Tests and Power Comparisons in Time-Dynamic Copula Models

Workshop "Limit Theorems in Number Theory and Probability Theory"
30./31.07.2012

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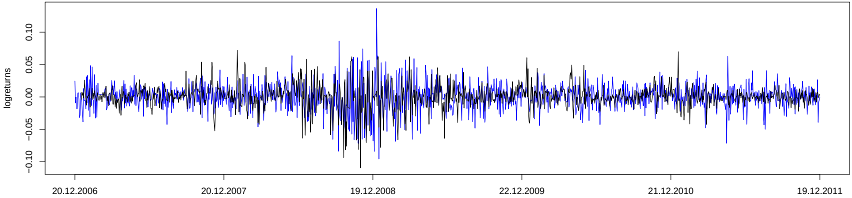
Outline

Heteroscedastic Time Series



$d \geq 2 \quad \boldsymbol{Y}_t = \boldsymbol{\Sigma}_t^{1/2} \boldsymbol{\varepsilon}_t \quad \text{with } \boldsymbol{\varepsilon}_t \sim \text{WN}(\mathbf{0}, \mathbf{C}^*) \text{ iid}$

$$\boldsymbol{\Sigma}_t = \text{Cov}\left(\boldsymbol{Y}_t \,|\mathcal{F}_{t-1}\right), \text{ where } \mathcal{F}_t = \sigma(\boldsymbol{Y}_t, \boldsymbol{Y}_{t-1}, \ldots)$$

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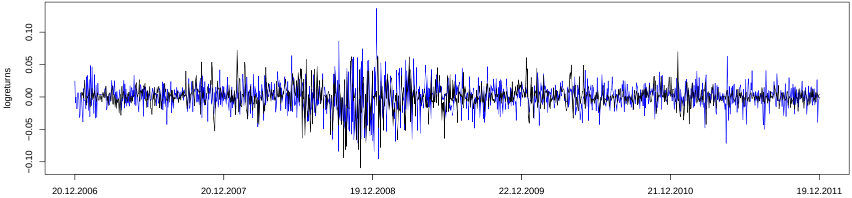

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$$\text{With} \quad \boldsymbol{\Sigma}_t = \text{diag}(\sigma_{1t}^2, \ldots, \sigma_{dt}^2)$$

the dependence is captured by the multivariate distribution of the residuals

$$\boldsymbol{\varepsilon}_t \sim F(\theta) = C(F_1, \ldots, F_d; \theta) \text{ independent for all } t.$$

Heterogeneous Dependence Structure

(Parsimonious) univariate Garch(p, q)-models for every $1 \leq j \leq d$

$$Y_{jt} = \sigma_{jt}\varepsilon_{jt},\ \sigma_{jt}^2 = \alpha_{j0} + \sum_{i=1}^{p} \alpha_{ji} Y_{j,t-i}^2 + \sum_{k=1}^{q} \beta_{jk}\sigma_{j,t-k}^2$$

are linked through the dependence structure of the residuals.

Heterogeneity in the dependence via time-dynamic parameters θ_t for every t

$$\boldsymbol{\varepsilon}_t \sim F(\theta_t),$$

such that $\boldsymbol{\varepsilon}_t$ are still independent, but not identically distributed.

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Parameter Stability

Test the null hypothesis

$$\mathrm{H}_0\colon\ \{\theta_t \equiv \theta_0\ \forall\, t\}$$

against the alternative that the parameter changes over time

$$\mathrm{H}_A\colon\ \{\theta_1 = \ldots = \theta_{t_1} \neq \theta_{t_1+1} = \ldots = \theta_{t_k} \neq \theta_{t_k+1} = \ldots\ldots = \theta_T\}\,.$$

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- estimation of parameters and construction of tests

1. DeGarching

Let the corresponding observations of $\boldsymbol{Y}_t = (Y_{1t}, \dots, Y_{dt})^\top$ be

$$(y_{1t}, \dots, y_{dt})^\top,\ 1 \leq t \leq T.$$

The parameter vector $\gamma_j = (\alpha_{j0}, \alpha_{j1}, \dots, \alpha_{jp}, \beta_{t1}, \dots, \beta_{tq})^\top$ for every component $1 \leq j \leq d$ is estimated via quasi-maximum-likelihood (QMLE):

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- assume that $\varepsilon_{jt} \sim \mathcal{N}(0,1)$ for all $1 \le t \le T$
- maximize the likelihood function L_T to obtain $\hat{\gamma}_j$
  - L_T as function of observations and some starting values y_0, σ_0^2

