Online Appendix

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THE IMPACT OF A FIRM'S SHARE OF EXPORTS ON REVENUE, WAGES, AND MEASURE OF WORKERS HIRED

— THEORY AND EVIDENCE

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A Mathematical Derivations

Domestic demand y_d and revenue r_d

The preferences of a representative consumer are given by a C.E.S. utility function over a continuum of goods indexed by ω:

$$U = \left[\int_{\omega\in\Omega} y(\omega)^\rho d\omega\right]^{\frac{1}{\rho}} ,$$

where the measure of the set Ω represents the mass of available goods within the sector U. These goods are substitutes, implying $0 < \rho < 1$ and an elasticity of substitution between any two goods of

$$\sigma = \frac{1}{1-\rho} > 1 \quad \Leftrightarrow \quad \rho = 1 - \frac{1}{\sigma} = \frac{\sigma - 1}{\sigma} .$$

The consumer's constrained maximization problem may be solved by the Lagrangian

$$\mathcal{L} = U^\rho - \lambda\left(\int_{\omega\in\Omega} p(\omega)y(\omega)d\omega - I\right) ,$$

where U^ρ is a strictly increasing transformation of U and I the consumer's income. Which yields the following first-order condition

$$\frac{\partial\mathcal{L}}{\partial y(\omega)} = \rho y(\omega)^{\rho-1} - \lambda p(\omega) = 0 .$$

By dividing the first-order condition of one variety ω_1 by the first-order condition of another variety ω_2 we obtain relative demand

$$\frac{y(\omega_1)}{y(\omega_2)} = \left(\frac{p(\omega_1)}{p(\omega_2)}\right)^{\frac{1}{\rho-1}} .$$

Multiply both sides with $y(\omega_2)$ yields

$$y(\omega_1) = y(\omega_2)\left(\frac{p(\omega_1)}{p(\omega_2)}\right)^{-\sigma}.$$

By multiplying both sides with $p(\omega_1)$ and taking the integral with respect to ω_1 we get

$$\int_{\omega\in\Omega} p(\omega_1)y(\omega_1)d\omega_1 = \int_{\omega\in\Omega} y(\omega_2)p(\omega_1)^{1-\sigma}p(\omega_2)^{\sigma}d\omega_1 .$$

On the left-hand side we now have the consumer's total expenditure on all varieties R, which is equal to his income I, i.e.

$$R = I = y(\omega_2)p(\omega_2)^{\sigma}\int_{\omega\in\Omega} p(\omega_1)^{1-\sigma}d\omega_1 .$$

Solving for $y(\omega_2)$ yields the Marshallian demand for ω_2

$$y(\omega_2) = \frac{Ip(\omega_2)^{-\sigma}}{\int_{\omega\in\Omega} p(\omega_1)^{1-\sigma}d\omega_1}.$$

Let the aggregate price be given by

$$P = \left[\int_{\omega\in\Omega} p(\omega)^{1-\sigma}d\omega\right]^{\frac{1}{1-\sigma}},$$

Marshallian demand is equal to

$$y(\omega) = p(\omega)^{-\sigma}P^{\sigma-1}I = \left(\frac{p(\omega)}{P}\right)^{-\sigma}\frac{I}{P}.$$

With domestic output $y_d(\omega)$ being equal to $y(\omega)$ for non-exporting firms, domestic firm revenue can then be written as

$$r_d(\omega) = y_d(\omega)\cdot p(\omega) = I\left(\frac{p(\omega)}{P}\right)^{1-\sigma},$$

where I_d and P_d indicate domestic income and the domestic aggregate price, respectively. Note that with $p_d(\omega) = y_d(\omega)^{\frac{1}{-\sigma}}I_d^{\frac{1}{\sigma}}P_d^{\frac{\sigma-1}{\sigma}}$ domestic revenue can also be written as in HIR, i.e.

$$r_d(\omega) = y_d(\omega)^{1-\frac{1}{\sigma}}I_d^{\frac{1}{\sigma}}P_d^{\frac{\sigma-1}{\sigma}} = y_d(\omega)^{\rho}I_d^{1-\rho}P_d^{\rho} = y_d(\omega)^{\rho}A_d ,$$

where A_d is called the domestic demand shifter.

