Securitized Senior Life Settlements' Macauley Duration and Longevity Risk

Carlos E. Ortiz
Arcadia University
Department of Mathematics and Computer Science
ortiz@arcadia.edu

Charles A. Stone
Brooklyn College, City University of New York
Department of Economics
zfinance@interserv.com

Anne Zissu
Temple University
Department of Finance
zissu@sprynet.com

In this paper we illustrate that Macaulay duration is an inaccurate metric of interest rate risk for fixed income securities that are backed by pools of senior life settlement contracts. The inaccuracy is due to the fact that the timing of cash flows from senior life settlement instruments, both the life insurance premiums and the death benefits, are a function of how long the associated pool of insured actually lives. Deviations in actual life from life expectancy make statements about the interest rate risk of a security backed by senior life settlements conjectural. It is only possible to summarize the interest rate risk of a pool of senior life settlements if the statement is modified to account for the uncertainty of cash flow timing and magnitude. We calculate the relationship between duration and changes in the timing of cash flows to and from senior life settlement contracts to illustrate the weakness of using duration to quantify interest rate risk for life settlement securities. Once we have illustrated the weakness of using the duration metric for this class of securities we offer a solution to the problem of not being able to summarize interest rate risk with duration. Our approach is similar to what has been done in the market for collateralized mortgage obligations. We demonstrate that it is possible to carve out a class of securities from a pool of senior life settlements that will have a stable duration across a wide range of weighted average lives of a pool of senior life settlement contracts.

Key words: Senior life settlements, longevity risk, mortality tables, duration.

Introduction and Motivations

In 2004 the first securities backed by a pool of senior life settlement contracts was structured and issued. It was a $63 Million *class A* senior life settlement-securitization backed by $195 million in face value of life insurance policies, issued by Tarrytown Second, LLC. If the market for asset-backed securities collateralized by senior life settlements is to grow to its full potential, the measurements of risks specific to this class of securities must be developed and refined.

Senior life settlements are contracts that transfer a life insurance policy from the insured to a financial entity. The financial entity purchases the policy at a discount from the owner. The discount will be a function of the present value of the premiums due and the death benefit both of which depend upon the life expectancy of the insured and the yield curve. Once the life insurance policy is transferred from the owner to the financial entity that has bought it, the purchaser of the policy, the life settlement company, is liable for the premium payments and becomes the beneficiary. Upon the death of the insured the life settlement company receives the death benefit.

The value of the fixed stream of premium payments and future death benefits are exposed to interest rate risk. If the death date of the insured were known with certainty, duration would be a useful measure of interest rate risk like it is for any fixed income security whose cash flows are not tied to embedded options such as mortgage backed securities, callable bonds and bank deposits or to external factors such natural disasters or credit events.

The term "senior" in senior life settlement contracts refers to the age of those people who life settlement companies will buy insurance contracts from. In the current market the life expectancy of the supply of life insurance polices to the life settlement market is no more than twelve years. It is the demand for living benefits that is driving the supply of life insurance policies to the life settlement market and it is the search for investment value that is driving the demand for life insurance polices by life settlement companies.

As of February 2005 there was approximately $12.7 trillion of life policies in the U.S, of which approximately 10% for insured that are at least 70 years old. Each year $1.5 trillion in life policies lapses or is surrendered. The market for senior life settlement contracts was estimated to have amounted to between $6 billion and $8 billion in 2004, with a projection of $45 billion in face value by 2007 (source: www.calbrokermag.com). We do not discuss the reasons people choose to discontinue a life insurance policy. For our purposes the interesting point is that life settlement companies can offer owners of life insurance more value than the savings in premiums they achieve by letting a policy lapse.

The value of a life settlement contract is based on the life expectancy of the settler which in turn is a function of the age and health condition of the settler. The valuation of a life settlement is achieved by discounting the premia paid over the established settler's life expectancy and the benefit to be received at the time the settler dies. If a settler lives above life expectancy, premia need to be paid over a longer period, and it takes longer to receive the death benefit. Longevity reduces the value of senior life settlements.

Longevity risk is the key variable in the evaluation of life settlement contracts Life expectancy tables are always changing. Life expectancy in developed countries has been increasing steadily over time, although not uniformly across different age ranges, as suggested by Renshaw, Haberman, and Hatzoupoulos (1996). Lin and Cox (2005) compute the percentage change in the present values of annuity payments under different simulated mortality shocks. From their numerical applications we can see how sensitive securities with longevity risk can be to deviations from the life expectancy on which premiums were set. . We should note that longevity risk is the risk of living longer than expected and mortality risk is the risk of dieing sooner than expected. Obviously these risks are the two sides of the same coin.

We contribute to Lin and Cox's research by computing the Macaulay duration's sensitivity to longevity risk. Shifts in life expectancy tables disturb the reliability of the Macaulay duration. Without duration as a useful metric of interest rate risk, investments in senior life settlement contracts can not be readily gauged to other fixed income securities. Without duration as a reliable measure of interest rate risk senior life settlement contracts will have to be discounted at a higher rate to compensate for this uncertainty. This loss of value will constrain the growth of the market.

We find the conditions for which the Macaulay duration of a life settlement contract is not affected by changes in the insured person's life above or below that of his life expectancy. Once we calculate the combinations of discount rates and deviations in life from life expectancy for which duration remains a stable measure of interest rate risk for a life settlement contract, we use this information to structure a class of senior life settlement backed securities whose interest rate risk can be summarized using the standard measure of duration.

Of course by deleveraging one class of securities we are left with a highly leveraged class of securities. For this smaller but more volatile class with respect to deviations on the actual life of a pool of insured from the life expectancy, the pool duration becomes even more unreliable as a measure of intest rate risk. Just as the most volatile classes of collateralized mortgage obligations with respect to prepayment risk must be placed with fund managers and risk managers who specialize in estimating and assuming prepayment risk, our solution to the duration problem for senior life settlements relies on the existence of group of investors who would be willing to take positions in securities that are leveraged with respect to life extension risk. It is important to keep in mind that an unexpected increase in longevity perhaps due to a new pharmaceutical that is approved will reduce the value of a pool of life settlements as the pool of insured lives longer and pushes out the date that death benefits are received. The risk cuts both ways, an extreme heat wave or flu epidemic would reduce the average life of a life settlement pool increasing the yield on the life settlement contracts.