Even if the normality assumption fails, but $\mathbb{E}\,\varepsilon_{jt}^4 < \infty$, it holds that for $T \to \infty$

1. $\hat{\gamma}_j \xrightarrow{a.s.} \gamma_j$
2. $\sqrt{T}(\hat{\gamma}_j - \gamma_j) \xrightarrow{d} \mathcal{N}(0, \mathrm{v})$

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Since $\sigma_{jt}^2 = w_t(\gamma_j)$, we obtain empirical residuals $\tilde{\varepsilon}_t$ with components

$$\tilde{\varepsilon}_{jt} = \frac{y_{jt}}{\hat{\sigma}_{jt}}, \quad \text{where} \quad \hat{\sigma}_{jt} = \sqrt{\hat{w}_t(\hat{\gamma}_j)}.$$

2. Canonical Maximum Likelihood

Semi-parametric estimation on $(\tilde{\varepsilon}_{1t}, \ldots, \tilde{\varepsilon}_{dt})^\top \sim C(F_1, \ldots, F_d; \theta)$

- rank-transformation for the marginal distributions $\tilde{u}_{jt} = \frac{1}{T+1} \sum_{s=1}^{T} \mathbb{1}_{\{\tilde{\varepsilon}_{js} \leq \tilde{\varepsilon}_{jt}\}}$
- maximum likelihood estimation

$$\hat{\theta}_t = \underset{\theta}{\operatorname{argmax}} \sum_{s=t-b+1}^{t} \log c(\tilde{u}_{1s}, \ldots, \tilde{u}_{ds}; \theta)$$

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According to Chan et al. (2009) it holds that for $T \to \infty$ and if $b = o(T)$

1. $\hat{\theta}_t \xrightarrow{P} \theta$ for all t
2. $\sqrt{b}\,(\hat{\theta}_t - \theta) \xrightarrow{d} \mathcal{N}(0, \Sigma)$

with variance due to Genest et al. (1995)

$$\Sigma = \frac{\operatorname{Var}\left(\frac{\partial}{\partial\theta} \log c(F_1(\varepsilon_1), \ldots, F_d(\varepsilon_d); \theta) + \sum_{j=1}^{d} W_j(\varepsilon_j)\right)}{\mathbb{E}\left([\frac{\partial}{\partial\theta} \log c(F_1(\varepsilon_1), \ldots, F_d(\varepsilon_d); \theta)]^2\right)^2} =: \frac{\sigma^2}{\beta^2}$$

where $W_j(\varepsilon_j) = \int_{[0,1]^d} \mathbb{1}_{\{F_j(\varepsilon_j) \leq u_j\}} \frac{\partial^2}{\partial\theta \partial u_j} \log c(u_1, \ldots, u_d; \theta) dC(u_1, \ldots, u_d; \theta)$.

Simple Test - Construction and Power

Under the simple hypothesis $\mathrm{H}_{0,k}\colon \{\theta_{t_k} = \theta_0\}$ consider the statistic

$$T_{bk} = \frac{\sqrt{b}(\hat{\theta}_{t_k} - \hat{\theta})}{\sqrt{\hat{\Sigma}}} \xrightarrow[(T\to\infty)]{d} \mathcal{N}(0,1)$$

with $\hat{\theta}$ a global estimate and $\hat{\Sigma}$ a consistent variance estimate.

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Under the local alternative $\mathrm{H}_{A,k}\colon \{\theta_{t_k} = \theta_b^{\pm}(x) = \theta_0 \pm \frac{x}{\sqrt{b}}\}$ and for $x \neq 0$ it holds

$$\mathbb{P}\left(|\hat{T}_{bk}| > q_{\alpha/2} \,\middle|\, \mathrm{H}_{A,k}\right) \xrightarrow[(T\to\infty)]{} \gamma(x) > \alpha$$

with $q_{\alpha/2} = \mathcal{N}_{0,1}^{-1}(1-\alpha/2)$.

The local test $|\hat{T}_{bk}| > q_{\alpha/2}$ has an asymptotic local power γ of size $\sqrt{b}$.

