Mortality maps based on spatial extrapolation

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Abstract. Mortality maps display the spatial demographical structure of mortality which is commonly described via death rates. In the present paper, mortality maps of the population in Germany are considered over the time horizon from 1996 to 2003. We address regional differences and similarities of the mortality in South, East, and West Germany over various age groups. Furthermore, we analyze the time dynamics of the mortality across Germany. The spatial structure of the mortality maps is investigated by means of binary images. They are obtained via thresholding with respect to weighted averages of death rates, which is a common technique in morphological image analysis. Moreover, considering the differences of such binary images, we show that it is possible to analyze the changes of mortality in space and time. The consequences of our observations are discussed for the risk based pricing of life insurances.

JEL classification: J11, C51, C88

Key words: Stochastic mortality, space–time analysis, extrapolation, life insurance pricing

1 Introduction

The mortality of a population is typically described via death rates depending on age, gender, and year. Death rates form an important input factor for contractual calculations of life insurances or pension plans. For example the insurance industry in Germany uses so–called mortality tables (see [7]) which form a summary of projected current and future death rates. In contrast to the premium calculation of third party liability insurances of motor vehicles which depend on the region the insurant lives, the premium of German life insurances do usually not depend on the regional characteristics of mortality. This paper analyzes the differences and similarities in the spatial demographical structure of mortality in Germany via so–called mortality maps. Moreover, changes of mortality in space and time are also discussed.

The paper is organized as follows. After introducing the data in Section 2.1, we construct mortality maps via spatial extrapolation for several (sub–) populations within Germany. In Section 2.2, we explain the utilized extrapolation technique which is based on an inverse distance method. Then, in Section 2.3, an explorative analysis of the constructed mortality maps follows, where the spatial structure of these maps is investigated by means of binary images. They are obtained via thresholding, which is a common technique in morphological image analysis. Moreover, considering the differences of such binary images, we show in Section 2.4 that it is possible to analyze the changes of mortality in space and time. Thus, our results refer to the regional structure of mortality as well as to its dynamics over time.

In Sections 3.1 and 3.2, the implications of our observations are discussed for the risk based pricing of life insurance contracts. Finally, in Section 3.3, we still mention another, more sophisticated method for the statistical analysis of mortality maps which will be discussed in detail in a forthcoming paper (see [2]). This method is based on mathematical models from stochastic geometry and spatial statistics. It can be used to build asymptotic significance tests for the evaluation of structural differences and similarities in mortality maps.

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2 Construction and explorative analysis of mortality maps

2.1 Data processing

Our analysis is based on a demographical data set provided by the Federal Statistical Office in Wiesbaden, Germany. It consists of the population sizes and the numbers of deaths per administrative district and non–district town in Germany – we refer to both as districts – over the time horizon from 1995 to 2003. The data are given by gender and age groups. Death rates – per district and year – are calculated as the ratio of the number of deaths with respect to the size of the considered (sub–) population in the preceding year, i.e. the death rates m_t are calculated by

$$m_t = \frac{d_t}{p_{t-1}}\ , \qquad t = 1996, \ldots, 2003,$$

where d_t and p_t, respectively, denote the number of deaths and the size of the population in year t. The age and gender indexing is suppressed for notational convenience. We did analyze the impact of migration on the calculation of death rates and came to the conclusion that it is negligible.

For the creation of mortality maps, the death rates of an administrative district were projected to the city in which the local government is seated. For those cities and non–district towns, the Euclidean coordinates (per postal code) were taken to get their topological location within Germany. If a city had several postal codes, the Euclidean coordinates were averaged. There were two federal states, namely Hamburg and Berlin, which were projected to themselves as cities. Thus, death rates are given for 322 administrative districts and 106 non–district towns, spread over Germany; see Figure 1.

Figure 1: Topology of the administrative districts and the non–district towns.

Data analysis was done using the GeoStoch library system. GeoStoch is a Java–based open–library system developed by the Department of Applied Information Processing and the Department of Stochastics at the University of Ulm which can be used for stochastic–geometric data analysis and spatial statistics ([12]). See also the internet description of this project under *http://www.geostoch.de.*

2.2 Extrapolation and grouping

The extrapolation method we utilize is a modification of the so–called *inverse distance method*; see [6],[11]. For each location within a certain region, say Germany, the method calculates a weighted average of the given death rates by putting a higher weight on those measurement points which are closer to the actually considered location.