One class is structured so that its duration is a relatively stable measure of interest rate risk while the second smaller class whose design is constrained by the design of the first class has interest rate risk that can not be measured by duration because it is highly leveraged with respect to changes in the life of the insured.

In the following section of the paper we derive the changes in the duration of senior life settlement contracts with respect to changes in the average life of the pool of insured who have sold their policies in the life settlement market. We show that for a set of combinations of discount rates and deviations of the actual life of the insured from his life expectancy, duration is a stable consistent measure of interest rate risk for life settlement contracts and securities backed by senior life settlements. We demonstrate our results with numerical examples.

Modeling framework: Macaulay Duration and Its Sensitivity to Shifts in Life Expectancy Tables

The value of an individual senior life settlement is equal to the present value of the premia *p* paid periodically during the life of the senior life settler, plus the discounted value of the benefit *B* received at death of the settler.

$$P = -p[1/(1+y)^1 + 1/(1+y)^2 + \ldots\ldots + 1/(1+y)^n] + B/(1+y)^n \qquad (1)$$

Where *p* stands for the insurance "premium" paid each year- first by the original owner of the policy and then by the life settlement company, *B* is the death benefit at the time of death of the life settler, *y* is the discount rate and *P* is the present value of the life settlement contract.

The general formula for the Macaulay Duration, *D*, of a fixed income security is

$$D = \sum_{1=i}^{t} (i)CF_i/(1+y)^i/P$$

Where *i* is the time at which the cash flow is paid, *y* is the discount rate, *P* is the price of the security at the time the duration is computed, and CF_i represents the cash flow at time *i*. *t* is the time when the settler dies, and is unknown. The Macaulay duration can be rearranged so that it measures the percentage change in a security's price over the percentage change in yield, the price elasticity of the security.

J.M. Keynes (1936) introduced the concept of a bond's price elasticity with respect to interest rates. J.R. Hicks' (1938) research focused on that same elasticity concept. It was Macaulay (1938) that expanded Keynes's and Hick's work by developing the duration risk metric, which has since taken on his name, "*Macaulay duration".* While the Macaulay duration is calculated for parallel shits in a flat yield curve, other variations of duration have been developed to account for non-parallel shifts and for securities with embedded options. Macaulay and modified duration (which is simply the Macaulay duration divided by *1+y*), are widely used metrics by portfolio and asset/liability managers.

The cash flows to and from a life settlement contract are the yearly premia *p* and the death benefit *B* received at the time of the insured death, at time *t*.

Equation (2) is the calculation of the Macaulay duration *D* for a life settlement contract. The present value of each cash flow is multiplied by the time at which it is paid, *i*, where *i* runs from year *1* to year *t*, *t* being the time the premia stop being paid and when the death benefit is received. The sum of the present values, multiplied by the time at which they are received, is divided by the present value of the life settlement contract, *P*.

$$D = [\sum_{i=1}^{t} p(i)/(1+y)^i - tB/(1+y)^t]/P \qquad (2)$$

Changes in a Senior Life Settlement's Duration for shifts in life expectancy

Investors in mortgage-backed securities are exposed to the risk that the prepayment rate on the underlying pool of mortgages will be above or below the rate they use to price the mortgage-backed security.. Prepayments affect the timing and magnitude of cash flows generated by mortgage-backed securities. For example when prepayments accelerate, principal is prepaid faster so that the total interest paid declines, a change in magnitude, and the principal is collected faster, a change in timing. For MBS the cash flows generated by a pool of mortgages are always positive. Investors in pools of life settlements contracts are exposed to longevity risk—the risk that the pool of insured live longer than the expectation upon which the pool of contracts were priced. Unlike a pool of mortgages which generate positive cash flows composed of interest and principal payments, the underlying pool of life settlements contracts generates a stream of negative cash flows, the insurance premiums and a stream of positive cash flows, the death benefits. In the case of pool of senior life settlements, the timing, magnitude and direction of cash flows are affected by the longevity of underlying insured. The change in the direction of the cash flows from negative to positive for a pool of senior life settlements is the reason that under certain conditions, a deviation from life expectancy of the settlers does not affect the Macaulay duration of the pool.

In this section we calculate the first derivative of the Macaulay duration with respect to the change in time *t* above or below the expected death, and find the conditions for which the derivative is equal to zero. As a matter of simplifying the derivative calculation we set $a = (1)/(1+y)$ and equation (2) then becomes:

$$D = [(\sum_{i=1}^{t} pi(a)^i) - Bt(a)^t]/P$$

The next step is to isolate the term, that we call *f(t)*:

$$f(t) = \sum_{i=1}^{t} pi(a)^i$$

In the next section we find an expression for *f(t)*.

Expression of *f(t)*

Recall that for every real number *b* and natural number *k<n*, the following formula holds:

$$\sum_{i=k}^{t} b^i = (b^{n+1} - b^k)/(b-1) \qquad (3)$$

We can rewrite *f(t)* as

$$f(t) = \sum_{i=1}^{t} pi(a)^i = p\sum_{i=1}^{t} i(a)^i = \qquad (4)$$

$$\begin{aligned} p(a+a^2+a^3+\ldots a^t+ & \qquad (5)\\ a^2+a^3+\ldots a^t+ & \qquad (6)\\ a^3+\ldots a^t+ & \qquad (7)\\ \ldots + & \qquad (8)\\ a^t) & \qquad (9) \end{aligned}$$

By applying formula (3) to every single line of the previous segments (fragments (5), (6), etc.) we get that

$$f(t) = \sum_{i=1}^{t} pi(a)^i = p\sum_{i=1}^{t} i(a)^i =$$

$$p[(a^{t+1}-a^1)/(a-1) + (a^{t+1}-a^2)/(a-1) + (a^{t+1}-a^3)/(a-1) + \ldots + (a^{t+1}-a^t)/(a-1)]$$

The above expression can be simplified into

$$f(t) = \sum_{i=1}^{t} pi(a)i = p\sum_{i=1}^{t} i(a)^i =$$

$$p[ta^{t+1} - (a + a^2 + \ldots + a^t)]/(a-1)$$

Applying formula (3) again we get that

$$f(t) = \sum_{i=1}^{t} pi(a)^i = p\sum_{i=1}^{t} i(a)^i = p[ta^{t+1} - (a^{t+1}-a)/(a-1)]/(a-1)$$

$$[p/(a-1)][(ta^{t+2} - ta^{t+1} - a^{t+1}+a]/(a-1) = [p/(a-1)^2][ta^{t+2} -(t+1)a^{t+1} +a]$$

In summary, we proved that

$$f(t) = [p/(a-1)^2][ta^{t+2} - (t+1)a^{t+1} +a]$$

where $a=1/(1+y)$ and p, y, are constant with respect to t.