Multiple Test - Construction

For the local alternative $\mathrm{H}_{A,k}\colon \{\theta_{t_k} = \theta_b^{\pm}(x)\}$ we consider

$$X_k := \mathbb{1}_{\{|\hat{T}_{bk}|>q_{\alpha/2}\}} \xrightarrow{d} \begin{cases} \mathrm{Bin}(1,\alpha) & \text{under } \mathrm{H}_{0,k}, \\ \mathrm{Bin}(1,\gamma) & \text{under } \mathrm{H}_{A,k}. \end{cases}$$

For independent $\mathrm{H}_{0,k}$ the global null hypothesis $\mathrm{H}_0 = \bigcap_{k=1}^{n} \mathrm{H}_{0,k}$ corresponds to

$$\mathrm{H}_0\colon \{\theta_{t_1} = \ldots = \theta_{t_n} \equiv \theta_0 \ \text{ for } |t_i - t_j| > b, i \neq j\}.$$

Under H_0 a suitable test statistic $S_n = \sum_{k=1}^{n} X_k$ is asymptotically $\mathrm{Bin}(n,\alpha)$ and the test has proper size $\alpha^\star$ if

$$\mathbb{P}\left(S_n \geq \kappa \,\middle|\, \mathrm{H}_0\right) = \sum_{j=\kappa}^{n} \binom{n}{j} \alpha^j (1-\alpha)^{n-j} \leq \alpha^\star.$$

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Fix $\alpha, \alpha^\star \implies$ some $\kappa \notin \mathbb{N}$ in general, need to randomize the test.

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Fix $\kappa, \alpha^\star \Longrightarrow$ solution $\alpha \in (0,1)$, choose $c_{\alpha^\star} = \kappa - 1$ as critical value.

Multiple Test - Power

Consider the multiple alternative

$$\mathrm{H}_A\colon \{\theta_k = \theta_b^{\pm}(x), 1 \leq k \leq m\} \cap \{\theta_k = \theta_0, k > m\} = \bigcap_{k=1}^{m} \mathrm{H}_{A,k} \cap \bigcap_{k=m+1}^{n} \mathrm{H}_{0,k}.$$

Then under H_A the test statistic is a convolution of binomial distributions with different probabilities. In particular, it holds that

$$S_n \,\big|\, \mathrm{H}_A = S_m^{(\gamma)} + S_{n-m}^{(\alpha)} \overset{st}{\geq} S_m^{(\alpha)} + S_{n-m}^{(\alpha)} = S_n^{(\alpha)} = S_n \,\big|\, \mathrm{H}_0.$$

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Therefore, for $x \neq 0$ it holds that

$$\Gamma(x) = \lim_{T\to\infty} \mathbb{P}\left(S_n > c_{\alpha^\star} \,\middle|\, \mathrm{H}_A\right) \ge \lim_{T\to\infty} \mathbb{P}\left(S_n > c_{\alpha^\star} \,\middle|\, \mathrm{H}_0\right) = \alpha^\star.$$

The multiple test $S_n > c_{\alpha^\star}$ has also an asymptotic local power Γ of size $\sqrt{b}$.

Omnibus Test - Construction

The global null hypothesis $\mathrm{H}_0\colon \{\theta_1 = \ldots = \theta_T \equiv \theta_0\}$ implies $\mathrm{H}_0\colon \{\max_t \theta_t = \theta_0\}$ and can be tested against the alternative

$$\mathrm{H}_A^\star\colon \{\max_t \theta_t \neq \theta_0\}.$$

For this, define a thinned estimator series for all $k = 1, \ldots, N = \frac{T-b}{cb}$ with a thinning constant $c \in (1/b, 1]$

$$\xi_{t_k} := \frac{\sqrt{b}(\hat{\theta}_{t_k} - \hat{\theta})}{\sqrt{\hat{\Sigma}}} \xrightarrow{d} \mathcal{N}(0,1).$$

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Theorem (asymptotic extremal distribution)

For $N = N(T) \xrightarrow[(T\to\infty)]{} \infty$, $M_N = \max_k \xi_{t_k}$ *and if* $b > T^{7/9}$, *it holds that*

$$\lim_{T\to\infty} \mathbb{P}\left((M_N - d_N)/a_N \leq x\right) = \Lambda(x) = \mathrm{e}^{-\mathrm{e}^{-x}} \quad \forall\, x \in \mathbb{R},$$

where $a_N = \sqrt{2\log N}$ *and* $d_N = a_N - \frac{\log\log N - \log 2\pi}{2a_N}$.

Sketch of proof I

Consider the joint distribution of $\left(\hat{\theta}_{t_1}, \ldots, \hat{\theta}_{t_N}\right)^\top$

$$\Rightarrow \quad \boldsymbol{S}_b = \frac{1}{\sqrt{b}}\left(\boldsymbol{X}^{(1)} + \ldots\ldots + \boldsymbol{X}^{(b)}\right) = \frac{1}{\sqrt{b}}\left(\boldsymbol{Y}^{(1)} + \ldots + \boldsymbol{Y}^{(cb)}\right)$$

with

$$\mathbf{C}_N^2 = \operatorname{Cov} \boldsymbol{Y}^{(k)} = \begin{pmatrix} \kappa & \kappa-1 & \ldots & 1 & & 0 \\ & \ddots & \ddots & & \ddots & \\ & & \ddots & \ddots & & 1 \\ & & & \ddots & \ddots & \vdots \\ & & & & \ddots & \kappa-1 \\ & & & & & \kappa \end{pmatrix}$$

symmetric, positive definite, $(\kappa-1)$-banded, Toeplitz with decaying elements.