Suppose that in region $W \subseteq \mathbb{R}^2$ the death rates $m_t(u_i)$ at the locations $u_i \in W$, $i = 1, ..., n$, and years $t = 1996, \ldots, 2003$ are observed. These data are aggregated into three groups according to the three periods 1996–1998, 1999–2001 and 2002–2003, which we refer to as time horizons t_1, t_2, and t_3, respectively. Notice that we group the data in order to reduce the noise of the yearly observed death rates. In particular, the corresponding death rates are calculated by

$$m_{t_j}(u_i) = \frac{\sum\limits_{t \in t_j} p_{t-1}(u_i) m_t(u_i)}{\sum\limits_{t \in t_j} p_{t-1}(u_i)}\,, \qquad j = 1, 2, 3,$$

where $m_t(u_i)$ and $p_t(u_i)$ denote the mortality rate and the population size of the district, respectively, which is associated with location u_i in year t. Furthermore, the estimation $\hat{m}_{t_j}(u_0)$ of the value $m_{t_j}(u_0)$ at a non–observed location $u_0 \in W$ with $u_0 \neq u_i$ for all $i = 1, ..., n$ is given by the linear convex combination

$$\hat{m}_{t_j}(u_0) = \sum_{i=1}^{n} \lambda_{ij} m_{t_j}(u_i)\,, \qquad j = 1, 2, 3, \tag{1}$$

where $\lambda_{ij} \geq 0$ and $\sum_{i=1}^{n} \lambda_{ij} = 1$. In particular, under the assumption that $u_0 \neq u_i$ for all $i = 1, ..., n$, the weights λ_{ij} are defined by

$$\lambda_{ij} = \frac{p_{t_j}(u_i)}{|u_i - u_0|^3} \left(\sum_{k=1}^{n} \frac{p_{t_j}(u_k)}{|u_k - u_0|^3} \right)^{-1} \tag{2}$$

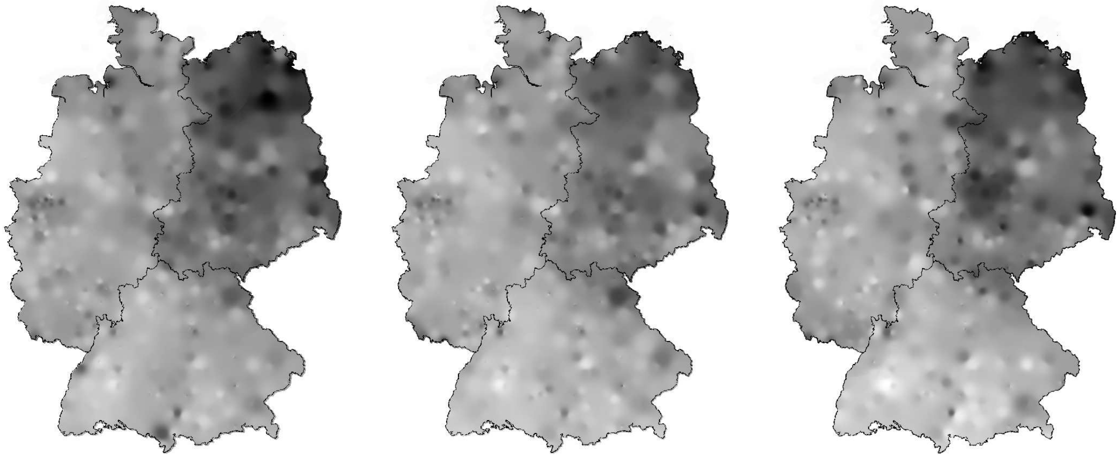
for all $j = 1, 2, 3$ and $i = 1, \ldots, n$, where $|\cdot|$ denotes the Euclidean norm and $p_{t_j}(u_i) = \sum_{t \in t_j} p_t(u_i)$. Notice that the larger the distance $|u_i - u_0|$ of the measurement point u_i from the considered location u_0, the smaller is its weight with respect to this measurement. The decay of the weights regarding the distance is cubical. Other rates of decay have been considered and the sensitivity of the descriptive and statistical analysis is reported in [2]. Moreover, we set the weights λ_{ij} equal to zero if $|u_i - u_0| > r$ for some maximum distance $r > 0$, representing the idea that there is only an influence from cities which are in the immediate neighborhood. The radius r is chosen such that for every pixel belonging to Germany a mortality rate can be estimated, i.e., $r = 80$ (km).

Figure 2 exemplarily presents three mortality maps. The mortality maps are grey scale images in which every pixel has a value between 0 and 255. A high value corresponds to a low death rate, i.e., light pixels refer to a low death rate, whereas dark pixels refer to a high death rate. The resolution of the images is 631×850 pixels.