Expression for D

We can add now the new expression of $f(t)$ to get the final expression for D:

$$D =[(\sum_{i=1}^{t} pi(a)^i) – Bt(a)^t]/P =$$

$$[f(t) – Bt(a)^t]/P = \{(p/(a-1)^2[ta^{t+2} – (t+1)a^{t+1} + a]) – Bt(a)^t\}/P =$$

$$(1/P)\{ta^t[a^2p/(a-1)^2 – ap/(a-1)^2 – B] – a^t[ap/(a-1)^2] + ap/(a-1)^2 \}=$$

$$(1/P)\{ta^t[ap/(a-1) – B] –a^t[ap/(a-1)^2] + ap/(a-1)^2\}$$

In summary, we get that

$$D = (1/P)\{ta^t[ap/(a-1) - B] – a^t[ap/(a-1)^2] + ap/(a-1)^2\} \qquad (10)$$

For the purpose of having an expression of D using the original parameters, we can now replace a by $1/(1+y)$ in *eq. (10)* to obtain that

$$D = (1/P)\{ta^t[ap/(a-1) - B] – a^t[ap/(a-1)^2] + ap/(a-1)^2\} =$$

$$(1/P)\{t[1/(1+y)^t][[p/(1+y)]/[(1/(1+y)-1]–B]–(1/(1+y))^t[p/(1+y)]/[(1/(1+y))–1]^2+ [(p/(1+y)]/[(1/(1+y)) – 1]^2$$

To conclude, we have the following close expression for the function D:

$$D = \{t[p(-y)^{-1} – B]/(1+y)^t - p/[y^2(1+y)^{t-1}] + p(1+y)/y^2\}/P \qquad (11)$$

The Derivative of Duration With Respect to Time

Note that the expression of D in eq.(10) is of the form

$$D = \{ta^t[C(a-1) – B] –a^tC + C\}/P =$$

$$(1/P)\{ta^t[C(a-1) – B] – a^tC + C\} \qquad (12)$$

Where a, B, P are constants and C is a constant defined as

$C = ap/(a-1)^2$

Now we take the derivative of duration as expressed in equation (12) with respect to t. The solution is given is equation 13.

$D' = (1/P)\{a^t[C(a-1) - B] + ta^t[C(a-1) - B]\,[\ln(a)] - a^tC[\ln(a)]\}$

Where C and a are the constants previously specified.

We simplify the last expression to arrive at equation (13).

$D' = (a^t/P)\{[C(a-1) - B] + t[C(a-1) - B]\,[\ln(a)] - C[\ln(a)]\} =$

$$\boxed{D' = (a^t/P)\{[t[C(a-1) - B]\ln(a)] + [C(a-1) - B - C[\ln(a)]]\} \qquad (13)}$$

Shifts in Senior Life Settlements' Life Expectancies

Data in exhibit 1 describes a typical distribution of life expectancies for insurance policies that are eligible for sale into the senior life settlement market.

Exhibit 1

Typical Distribution of Available Life Expectancies

Life Expectancy (LE)	**% of Insured in LE Category**
LE<= 36 months	1
36 months<LE<=72 months	12
72 months<LE<=108 months	30
108 months<LE<=144 months*	30
144 months<LE<=180 months	17
180 months<LE<=216 months	8
LE>=216 months	2

*As a practical matter, the life expectancies that are found most commonly in life settlement transactions are normally 12 years or less. Source: A.M. Best, September 2005.

We use the distribution from exhibit 1 to construct a hypothetical pool of senior life settlement contracts and examine how the duration of this pool changes with respect to deviations in the actual life of the insured from their expected life. We assume that the pool is composed of 100 life settlement contracts (m_0), the number of settlers from the pool dying at each point in time (d_i) is computed over a total of n periods (n = 225) as follows in Exhibit 2:

Exhibit 2
$d_1, \ldots, d_{36} = 1/36 = .02778$
$d_{37}, \ldots, d_{72} = 12/(72-36) = .3333$
$d_{73}, \ldots, d_{108} = 30/(108-72) = .83333$
$d_{109}, \ldots, d_{144} = 30/(144-108) = .83333$
$d_{145}, \ldots, d_{180} = 17/(180-144) = .47222$
$d_{181}, \ldots, d_{216} = 8/(216-180) = .22222$
$d_{217}, \ldots, d_{225} = 2/(225-216) = .22222$

Exhibit 2 is based on the data from exhibit 1. The rate of death based on the "Typical Distribution of Life Expectancies for senior life settlement contracts" (which we call scenario 1 throughout the paper) is graphed in Exhibit 3.

Exhibit 3: Death rate over time based on a "Typical Distribution of Life Expectancies for Senior Life Settlement Contracts"