Sketch of proof II

Due to Berenhaut and Bandyopadhyay (2005) the Cholesky square root

$$\mathbf{C}_N = \begin{pmatrix} c_{11} & & & & \\ \vdots & \ddots & & 0 & \\ c_{\kappa 1} & & \ddots & & \\ & \ddots & & \ddots & \\ 0 & & c_{N\kappa} & \dots & c_{NN} \end{pmatrix}$$

is lower triangular, $(\kappa - 1)$-banded, with monotone entries $c_{ij} \geq c_{i+1,j} \geq 0$

$\Longrightarrow \mathbf{C}^2 = \lim_{N\to\infty} \mathbf{C}_N^2$ and $\mathbf{C} = \lim_{N\to\infty} \mathbf{C}_N$ are bounded operators on $\ell^2(\mathbb{Z})$.

Due to Demko et al. (1984) the inverse matrix $\mathbf{C}^- = (c_{ij}^-)$ is lower triangular and exhibits off-diagonal exponential decay, i.e.

$$|c_{ij}^-| \leq const \cdot \lambda_1^{|i-j|}$$

$\Longrightarrow$ matrix entries c_{ij}^- are absolutely summable.

Sketch of proof III

Lyapunov-type Berry-Esseen bounds in a mv CLT due to Bentkus (2004)

$$\Delta_b = \sup_{\boldsymbol{x}\in\mathbb{R}^N} \left|F_{\boldsymbol{S}_b}(\boldsymbol{x}) - \Phi_{N,C^2}(\boldsymbol{x})\right| \leq const \cdot N^{1/4} \cdot \beta,$$

where $\beta = \beta_1 + \ldots + \beta_{cb}$ with $\beta_k = \mathbb{E}\left|\mathbf{C}^{-}\boldsymbol{Y}^{(k)}\right|^3$.

For $b > T^{7/9}$ it holds that

$$\Delta_b \xrightarrow[(T\to\infty)]{} 0.$$

The proof concludes with classical extreme value theory for weakly dependent normal random variables (e.g. Leadbetter et al., 1983). □

Omnibus Test - Power

For $x > 0$ specify the local alternative

$$\mathrm{H}_A^\star: \left\{ \max_k \theta_{t_k} = \theta_0 + x/\sqrt{b/\log N} \right\}.$$

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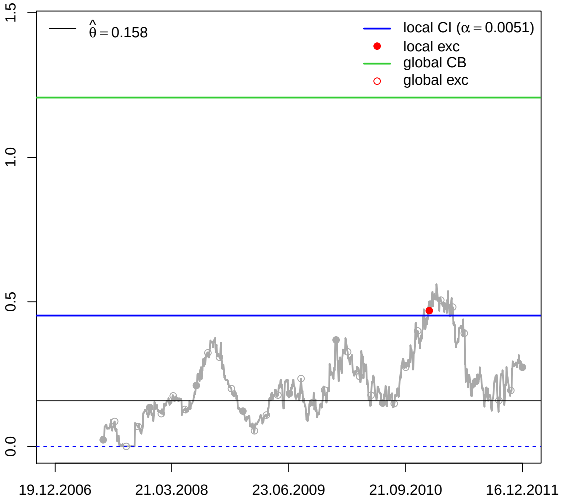
With $\lambda_{\alpha^\star} = \Lambda^{-1}(1-\alpha^\star)$, the quantile of the Gumbel extremal distribution, it holds that

$$\mathbb{P}\left((M_N - d_N)/a_N > \lambda_{\alpha^\star} \,\middle|\, \mathrm{H}_A^\star\right) \xrightarrow[(T\to\infty)]{} \Gamma^\star(x) > \alpha^\star.$$

The omnibus test $(M_N - d_N)/a_N > \lambda_{\alpha^\star}$ has a local power $\Gamma^\star$ of size $\sqrt{b/\log N}$.