From the mortality maps presented in Figure 2 it can be seen that there are regional and temporal differences, but also similarities of mortality. In Sections 2.3 and 2.4 below, we state some typical results which were obtained in an explorative analysis of mortality maps.

2.3 Regional mortalities

In our analysis of the spatial structure of mortality, we concentrate on three regions of Germany: South, east and west. The southern region of Germany consists of the states Bavaria and Baden–Württemberg, the eastern region denotes the states of formerly East Germany, and the remaining



(a) Time horizon: 1996–1998 (b) Time horizon: 1999–2001 (c) Time horizon: 2002–2003

Figure 2: Mortality maps based on extrapolated death rates for the German population of age 50 and younger over the time horizons 1996–1998 (left plot), 1999–2001 (middle plot) and 2002–2003 (right plot).

states form the western region of Germany. Several methods for the exploration of the regional differences and similarities of mortalities are thinkable. In the following we will focus on the so–called *threshold method*. For this method we consider a threshold μ_{t_j} which is defined via the following weighted average of death rates per period t_j :

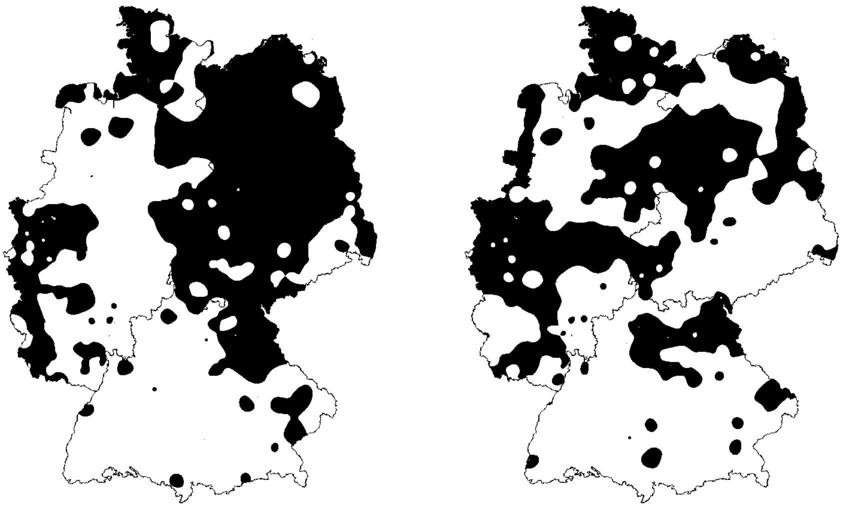
$$\mu_{t_j} = \frac{1}{\sum_{i=1}^{n} p_{t_j}(u_i)} \sum_{i=1}^{n} p_{t_j}(u_i) m_{t_j}(u_i), \qquad j = 1, 2, 3.$$

For example, in Figure 3 the black regions correspond to locations where (measured or estimated) death rates exceed the threshold μ_{t_j} and the white regions refer to locations where death rates are below the threshold. This results in a binary image where the decision between ‚black' and ‚white' is performed for every pixel.

Notice that Figure 3 reveals an interesting evolvement of the mortality for the female population aged between 50 and 65 over the time horizons 1996–1998 and 2002–2003. On the one hand it shows that the mortality levels in South Germany are lower than in East and West Germany in both time periods. On the other hand it illustrates the remarkable fact that the spatial distribution of mortality between East Germany and West Germany is more homogeneous in their deviations from the threshold μ_{t_j} in period 2002–2003 (t_3) than it is in period 1996–1998 (t_1). The latter development of the mortality can be already observed since the reunification of Germany, and its reasons are complex. Empirical studies such as [8], [9], [10], [15], and [16] examine a large list of explanatory variables and conclude that the higher mortality in East Germany – during and after the reunification – is mainly driven by the following reasons. First of all, diseases of the cardiovascular system occurred more frequently in East Germany than in the remaining parts. The cause of death due to these diseases was more pronounced for the female population. Moreover, [15] argues that the psycho–social stress caused by the social, political, and economic changes during the reunification are the most convincing rational for the higher mortalities in East Germany. A key variable in this context is in particular the effect of unemployment. This justifies the fact that the mortality was notably higher in the middle aged population. Further important factors were the poor economic conditions, the higher environmental pollution, increased consumption of alcohol, and more fatal traffic accidents in East Germany.