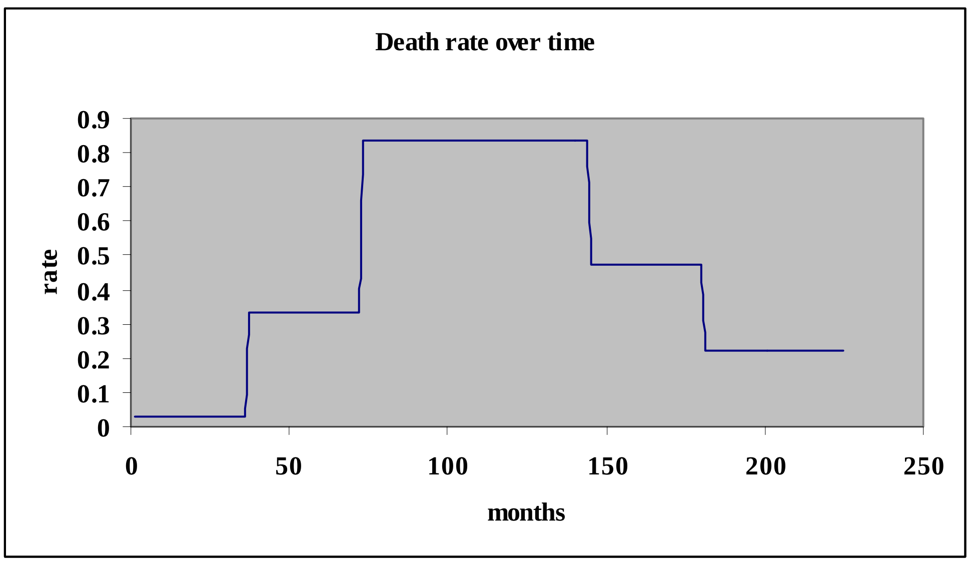
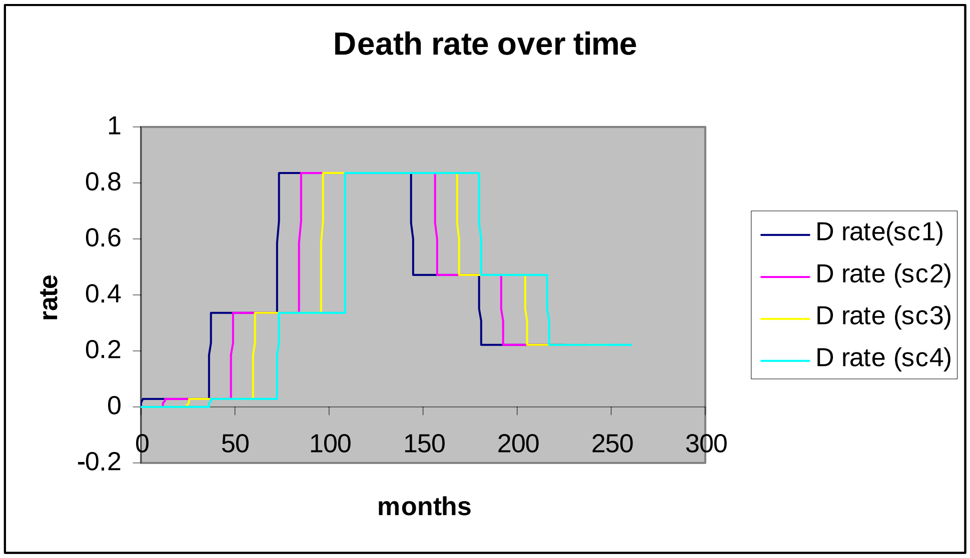


Exhibit 4 graphs the death rate over time under different scenarios. Scenario 1 is the base case, with "Typical Distribution of Life Expectancies"; scenario 2 shifts the distribution of the death rate in scenario 1 by twelve months (life extension by twelve months relative to the base case). Each subsequent scenario shifts/extends the distribution of death rate by another twelve months.

Exhibit 4



Conditions for a reliable Senior Life Settlement's Macaulay Duration

We now develop the conditions for which the Macaulay duration of a senior life settlement, or a pool of senior life settlements, is not affected by changes in the life of the insured above or below its life expectancy computed at the time the insurance contract was sold. The Macaulay duration of a pool of senior life settlements is the weighted average duration of each life settlement.

Using equation (13), we can now find the conditions for which *D' = 0*, that is the conditions for which the duration does not change when a settler lives above or below life expectancy.

Note that (a^t/P) can never be equal to *0* because *a* is always positive (1/1+y). This means that the derivative of duration with respect to time (t) equals zero (*D' = 0)* when

$$t(C(a-1) - B)(\ln(a)) + C(a-1) - B - C(\ln(a)) = 0$$

This is a simple linear equation that we solve for t.

$$t = [(-(C(a-1) - B) + C(\ln(a))]/[(C(a-1) - B)(\ln(a))] =$$

$$[-1/\ln(a)] + C/[C(a-1) - B]$$

Now we replace C with $(ap)/(a-1)^2$ to arrive at *D' = 0* when

$t = [-1/\ln(a)] + [ap/(a-1)^2]/\{[ap/(a-1)\ ]– B]\}$

This previous equation can be simplified to :

$t = [-1/\ln(a)] + \ [ap/(a-1)^2]/\{[ap(a-1)– B(a-1)^2]/(a-1)^2\} =$

$[-1/\ln(a)] + ap/[ap(a-1) – B(a-1)^2]$

Finally, we replace *a* by its value of *1/(1+y)* to show that *D'=0* when

$$\boxed{t = [1/\ln(1+y)] + [p(1+y)/y(-p-By)] \qquad \text{eq. 14}}$$

Numerical Applications

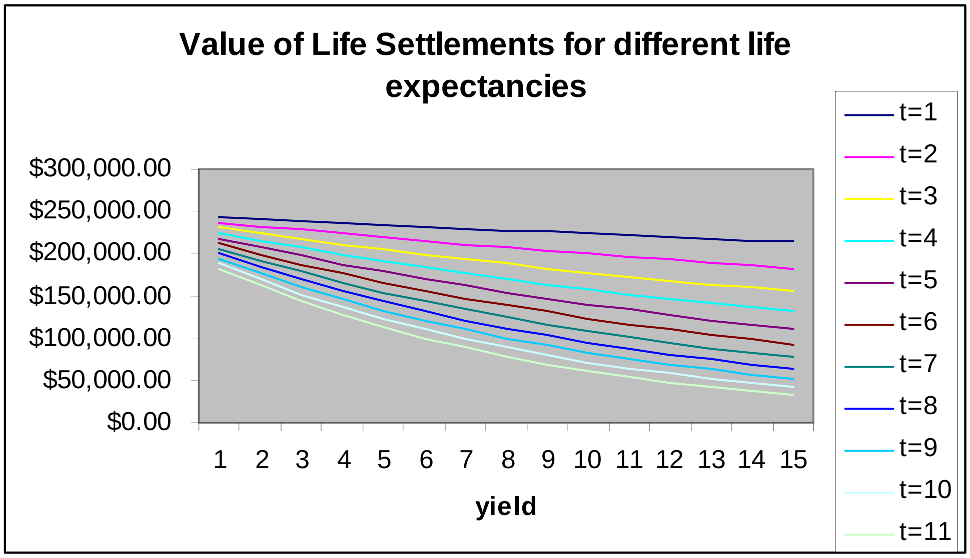
Using a premium *p* of *$4,000* and a death benefit *B* of *$250,000*, with equation (1) we obtain the value of a senior life settlements across life expectancies between 1 and 11 years, and across yields comprised between 1% and 15%. The values are displayed in exhibit 5.

