frac{p_2}{\rho_2}} \left(\rho_2 \frac{d_3}{p_2} - 1\right) df(d_3)}$$

In addition we get that $\omega_{i1} = 0$ is optimal for any realization of d_1, given $\omega_{i2} = 0$, if

$$-u'(d_1)p_1 + \beta \int_0^\infty u'(d_2)(d_2 + p_2) df(d_2) + \alpha \int_0^{\frac{p_1}{\rho_1} - p_2} \left(\rho_1 \frac{d_2 + p_2}{p_1} - 1\right) df(d_2) p_1 \leq 0$$

and

$$-u'(d_1)p_1 + \beta \int_0^\infty u'(d_2)(d_2 + p_2) df(d_2) + \alpha \int_{\frac{p_1}{\rho_1} - p_2}^\infty \left(\rho_1 \frac{d_2 + p_2}{p_1} - 1\right) df(d_2) p_1 \geq 0$$

which implies for p_1

$$p_1 \geq \frac{\beta \int_0^\infty u'(d_2)(d_2 + p_2) df(d_2)}{u'(d_1) + \alpha \int_{\frac{p_1}{\rho_1} - p_2}^\infty \left(\rho_1 \frac{d_2 + p_2}{p_1} - 1\right) df(d_2)}$$

and

$$p_1 \leq \frac{\beta \int_0^\infty u'(d_2)(d_2 + p_2) df(d_2)}{u'(d_1) + \alpha \int_0^{\frac{p_1}{\rho_1} - p_2} \left(\rho_1 \frac{d_2 + p_2}{p_1} - 1\right) df(d_2)}$$

□

4 Example

As we have shown above, there can be multiple equilibria with different prices in our exchange economy. In the following example, we demonstrate an equilibrium in which time-varying returns obtain. For simplicity we use risk neutral investors with $u(c) = c$.[2] In our model, the following asset prices constitute an equilibrium by Proposition 3:

$$\begin{aligned} p_1 &= \beta E_1(d_2 + p_2) \\ p_2 &= \beta \frac{E_2 d_3}{1 + \alpha \int_0^{\frac{p_2}{\rho_2}} \left(\rho_2 \frac{d_3}{p_2} - 1\right) df(d_3)} \end{aligned}$$

In this equilibrium, expected returns are given by

$$\begin{aligned} r_1 &= \frac{E_1(d_2 + p_2)}{p_1} = \frac{1}{\beta} \\ r_2 &= \frac{E_2 d_3}{p_2} = \frac{1 + \alpha \int_0^{\frac{p_2}{\rho_2}} \left(\rho_2 \frac{d_3}{p_2} - 1\right) df(d_3)}{\beta} < \frac{1}{\beta} \end{aligned}$$

In this equilibrium, the price is higher in period 2 and, therefore, the return is lower for each realization of the dividend. Thus the unconditional expectation of the return is lower in period 2, that is, the return is time-varying.

5 Conclusion

We have shown that investors who exhibit loss aversion over relative consumption may coordinate on holding the endowment of a risky asset over a range of prices around the prices predicted by standard asset pricing models. The choice between the prices is a coordination problem. If the endowment consists of a risk free asset, there is a unique equilibrium price that is identical to the price that would prevail without loss aversion over relative consumption.

[2]We are aware that this example does not fulfill all the assumptions we made in section 2. Still, in this case all results continue to hold.

Future research may use this model as a building block to introduce endogenous changes in asset returns into larger dynamic general equilibrium models. This would allow to investigate the consequences of misaligned asset prices, for example, for aggregate investment, monetary policy or corporate financial decisions.

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