Notice, however, that in Section 3.1, we point out that the average mortality of the middle aged female population in East Germany dropped below the corresponding mortality level in West

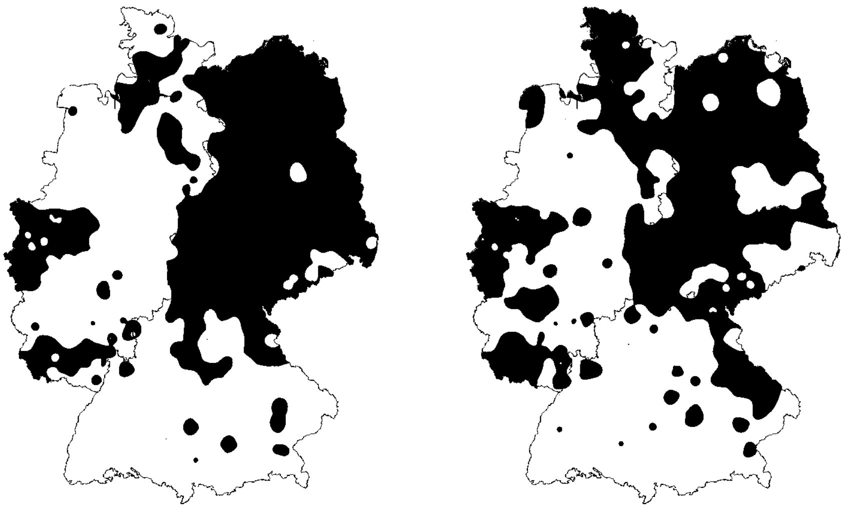
Germany during the time horizon 2002–2003.



(a) Time horizon: 1996–1998 (b) Time horizon: 2002–2003

Figure 3: Mortality maps based on the threshold method for the German female population aged between 50 and 65 over the time horizons 1996–1998 (left plot) and 2002–2003 (right plot).

Figure 4 shows that the effect of a more homogeneous mortality between East and West Germany in 2002–2003 is less pronounced for the older female population aged between 65 and 75.



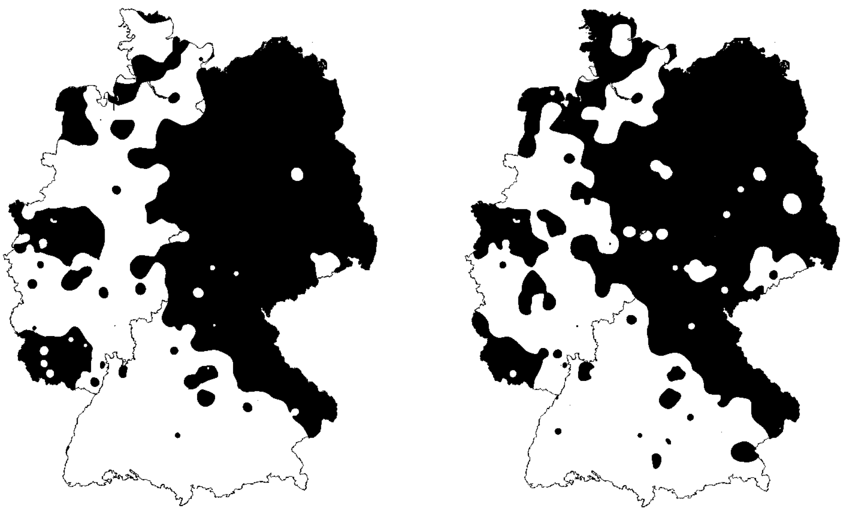
(a) Time horizon: 1996–1998 (b) Time horizon: 2002–2003

Figure 4: Mortality maps based on the threshold method for the female population aged between 65 and 75 over the time horizons 1996–1998 (left plot) and 2002–2003 (right plot).

In Figure 5 we examine the mortality of the German male population aged between 50 and 65 over the time horizons 1996–1998 and 2002–2003. The figure points out that nearly all death rates in East Germany of this subpopulation are above the threshold. This structure is representative for all (male) age groups and does not really change over the observed time horizons.

The above results are astonishing as they show a quite different behavior of the male population in contrast to the female population. The reasons for these observations are similar to the explanations given above, with the difference that the converging effects of the female mortality, as shown in Figure 3, can not be seen in the corresponding male subpopulation. Furthermore, the mortalities in South Germany are usually below the threshold except in the north–eastern part – the Bavarian forest. For this particular region, [3] found out that a higher proportion of the above subpopulation dies due to cancer in the respiratory or alimentary system. Moreover, the latter reference mentions the low economical development of this region as a possible determinant.