Exhibit 5

y\t	1	2	3	4	5	6	7	8	9	10	11
0.01	$243,564.36	$237,192.43	$230,883.60	$224,637.22	$218,452.70	$212,329.40	$206,266.74	$200,264.09	$194,320.89	$188,436.52	$182,610.42
0.02	$241,176.47	$232,525.95	$224,045.05	$215,730.44	$207,578.86	$199,587.12	$191,752.08	$184,070.67	$176,539.87	$169,156.73	$161,918.37
0.03	$238,834.95	$227,995.10	$217,470.97	$207,253.37	$197,333.37	$187,702.30	$178,351.75	$169,273.54	$160,459.75	$151,902.67	$143,594.82
0.04	$236,538.46	$223,594.67	$211,148.73	$199,181.47	$187,674.49	$176,610.08	$165,971.23	$155,741.57	$145,905.36	$136,447.46	$127,353.33
0.05	$234,285.71	$219,319.73	$205,066.41	$191,491.82	$178,563.63	$166,251.08	$154,524.84	$143,356.99	$132,720.94	$122,591.37	$112,944.17
0.06	$232,075.47	$215,165.54	$199,212.77	$184,162.99	$169,965.09	$156,570.84	$143,934.75	$132,013.92	$120,767.85	$110,158.35	$100,149.38
0.07	$229,906.54	$211,127.61	$193,577.21	$177,174.96	$161,845.76	$147,519.40	$134,130.28	$121,617.08	$109,922.51	$98,993.00	$88,778.50
0.08	$227,777.78	$207,201.65	$188,149.67	$170,508.96	$154,174.96	$139,050.89	$125,047.12	$112,080.67	$100,074.69	$88,958.05	$78,664.86
0.09	$225,688.07	$203,383.55	$182,920.69	$164,147.42	$146,924.24	$131,123.16	$116,626.75	$103,327.29	$91,125.96	$79,932.07	$69,662.45
0.1	$223,636.36	$199,669.42	$177,881.29	$158,073.90	$140,067.18	$123,697.44	$108,815.85	$95,287.14	$82,988.31	$71,807.55	$61,643.23
0.11	$221,621.62	$196,055.51	$173,022.99	$152,272.96	$133,579.24	$116,738.06	$101,565.82	$87,897.13	$75,583.00	$64,489.19	$54,494.77
0.12	$219,642.86	$192,538.27	$168,337.74	$146,730.12	$127,437.61	$110,212.15	$94,832.28	$81,100.25	$68,839.51	$57,892.42	$48,118.23
0.13	$217,699.12	$189,114.26	$163,817.93	$141,431.80	$121,621.06	$104,089.43	$88,574.72	$74,844.88	$62,694.59	$51,942.11	$42,426.65
0.14	$215,789.47	$185,780.24	$159,456.35	$136,365.22	$116,109.84	$98,341.97	$82,756.11	$69,084.31	$57,091.50	$46,571.49	$37,343.41
0.15	$213,913.04	$182,533.08	$155,246.16	$131,518.40	$110,885.56	$92,943.97	$77,342.58	$63,776.16	$51,979.27	$41,721.10	$32,800.96

In exhibit 6 we graph the values of the senior life settlement that were calculated in exhibit 5. We observe that value is a decreasing function of yield and the decline in value is steeper for longer life expectancies.

Exhibit 6



In exhibit 7 we graph the changes in the value of life settlement contracts for different initial yield scenarios, from 5% to 10%, when life expectancy moves from 1 to 11 years. We observe that the higher the life expectancy of a senior life settler is, the lower the value of the life settlement becomes. Of course this is because premia would have to be paid over a longer period of time before the death benefit is paid. The inverse relationship between life expectancy and the value of the life settlement contract is accentuated for higher yields.

Exhibit 7

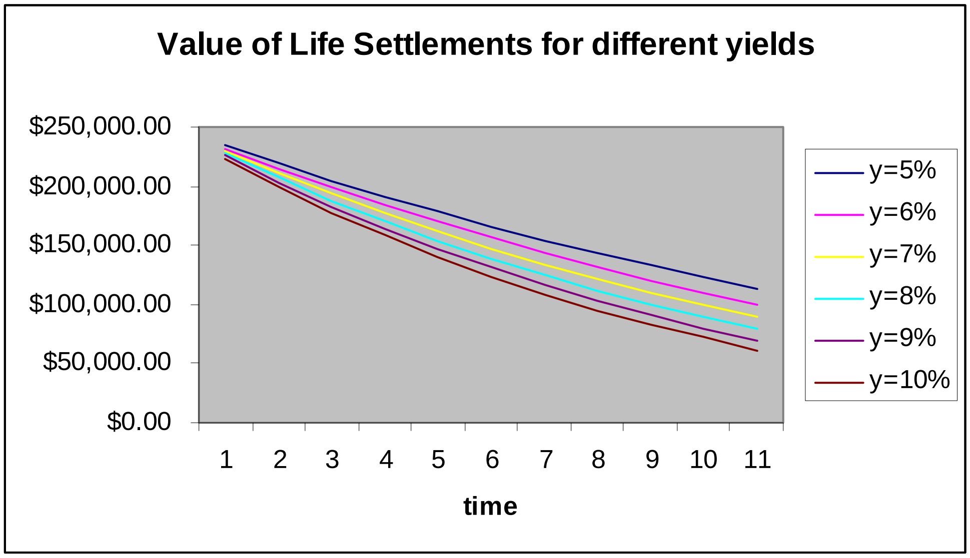
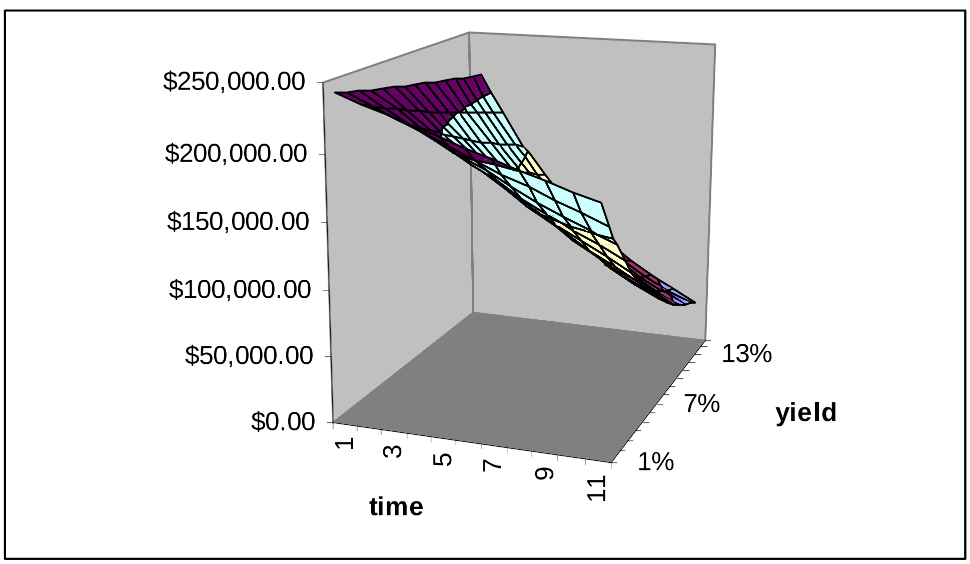