Baseload vs. peakload electricity in the UK

Coal vs. German Electricity



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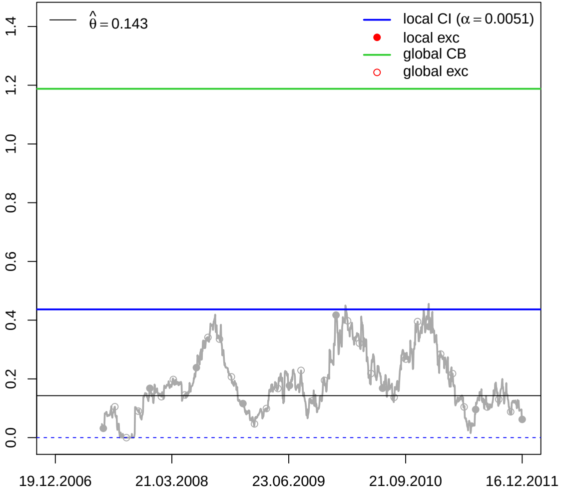

Summary & Outlook

- In most cases H_0 isn't rejected (especially for shorter time series)
- Some of the exceedances can be remedied by univariate regimes
- Some can't $\Longrightarrow$ Change Point test

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Regime change at $T/2$

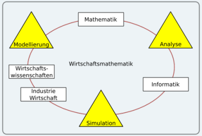
Hypotheses: $\mathrm{H}_0 : \{\theta_t \equiv \theta_0\}$ vs. $\mathrm{H}_A : \{\theta_t = \theta_0 \pm \frac{x}{\sqrt{T}}$ for $t > T/2\}$

Statistic: $$\bar{S}_{Tb} = \frac{1}{\sqrt{T}} \sum_{t=b}^{T/2} (\hat{\theta}_t - \hat{\theta}_{t+T/2}) \xrightarrow{d} \mathcal{N}(0, \Sigma)$$

Test: $\left| \bar{S}_{Tb} / \sqrt{\hat{\Sigma}} \right| > q_{\alpha/2}$ has an asymptotic local power of size $\sqrt{T}$

test	local power	contras	pros
Extremal	$\sqrt{b/\log N}$	low power	omnibus test
Binomial	$\sqrt{b}$	independent hypotheses	hints at location of cp
Change Point	$\sqrt{T}$	a priori knowledge of cp	high power

Contact



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Thank you for your attention

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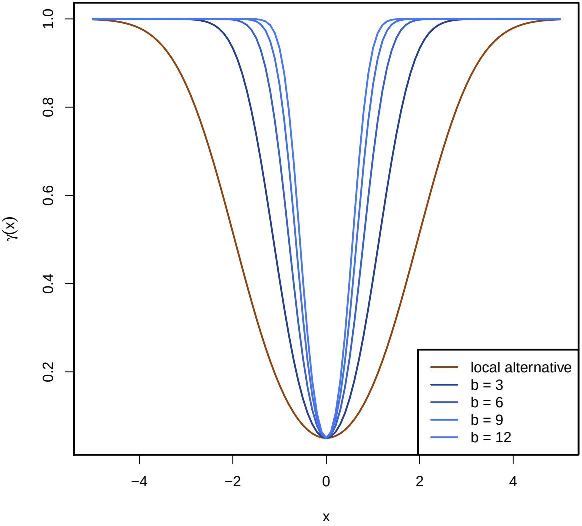
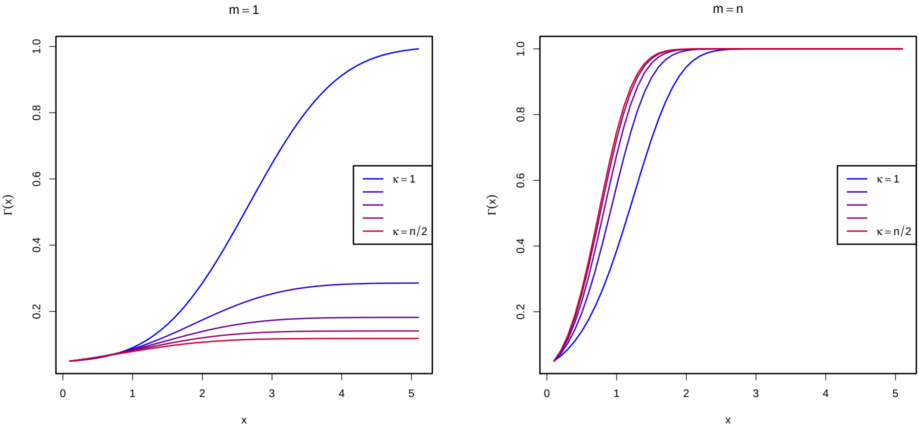
Local Power (I)



Local Power (II)



(a) "One false" alternative (b) "All false" alternative