However, the overall fraction of black areas in South Germany (death rates above the threshold) seems to be much smaller than in West Germany. In Section 3.3, we briefly explain how one can build a formal test in order to detect such differences of mortality automatically, using methods of stochastic geometry and spatial statistics. A more detailed description of this model–based approach to statistical analysis of mortality maps is undertaken in the forthcoming paper [2]. This also includes the statistical analysis of other combinations of regions, genders and age groups.

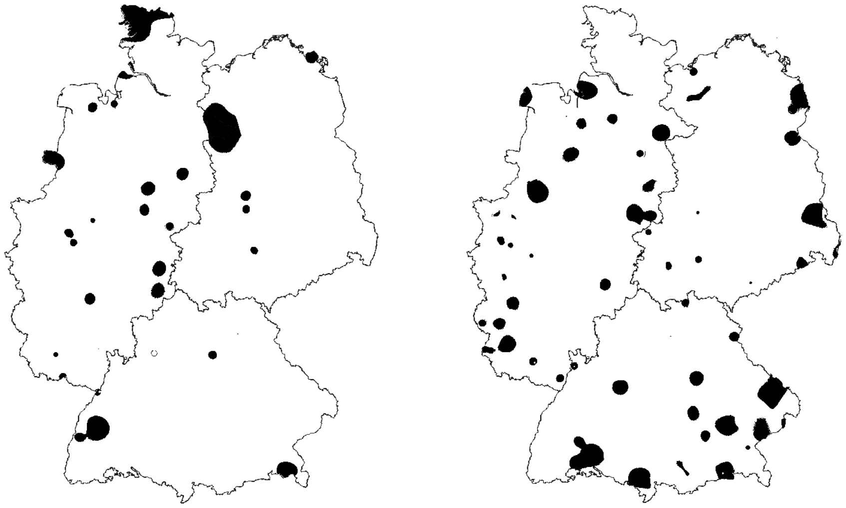


(a) Time horizon: 1996–1998 (b) Time horizon: 2002–2003

Figure 5: Mortality maps based on the threshold method for the male population aged between 50 and 65 over the time horizons 1996–1998 (left plot) and 2002–2003 (right plot).

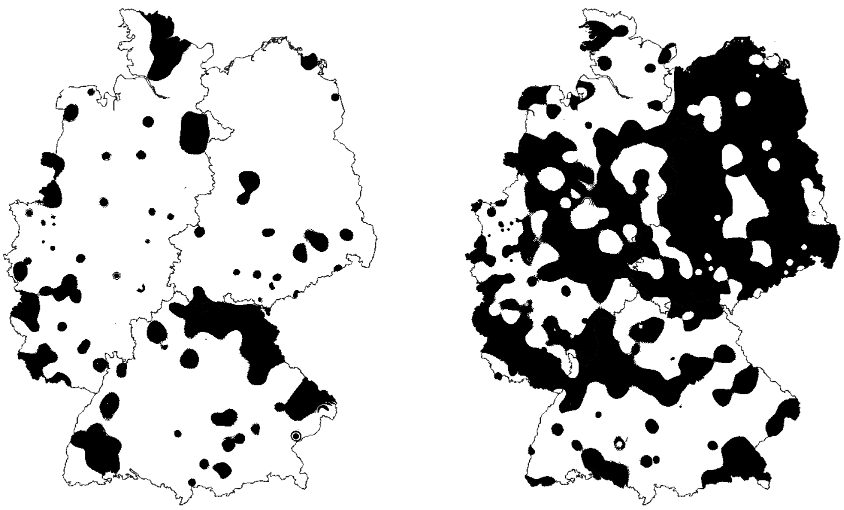
2.4 Dynamics of the mortality over time

Supplementary to the regional structure of mortality, as considered in the preceding section, we now investigate the dynamics of mortality over time. For that we calculate the point– or pixel–wise differences of two mortality maps, which we call *difference method*. Figures 6 and 7 illustrate the resulting signs of the differences for time horizons 1996–1998 minus 1999–2001 (t_1 minus t_2) and 1999–2001 minus 2002–2003 (t_2 minus t_3). More precisely, the black area refers to a negative sign, corresponding to an increasing death rate, and the white area stands for a positive sign, representing a decreasing death rate and, thus, mortality improvement. A precise analysis of the development of mortality is essential for the stability of many industries, in particular, for the life insurance industry and pension funds. It is well known, see e.g. [1], that the average mortality in Germany has followed a decreasing trend during the last 40 years. In order to account for this trend, the German Actuarial Institute (DAV) regularly adjusts mortality tables which serve as a basis for the pricing of life insurance contracts/pension plans and for the calculation of related risk reserves. The latest adjustment of German mortality tables was implemented in 2004; see [7]. Due to the long–term binding character of e.g. pension plans, the profit of those contracts depend strongly, among others, on the (future) development of mortality. Mortality maps, which are based on the difference method, provide a useful graphical tool in order to assess the space–time dynamics of the mortality. The corresponding statistical tests for such changes in time are indicated in Section 3.3.