Exhibit 8 combines exhibits 6 and 7 in a 3-D framework. We can see how the value of a life settlement is simultaneously negatively affected by an increase in yield and an increase in life expectancy. This is captured in the downward slope of the plane in value/time/yield space. Value is more sensitive to increases in yields for higher life expectancies (the east edge of the plane is steeper than the west edge), and it is more sensitive to increases in life above initial life expectancy for higher yields (the back edge of the plane has a steeper slope than the front edge).

Exhibit 8



We previously derived the conditions for which *D' = 0*. This was the result of eq. (14). We restate this formula below.

$t^* = [1/\ln(1+y)] + [p(1+y)/y(p-By)]$ eq. (14)

We call t^* the specific life expectancy that for an initial yield, the Macaulay duration remains the same for changes in the insured's life above or below t^*. We calculate t^* using a premium *p* equal to *$4000* and a death benefit *B* equal to $250,000, for yields between 1% and 15%. The results of this calculation are displayed in exhibit 9.

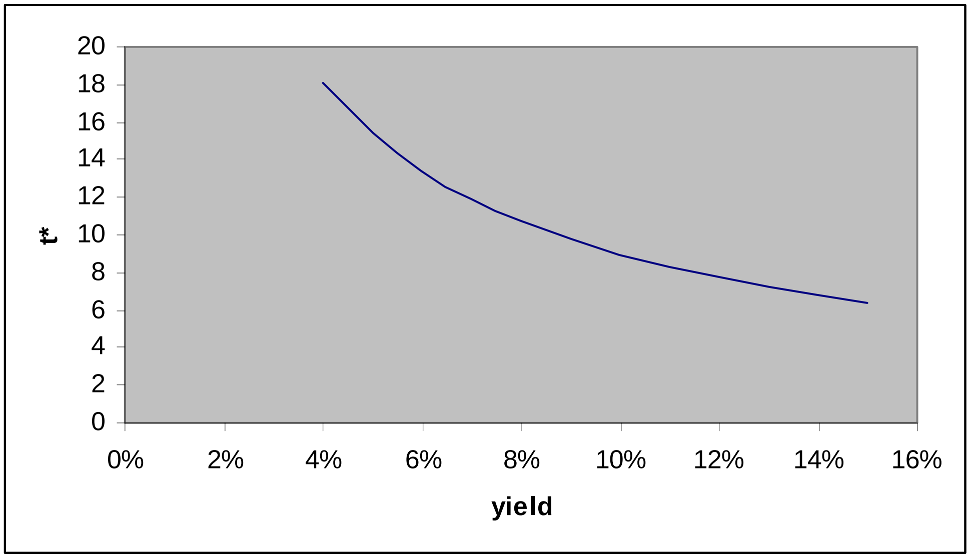
Exhibit 9

y	**t***
1%	38.345325
2%	27.831683
3%	21.888841
4%	18.06816
5%	15.405025
6%	13.442513
7%	11.936223
8%	10.743587
9%	9.7758383
10%	8.9748173
11%	8.3008287
12%	7.7258521
13%	7.2295438
14%	6.7967764
15%	6.4160678

The data from exhibit 9 can be interpreted as those life expectancy/yield combinations used to value life settlement contracts for which duration is a stable measure of interest rate risk, that is when *D'=0*, for life settlers living above or below life expectancy .

Exhibit 10 is the graph of the t* for yields ranging from 4% to 15%.

Exhibit 10



For example, a life settlement contract with a life expectancy of *8.98 years*, as highlighted in exhibit 9 (we round it to *9* for computational purpose) valued at a market rate (or yield) of *10%*, will have a Macaulay duration of *-9.3*. There is always the chance that the life settler lives *10 years*, one year above the settler's life expectancy, (an *11%* change in *t*). With an unchanged yield, in our example, *10%*, the Macaulay duration would not be affected by this life extension, and would remain at *-9.3*. Again using exhibit 9 we observe the duration measure is robust when the life settlement contract is valued over 8 years, one year below life expectancy. Duration remains unchanged at *-9.3*. This means that an investor purchasing securitized senior life settlements with a life expectancy of t^*, can rely on the computed Macaulay duration as a measure of interest risk knowing that the insured may actually live longer or shorter than the expectancy on which the original valuation was based.

We now illustrate that for a life expectancy other than t^*, calculations of duration will be unstable for deviations in life from the expected life of the insured. Using same premium $4,000 and death benefit, $250,000 as in the last example and a life expectancy of *5 years*, with current market rates of *5%*, we obtain duration of -5.2. If the settler lives 6 years instead of the expected 5 years, the duration goes to -5.9. If on the other hand he dies one year earlier than expected, after 4 years instead of 5 years, the duration changes from -5.2 to -4.4.

In Exhibit 12 we graph the price/yield relationship for three different life expectancies across a range of yields. The data is presented in exhibit 11. The middle curve is the plot of a 9-year expected life across yields ranging from 6% to 13%. The life settlement contracts are discounted at a yield of 10%, *initial point* (highlighted in exhibit 11) on the curve. The lowest curve is drawn for an extension of life by one year and the highest curve illustrates what happens to the price/yield relationship when life is shortened by one year. The curves shift parallel to one another. This parallel shift indicates that the duration of the life settlement contracts does not change and is still equal to -9.3. The slope of the tangent to the price/yield curve along the 10% yield remains constant. The slope of the tangent to the price/yield curve at an *initial point* is computed by taking the first derivative of the life settlement price with respect to yield. When the first derivative is multiplied by *(1+y)/P* we obtain the Macaulay duration in equation (2).