Link between A_d and $\tilde{\varphi}_t$

We can link the demand shifter A_d to the total average productivity $\tilde{\varphi}_t$ as defined in Melitz (2003), p.1710, by using equation (17) and (18) from ibid.:

$$A_d = R^{1-\rho}P^{\rho} = (M_t\bar{r})^{1-\rho}\left(M_t^{\frac{1}{1-\sigma}}\frac{w(\tilde{\varphi}_t)}{\rho\tilde{\varphi}_t}\right)^{\rho} = \bar{r}^{1-\rho}\left(\frac{w(\tilde{\varphi}_t)}{\tilde{\varphi}_t\rho}\right)^{\rho}. \qquad \text{(A.1)}$$

Υ and a derivation of $y_d(\varphi) = y(\varphi)/\Upsilon$

Using (4) we can write

$$y(\varphi) = y_d(\varphi) + y_{x,1}(\varphi) + \ldots + y_{x,c'}(\varphi)$$

as

$$y(\varphi) = y_d(\varphi) + \tau_1^{\frac{\rho}{\rho-1}} y_d(\varphi) \left(\frac{A_{x,1}}{A_d}\right)^{\frac{\rho}{1-\rho}} + \ldots + \tau_{c'}^{\frac{\rho}{\rho-1}} y_d(\varphi) \left(\frac{A_{x,c'}}{A_d}\right)^{\frac{\rho}{1-\rho}}$$
$$y(\varphi) = y_d(\varphi) \left(1 + \tau_1^{\frac{\rho}{\rho-1}} \left(\frac{A_{x,1}}{A_d}\right)^{\frac{\rho}{1-\rho}} + \ldots + \tau_{c'}^{\frac{\rho}{\rho-1}} \left(\frac{A_{x,c'}}{A_d}\right)^{\frac{\rho}{1-\rho}}\right).$$

By defining $\Upsilon = 1 + \tau_c^{\frac{\rho}{\rho-1}} \left(\frac{A_{x,1}}{A_d}\right)^{\frac{\rho}{1-\rho}} + \ldots + \tau_{c'}^{\frac{\rho}{\rho-1}} \left(\frac{A_{x,c'}}{A_d}\right)^{\frac{\rho}{1-\rho}}$ we obtain

$$y_d(\varphi) = y(\varphi)/\Upsilon\,.$$

A firm's total revenue r

$$r(\varphi) = y_d(\varphi)^\rho A_d + \tau_1^{-\rho} y_{x,1}(\varphi)^\rho A_{x,1} + \ldots + \tau_{c'}^{-\rho} y_{x,c'}(\varphi)^\rho A_{x,c'}$$

using the first-order conditions (4) this can be written as

$$= y_d(\varphi)^\rho A_d + \tau_1^{\frac{\rho}{\rho-1}} y_d(\varphi)^\rho A_{x,1} \left(\frac{A_{x,1}}{A_d}\right)^{\frac{\rho}{1-\rho}} + \ldots + \tau_{c'}^{\frac{\rho}{\rho-1}} y_d(\varphi)^\rho A_{x,c'} \left(\frac{A_{x,c'}}{A_d}\right)^{\frac{\rho}{1-\rho}}$$
$$= y_d(\varphi)^\rho A_d \left(1 + \tau_1^{\frac{\rho}{\rho-1}} \left(\frac{A_{x,1}}{A_d}\right)^{\frac{1}{1-\rho}} + \ldots + \tau_{c'}^{\frac{-\rho}{1-\rho}} \left(\frac{A_{x,c'}}{A_d}\right)^{\frac{1}{1-\rho}}\right)$$
$$= y(\varphi)^\rho A_d \left(1 + \sum_{c=1}^{c'} \tau_c^{\frac{\rho}{\rho-1}} \left(\frac{A_{x,c}}{A_d}\right)^{\frac{1}{1-\rho}}\right)^{1-\rho}.$$

Revenue as a function of a firm's productivity

Using the earlier definition of $r(\varphi)$ in (3), the production function, and the first-order conditions (7) and (8), we are now able to express revenue as

$$r(\varphi) = \Upsilon(\varphi)^{\frac{1-\rho}{\Gamma}} A^{\frac{1}{\Gamma}} \left(\frac{\zeta}{\zeta-1} a_{\min}^{\gamma\zeta} \varphi \left(\frac{\rho\gamma}{(1+\rho\gamma)b}\right)^\gamma \left(\frac{\rho(1-\gamma\zeta)}{\varepsilon(1+\rho\gamma)}\right)^{\frac{1-\gamma\zeta}{\delta}}\right)^{\frac{\rho}{\Gamma}}, \tag{A.2}$$

where $\Gamma = 1 - \rho\gamma - \rho(1-\gamma\zeta)/\delta$. In the next step we compute the firm's profits by making once more use of the first-order conditions

$$\pi(\varphi) = \frac{\Gamma}{1+\rho\gamma} r(\varphi) - f_d - \sum_{c=1}^{c'} f_{x,c}\,.$$

Furthermore we know that the firm with the lowest productivity φ_d makes exactly zero profits (and is not exporting). Thence it follows[17]

$$\frac{\Gamma}{1+\rho\gamma} r(\varphi_d) = f_d \quad \Rightarrow \quad r(\varphi_d) \equiv r_d = \frac{1+\rho\gamma}{\Gamma} f_d \,. \tag{A.3}$$