(a) Difference: 1996–1998/1999–2001 (b) Difference: 1999–2001/2002–2003

Figure 6: Mortality maps based on the difference method for the German population aged between 65 and 75 between the time horizons 1996–1998 and 1999–2001 (left plot) and 1999–2001 and 2002–2003 (right plot).



(a) Difference: 1996–1998/1999–2001 (b) Difference: 1999–2001/2002–2003

Figure 7: Mortality maps based on the difference method for the German population aged 50 and younger between the time horizons 1996–1998 and 1999–2001 (left plot) and 1999–2001 and 2002–2003 (right plot).

From Figures 6 and 7 we make two main observations. First, the mortality of the German population aged between 65 and 75 is improving over the considered time horizons; see Figure 6. Commonly accepted reasons for that trend are innovations in medicine and health care and an improved welfare system.

However, there is currently a controversial debate whether mortality improvements will continue in this extent as we have experienced it during the last decades. In this context, Figure 7 yields an interesting fact by showing a declining mortality trend within the younger population aged 50 and younger between the time horizons 1999–2001 and 2002–2003. This can be seen by the extent of black areas in the right mortality map of Figure 7.

3 Concluding remarks and possible extensions

3.1 Risk based pricing in life insurance

The regional differences of mortality in South, East, and West Germany, as detected in Section 2.3, have implications on the premium of life insurance contracts or pension plans. Consider an initial cohort of size l_{x_0,I_j} all aged x_0. The indexing I_j refers to one of the three regions: South (I_1), East (I_2), and West Germany (I_3). We consider a simple term insurance which pays an amount P at death occurring up to a maximum age of $x_0 + N$ years. Furthermore, we distinguish for gender and the time horizon t_j, but suppress the corresponding index for notational convenience. Let $\bar{m}_{x,I_j}$ denote the average – population weighted – death rate in region I_j at age x. Then, $d_{x,I_j} = l_{x,I_j} \cdot \bar{m}_{x,I_j}$ is the expected number of deaths at age x, where $l_{x+1,I_j} = l_{x,I_j} \cdot \big(1 - \bar{m}_{x,I_j}\big)$. Let ρ denote the risk free rate, then the premium A_{x_0,I_j} for this insurance, calculated via the equivalence principle, is given by

$$A_{x_0,I_j} = P \cdot \sum_{x=x_0}^{x_0+N} (1+\rho)^{x_0-x-1} \cdot \frac{d_{x,I_j}}{l_{x_0,I_j}} \,. \tag{3}$$

Notice that the premium A_{x_0,I_j} given in (3) does not depend on the size of the initial cohort l_{x_0,I_j} if the values of l_{x,I_j} and d_{x,I_j} are not rounded.

3.2 Numerical example

In order to illustrate the practical use of formula (3), we consider the following numerical example. Table 1 contains the resulting premiums for men and women depending on period and region. The observed death rates in each period are assumed to be constant in the future (therefore we do not forecast the death rates for the next $N = 35$ years), the risk free rate is set to $\rho = 0.0325$ and the payoff is $P = 30000$.

Notice that the premiums A_{x_0,I_j} computed according to (3) serve also as an indicator for the space–time structure of the (average) mortality. In particular, the results in Table 1 confirm the findings of the previous analysis and allow the following conclusions:

1. The premiums for the male population in East Germany do always exceed the corresponding premiums in West and South Germany. By contrast, the premiums for the female population in East Germany fall below the corresponding premiums in West Germany in the time horizons 1999–2001 and 2002–2003 for the initial cohort aged $x_0 = 30$.
2. All premiums for a specific combination of age and region decrease from 1996–1998 to 2002–2003.
3. Overall, the premiums for the male population are greater than the corresponding premiums for the female population.

	men			women		
	1996–1998	1999–2001	2002–2003	1996–1998	1999–2001	2002–2003
Initial cohort all aged $x_0 = 30$						
East I_1	3168	2799	2651	1456	1255	1189
West I_2	2626	2402	2318	1364	1298	1252
South I_3	2413	2203	2068	1222	1143	1091
Initial cohort all aged $x_0 = 35$						
East I_1	4686	4424	4205	2601	2184	2145
West I_2	4120	3954	3792	2375	2175	2148
South I_3	3804	3639	3420	2148	1943	1895
Initial cohort all aged $x_0 = 40$						
East I_1	6206	6051	5764	3797	3166	3157
West I_2	5657	5543	5320	3439	3108	3107
South I_3	5243	5122	4830	3130	2793	2760

Table 1: Term–insurance premium A_{x_0,I_j} based on death rates for various time horizons and regions I_j (East, West, and South Germany).