Exhibit 11

y/t	8 years	9 years	10 years
0.06	$132,013.92	$120,767.85	$110,158.35
0.07	$121,617.08	$109,922.51	$98,993.00
0.08	$112,080.67	$100,074.69	$88,958.05
0.09	$103,327.29	$91,125.96	$79,932.07
0.1	$95,287.14	$82,988.31	$71,807.55
0.11	$87,897.13	$75,583.00	$64,489.19
0.12	$81,100.25	$68,839.51	$57,892.42
0.13	$74,844.88	$62,694.59	$51,942.11
0.14	$69,084.31	$57,091.50	$46,571.49

Exhibit 12

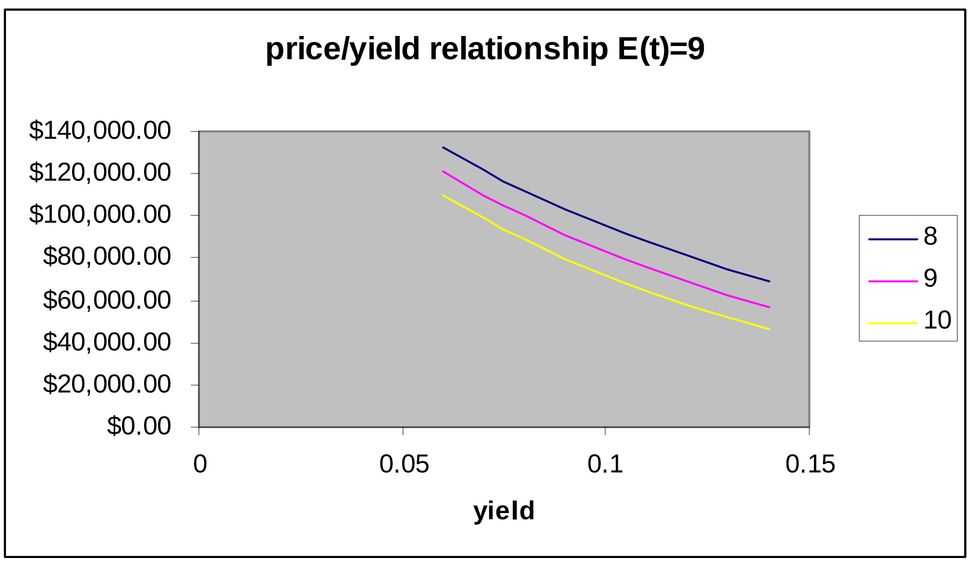


Exhibit 14 is the same space as exhibit 12 but the price/yield relationships are plotted for an expected life of five years across yields ranging from 1% to 9%. The *initial point* (highlighted in exhibit 13) along the curve is at a yield of 5%. When the life of the insured pool deviates by one year above or below the initial five-year life expectancy, the new curves have a tangent to the 5% yield with different slopes. When the life span increases from five to six years, the curve falls and rotates so that the new slope of the tangent to the curve at the yield of 5% is higher, and the initial duration of -5.2 changes to -5.9. When the life span of the pool of insured falls from five to four years the price/yield curve shifts upwards and the slope decreases. There is a decline in the duration of the pool of life settlements contracts from -5.2 to -4.4.

Exhibit 13

y/t	4 years	5 years	6 years
0.01	$224,637.22	$218,452.70	$212,329.40
0.02	$215,730.44	$207,578.86	$199,587.12
0.03	$207,253.37	$197,333.37	$187,702.30
0.04	$199,181.47	$187,674.49	$176,610.08
0.05	$191,491.82	$178,563.63	$166,251.08
0.06	$184,162.99	$169,965.09	$156,570.84
0.07	$177,174.96	$161,845.76	$147,519.40
0.08	$170,508.96	$154,174.96	$139,050.89
0.09	$164,147.42	$146,924.24	$131,123.16

Exhibit 14

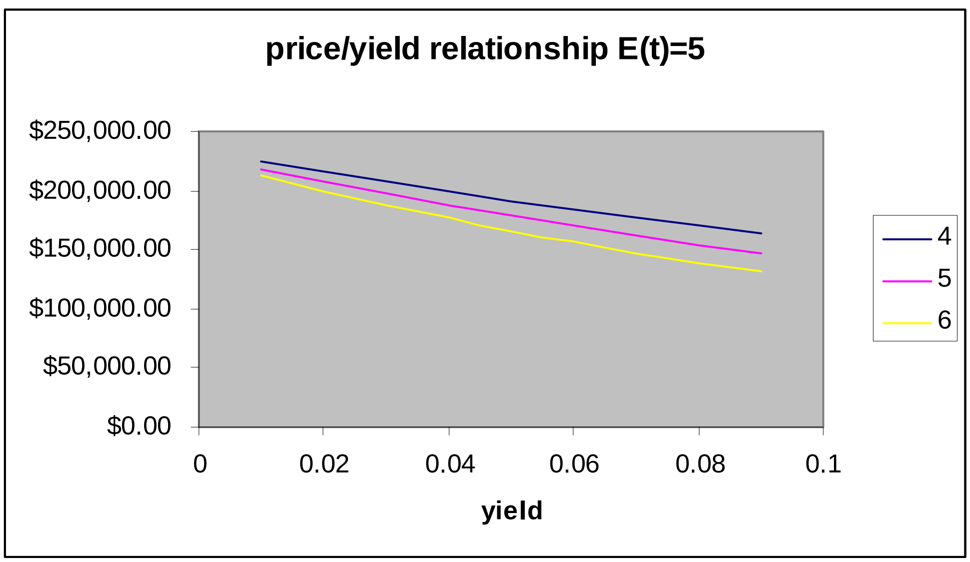
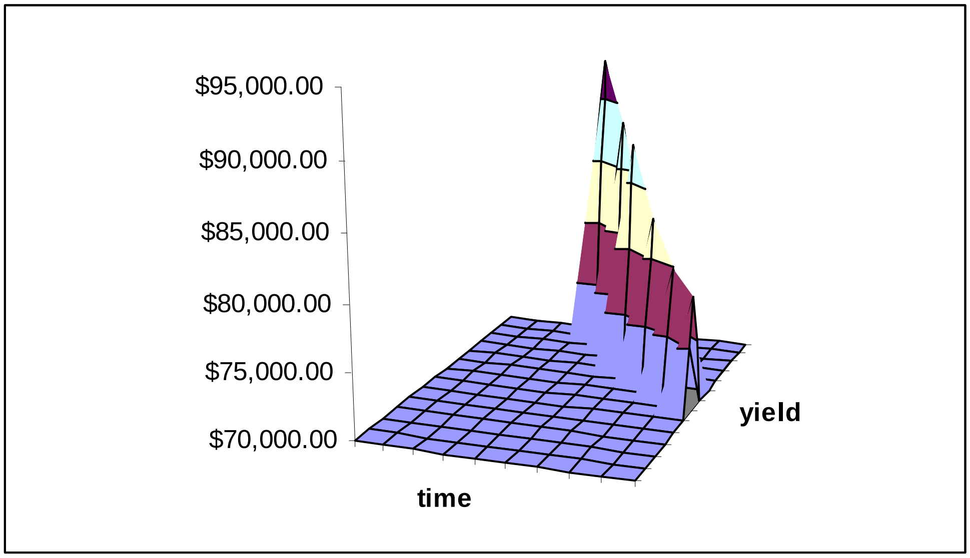