In the following, we use the expression for $r(\varphi)$ from (A.2) and determine the relative revenue of a firm in comparison to the firm with the lowest productivity φ_d:

$$\frac{r(\varphi)}{r_d} = \Upsilon^{\frac{1-\rho}{\Gamma}} \left(\frac{\varphi}{\varphi_d}\right)^{\frac{\rho}{\Gamma}} \quad \Rightarrow \quad r(\varphi) = r_d \cdot \left(\frac{\varphi}{\varphi_d}\right)^{\frac{\rho}{\Gamma}} \cdot \Upsilon^{\frac{1-\rho}{\Gamma}}$$

Wage as a function of a firm's productivity

By the same token we are able to compute $a_\varepsilon(\varphi)$. We again employ the first-order conditions (7) and (8) and get

$$\frac{a_\varepsilon(\varphi)^\delta}{a_\varepsilon(\varphi_d)^\delta} = \Upsilon^{\frac{1-\rho}{\Gamma}} \left(\frac{\varphi}{\varphi_d}\right)^{\frac{\rho}{\Gamma}} \quad \Rightarrow \quad a_\varepsilon(\varphi) = a_\varepsilon(\varphi_d) \Upsilon^{\frac{1-\rho}{\delta\Gamma}} \left(\frac{\varphi}{\varphi_d}\right)^{\frac{\rho}{\delta\Gamma}} \,. \tag{A.4}$$

Using (A.3) with the first-order conditions and (8), we can compute

$$a_\varepsilon(\varphi_d) = \left(\frac{\rho(1-\gamma k)}{(1+\rho\gamma)\varepsilon} \frac{1+\rho\gamma}{\Gamma} f_d\right)^{\frac{1}{\delta}} = \left(\frac{\rho(1-\gamma\zeta_d)}{\varepsilon\Gamma} f_d\right)^{\frac{1}{\delta}} \,.$$

With the wage condition from (9), the lowest wage paid by a domestic firm is then

$$w(\varphi_d) \equiv w_d = b \left(\frac{a_\varepsilon(\varphi_d)}{a_{\min}}\right)^{\zeta_d} = \left(\frac{\rho(1-\gamma\zeta_d)}{\varepsilon\Gamma a_{\min}^\delta} f_d\right)^{\frac{\zeta_d}{\delta}} \,.$$

This yields a wage relation that is solely dependent on φ, $\Upsilon(\varphi)$, φ_d, b, and parameters:

$$\frac{w(\varphi)}{w_d} = \left(\frac{a_\varepsilon(\varphi)}{a_\varepsilon(\varphi_d)}\right)^{\zeta_d} = \Upsilon^{\frac{\zeta_d(1-\rho)}{\delta\Gamma}} \left(\frac{\varphi}{\varphi_d}\right)^{\frac{\rho\zeta_d}{\delta\Gamma}} \quad \Rightarrow \quad w(\varphi) = w_d \cdot \left(\frac{\varphi}{\varphi_d}\right)^{\frac{\rho\zeta_d}{\delta\Gamma}} \cdot \Upsilon^{\frac{\zeta_d(1-\rho)}{\delta\Gamma}} \,.$$

As can be seen from this last equation, wages increase with firm productivity and are always higher for exporting firms than for non-exporting firms.

Measure of workers hired as a function of a firm's productivity

By the same token, we can derive the lowest measure of workers hired

$$h(\varphi_d) \equiv h_d = m(\varphi_d) \left(\frac{a_{\min,d}}{a_\varepsilon(\varphi_d)}\right)^{\zeta_d} = \frac{\rho\gamma}{1+\rho\gamma} \frac{r_d}{b} \left(\frac{a_{\min,d}}{a_\varepsilon(\varphi_d)}\right)^{\zeta_d} \,.$$

Using (A.4) and (6), the relation to $h(\varphi)$ is then given by

$$\begin{aligned} \frac{h(\varphi)}{h_d} = \frac{r(\varphi)}{r_d} \left(\frac{a_\varepsilon(\varphi_d)}{a_\varepsilon(\varphi)}\right)^{\zeta_d} &= \Upsilon^{\frac{1-\rho}{\Gamma}} \left(\frac{\varphi}{\varphi_d}\right)^{\frac{\rho}{\Gamma}} \Upsilon^{\frac{\zeta_d(\rho-1)}{\delta\Gamma}} \left(\frac{\varphi}{\varphi_d}\right)^{\frac{-\zeta_d\rho}{\delta\Gamma}} \\ &= \Upsilon^{\frac{(1-\rho)(1-\zeta_d/\delta)}{\Gamma}} \left(\frac{\varphi}{\varphi_d}\right)^{\rho\left(1-\frac{\zeta_d}{\delta}\right)} \,. \end{aligned}$$

[17] Note, while $r_d(\varphi)$ is the domestic revenue for a firm with productivity φ, r_d is the revenue of a non-exporting firm with zero profits.

Which ultimalety leads to

$$h(\varphi) = h_d \cdot \left(\frac{\varphi}{\varphi_d}\right)^{\rho\left(1-\frac{\zeta_d}{\delta}\right)} \cdot \Upsilon^{\frac{(1-\rho)(1-\zeta_d/\delta)}{\Gamma}} .$$