4. The converging effect of the East and West German mortality dispersion around the average mortality (threshold) is more pronounced for the female population than for the male population. Further, this convergence is less evident between South Germany and the remaining parts of Germany. In fact, the differences between the mortality levels in West and South German are notably increasing.

5. The effects of a mortality improvement within the female population in East Germany are less pronounced with increasing age of the initial cohort.

3.3 Model–based analysis of mortality maps

Following the explorative analysis of mortality maps which has been developed in the previous sections, we now explain how one can turn to a model–based statistical analysis of the binary images obtained by the threshold and difference methods discussed in Sections 2.3 and 2.4, respectively.

This approach is based on models and methods from stochastic geometry and spatial statistics; see [13], [17], [18]. In particular, we assume that the black part of any given binary image is a realization of a so–called stationary random set Ξ in $\mathbb{R}^2$ which is observed in a bounded sampling window $W \subseteq \mathbb{R}^2$. Among others, the area fraction p_Ξ of the random set Ξ will be considered which is the expected area of Ξ per unit area, i.e., $p_\Xi = \mathrm{E}\,|\Xi \cap [0,1]^2|$. By the ratio

$$\widehat{p}_\Xi(W) = \frac{|\Xi \cap W|}{|W|}$$

a natural (unbiased) estimator for p_Ξ is given which can be easily computed from a single realization of $\Xi \cap W$. Under some additional model assumptions, it can be shown that $\widehat{p}_\Xi(W)$ is asymptotically normal distributed and the asymptotic variance

$$\sigma^2 = \lim_{|W|\to\infty} \sqrt{|W|}\ \mathrm{Var}\,\widehat{p}_\Xi(W)$$

of $\widehat{p}_\Xi(W)$ can be consistently estimated as the size $|W|$ of the sampling window W becomes unboundedly large; see e.g. [4], [5], [14]. The estimator $\widehat{\sigma}^2_\Xi(W)$ for σ^2 we utilize has the form

$$\widehat{\sigma}^2_\Xi(W) = \int_{\mathbb{R}^2} \widehat{\mathrm{Cov}}_{\Xi,W}(x)\,\gamma_W(x)\,dx$$

for some weighting function $\gamma_W(x) \geq 0$ and a consistent estimator $\widehat{\mathrm{Cov}}_{\Xi,W}(x)$ for the covariance $\mathrm{Cov}(Y_\Xi(o), Y_\Xi(x))$ of the stationary random field $\{Y_\Xi(x),\, x \in \mathbb{R}^2\}$, where

$$Y_\Xi(x) = \begin{cases} 1 & \text{if } x \in \Xi, \\ 0 & \text{if } x \notin \Xi. \end{cases}$$

Then, asymptotic Gaussian significance tests can be constructed to verify hypotheses, e.g., of the form $H_0:\ p_\Xi = c$ (versus $H_1 : p_\Xi < c$ or $H_1 : p_\Xi > c$) for some hypothetical area fraction $c \in (0,1)$. For example, this type of test can be used to analyze the difference maps considered in Section 2.4. Putting $c = 0.5$, it can be checked whether the mortality of a (sub-) population within given age group and region is significantly improving or not.

In a similar way, asymptotic Gaussian significance tests can be constructed to verify the hypothesis whether the area fractions p_{Ξ_1}, p_{Ξ_2} of two (independent) stationary random sets Ξ_1 and Ξ_2 are equal or not, i.e.,

$$H_0:\ p_{\Xi_1} = p_{\Xi_2} \qquad \text{versus} \qquad H_1:\ p_{\Xi_1} \neq p_{\Xi_2}\,.$$

Then, modelling mortality in two disjoint regions (say, I_1 and I_2) by Ξ_1 and Ξ_2, respectively, a rejection of the null hypothesis $H_0:\ p_{\Xi_1} = p_{\Xi_2}$ may lead, e.g., to a different regional pricing of life insurance contracts based on the actual death rates.