Exhibit 15



We construct Exhibit 15 by carving out from exhibit 8 the yield/time combinations for which the duration calculation for the life settlement contracts is immune to changes in the life of the insured person above or below the life expectancy used to initially value the contract, and by graphing the combinations against the appropriate value.

Conclusion

The derivation of the time/yield combinations for which the duration of a pool of life settlement contracts does not change as the age of the life settlers deviates from the life expectancy, opens up the possibility of structuring two classes of securities to finance a pool of senior life settlement contracts; one class would be designed to have a stable duration measure across a spectrum of pool longevity and the other class would pick up the slack by being designed to have a duration that was very sensitive to small changes in pool longevity. In fact the design of the second class is imposed by the design of the first. When financing a fixed pool of assets such as senior life settlements with various classes of asset-backed securities, the deleveraging of one class with respect to a risk dimension, in this case longevity risk, must be accompanied by a leveraging of another class. The first class would be structured to address the needs of those investors looking for investments with fairly certain durations, we call this class the *Sure Duration Class (SDC)*. This is done by carving out cash flows from the pool of life settlement contracts that satisfy equation (14). Cash flows generated by the life settlement pool but not allocated to the *Sure Duration Class* would be directed to the *Duration Companion Class* (*DCC*). This can be achieved by combining seasoned with unseasoned portfolios of senior life settlement contracts. In exchange for assuming the longevity risk of the pool, investors in the *DCC class* would be offered a higher yield.

References

A.M. Best, "Life Settlement Securitization", September 1, 2005.

Ballotta Laura and Haberman Steven, "The fair valuation problem of guaranteed annuity options: The stochastic mortality environment case", *Insurance: Mathematics and Economics*, February 2006.

Bhattacharga Jay, Goldman Dana, Sood Neeraj, "Price Regulation in Secondary Insurance Markets", *The Journal of Risk and Insurance*, 2004, Vol. 71, No.4, 643-675.

D. Blake, A. J. G. Cairns and K. Dowd, "Living With Mortality: Longevity Bonds And Other Mortality-Linked Securities", Presented to the Faculty of Actuaries, 16 January 2006.

Cowley, Alex; Cummins, J. David, "Securitization of Life Insurance Assets and Liabilities", *Journal* of *Risk* and *Insurance*, June 2005, v. 72, iss. 2, pp. 193-226.

Cummins David J**., "**Securitization of Life Insurance Assets and Liabilities", Wharton 2004.

Doherty Neil A. and Singer Hal J. "The Benefits of a Secondary Market for Life Insurance Policies", *The Wharton Financial Institutions Center*, November 14, 2002.

Dowd Kevin, Cairns Andrew J.G.and Blake David, "Mortality-dependent financial risk measures", *Insurance: Mathematics and Economics*, Volume 38, Issue 3, Pages 427-642 (15 June 2006).

Goldstein Matthew, "Dying for 8% - Investors Beware", *Tavakoli Structured Finance, Inc.*, August 2004.

Ingraham Harold G. and Salani Sergio S., "Life Settlements as Viable Option", *Journal of Financial Service Professionals*, September 2004.

J. Hicks, *Value and Capital: An inquiry into some fundamental principles of economic theory*. 1946 Edition, Oxford: Clarendon Press, 1939.

Keynes, J.M. *The General Theory of Employment Interest and Money*, New York: Harcourt, Brace & World, Inc., 1936.

Lin, Yijia and Cox, Samuel H., "Securitization of Mortality Risks in Life Annuities", *Journal* of *Risk* and *Insurance*, June 2005, v. 72, iss. 2, pp. 227-52.

Macaulay, F. Some Theoretical Problems Suggested by the Movements of Interest Rates, Bond Yields, and Stock Prices in the United States Since 1856. New York: National Bureau of Economic Research, 1938.

McNealy Sean and Marlene Frith, "The Life Settlement Broker: An Integral Player in the Life Settlement Supply Chain", *California Broker,* February 2005.

Milevsky, Moshe A., "The Implied Longevity Yield: A Note on Developing an Index for Life Annuities", *Journal* of *Risk* and *Insurance*, June 2005, v. 72, iss. 2, pp. 301-20.

Modu Emmanuel, "Life Settlement Securitization", *A.M. Best Report October* 18, 2004.

Questa, Giorgio S. (1999). Fixed-income Analysis for the Global Financial Market: Money Market, Foreign Exchange, Securities and Derivatives. (Wiley Series in Financial Engineering).

Richard Christine, "With $70M Bond Deal, Wall St. Manages to Securitize Death", *The Wall Street Journal*, April 30, 2004.

Stone Charles A. and Zissu Anne, "Securitization of Senior Life Settlements: Managing Extension Risk**",** *The Journal of Derivatives*, Spring 2006.

Stone Charles A. and Zissu Anne, "Risk Management – Risk Measurement" Letter from the Editors, *The Financier*, Vol.3, No. 4 & 5, 1996.

Taglich Brothers, Inc. *Research Report on Life Partners Holdings, Inc.* December 2004.

Weil Roman L., "Macaulay's Duration: An Appreciation." *Journal of Business*, 1973, *46*(4), pp. 589-92.