We also remark that, instead of analyzing single binary images extracted from mortality maps, images with three and even more gray levels can be investigated which are obtained if the mortality maps are segmented by several (say, $\ell > 1$) increasing thresholds simultaneously. Notice that such images can be described by ℓ–dimensional vectors of binary images. In other words, the mortality in a given region can then be modelled by a vector $(\Xi_1, \ldots, \Xi_\ell)$ of ℓ (non–independent) stationary random sets with the respective vector of area fractions $p_{\Xi_1}, \ldots, p_{\Xi_\ell}$ such that $p_{\Xi_1} \geq \ldots \geq p_{\Xi_\ell}$. Again, asymptotic significance tests can be constructed to verify hypotheses of the form $H_0:\ (p_{\Xi_1}, \ldots, p_{\Xi_\ell}) = c$ for some $c \in (0,1)^\ell$, or to verify whether the vectors of area fractions coincide which correspond to two independent ℓ–dimensional vectors of (non–independent) stationcary random sets.

A more detailed elaboration of these methods and the related results are the subject of the forthcoming paper [2].

Acknowledgement

This project is partly supported by a grant of the Deutsche Forschungsgemeinschaft (DFG–Graduiertenkolleg 1100). Stefanie Eckel holds a scholarship of this Graduiertenkolleg.

References

[1] B. Babel, E. Bomsdorf and R. Schmidt (2005) On the future dynamics of German mortality. *Journal of Population Economics* (under revision).

[2] B. Babel, S. Eckel, R. Schmidt and V. Schmidt (2006) Model–based analysis of mortality maps. Working paper (under preparation).

[3] Bayerisches Landesamt für Gesundheit und Lebensmittelsicherheit (2004) Bericht: Regionale Unterschiede der Sterblichkeit in Bayern. München.

[4] S. Böhm (2003) Asymptotische Signifikanztests für stationäre zufällige Mengen und ihre Anwendung bei der Untersuchung der Grundwasserbeschaffenheit in Baden–Württemberg. Doctoral Dissertation, University of Ulm.

[5] S. Böhm, L. Heinrich and V. Schmidt (2004) Asymptotic properties of estimators for the volume fractions of jointly stationary random sets. *Statistica Neerlandica* **58**, 388–406.

[6] P.A. Burrough (1986) *Principles of Geographical Information Systems for Land Resource Assessment*. Monographs on Soil and Resources Survey, Vol. **12**, Clarendon Press, Oxford.

[7] Deutsche Aktuarvereinigung (2004) Herleitung der DAV–Sterbetafel 2004R für Rentenversicherungen. Internal Report.

[8] C. Gräb (1994) Sterbefälle 1993 nach Todesursachen. *Wirtschaft und Statistik* **12**, 1033–1041.

[9] B. Häussler, E. Hempel and P. Reschke (1995) Die Entwicklung von Lebenserwartung und Sterblichkeit in Ostdeutschland nach der Wende (1989-1992). *Öffentliches Gesundheitswesen* **57**, 365–372.

[10] C. Höhn and J. Pollard (1991) Mortality in the two Germanies in 1986 and trends 1976–1986. *European Journal of Population* **7**, 1–28.

[11] T.A. Jones, D.E. Hamilton and C.R. Johnson (1986) *Contouring Geologic Surfaces with the Computer*. Van Nostrand Reinhold, London.

[12] J. Mayer, V. Schmidt and F. Schweiggert (2004). A unified simulation framework for spatial stochastic model. *Simulation Modelling Practice and Theory*, **12**, 307–326.

[13] I. Molchanov (1997) *Statistics of the Boolean Model for Practitioners and Mathematicians*. J. Wiley & Sons, Chichester.

[14] U. Pantle, E. Spodarev and V. Schmidt (2006) Central limit theorems for functionals of germ–grain models. *Advances in Applied Probability* (in press).

[15] R.T. Riphahn and K.F. Zimmermann (1998) The Mortality Crisis in East Germany. IZA Working Paper No. 6, http://ssrn.com/abstract=166329.

[16] R.T. Riphahn and K.F. Zimmermann (2000) The Mortality Crisis in East Germany. In: G.A. Cornia, R. Paniccià (eds), *The Mortality Crisis in Transitional Economies*. Oxford University Press, Oxford, 227–252.

[17] R. Schneider and W. Weil (2000) *Stochastische Geometrie*. Teubner, Stuttgart.

[18] D. Stoyan, W.S. Kendall and J. Mecke (1995) *Stochastic Geometry and its Applications* (2nd ed.). J. Wiley & Sons, Chichester.