Understanding the Behaviour of Credit Correlations Under Stress

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LRC - Risk Analytics & Instruments
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A Passion to Perform. Deutsche Bank

Agenda

1.	Stress Testing in DB's Credit Portfolio Model
2.	Analysis of Stressed Model Correlations
3.	Empirical Examples

Bonti, G., Kalkbrener, M., Lotz, C., and Stahl, G. (2006).
Credit risk concentrations under stress.
Journal of Credit Risk 2(3), 115-136.

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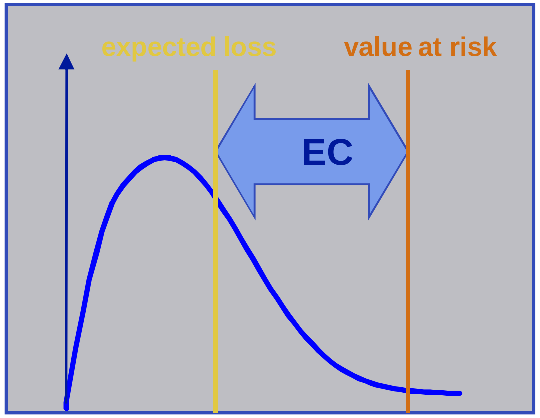
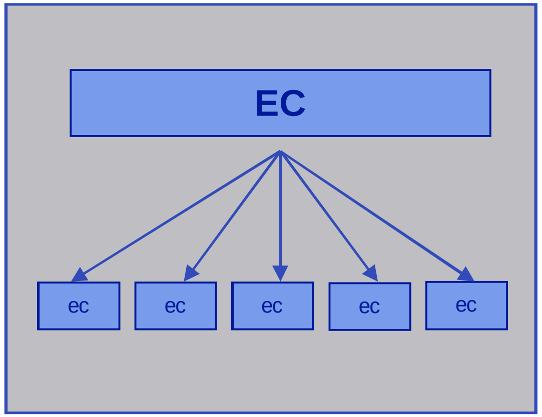
Deutsche Banks' Economic Capital Framework

- **Expected Loss (EL)** is the average expected loss over one year
- **Economic Capital (EC)** is the amount of capital needed to cover accumulated excess ("unexpected") losses over one year with a confidence level of 99.98%



- Economic Capital is allocated to business areas
- Main applications of Economic Capital:
  - Capital management
  - Performance measurement



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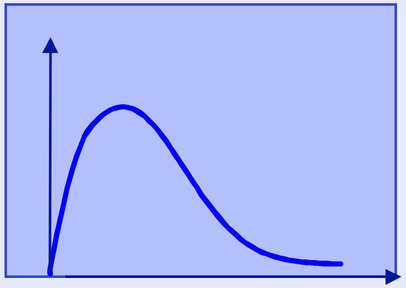
Economic Capital Modelling

Credit Risk

Counterparty defaults

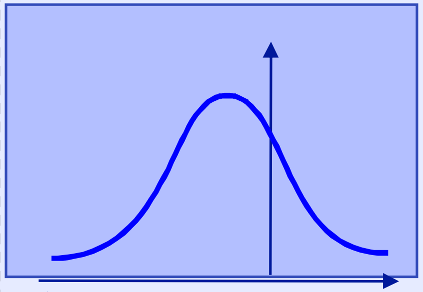


Transaction Level

- Simulation of portfolio loss distribution
  - DB's 45 factor model
- Risk Drivers:
  - EAD, PD, LGD, R^2, Collateral
  - Industry/Country weights

Market Risk

Market movements

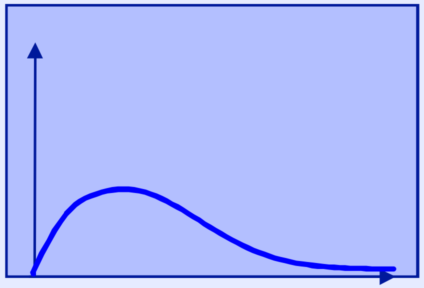


Business Area

- **Scenarios**
  - **Shocks from historic data & risk management assessment**
- **Stress Test Aggregation**
  - **Correlations from historical data (mixed copula)**
- **Risk Drivers**
  - **FX, IR, Credit, Equity**
  - **Transaction size, structure, hedging, basis**

Operational Risk

Adverse operational events

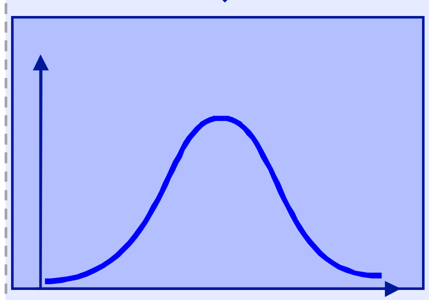


Business Area

- Simulation of loss events and their impact
- Risk Drivers:
  - Potential Loss Sizes Loss Frequencies
  - Business structure and organization

Business Risk

P&L volatility



Business Area

- Stress tests / Simulation of tax legislation impact
- Scenarios defined by business & risk managers
- Risk Drivers:
  - Revenues & Costs (actual vs. stressed)
  - Tax events

Risk Type Diversification

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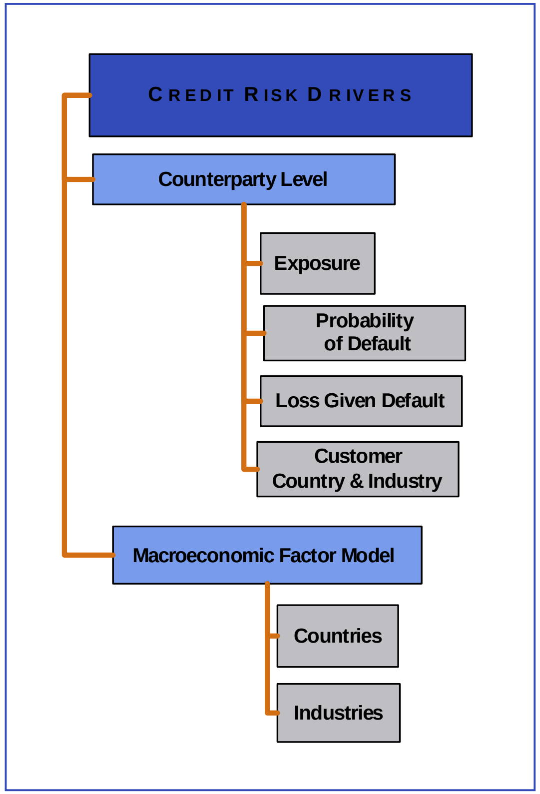
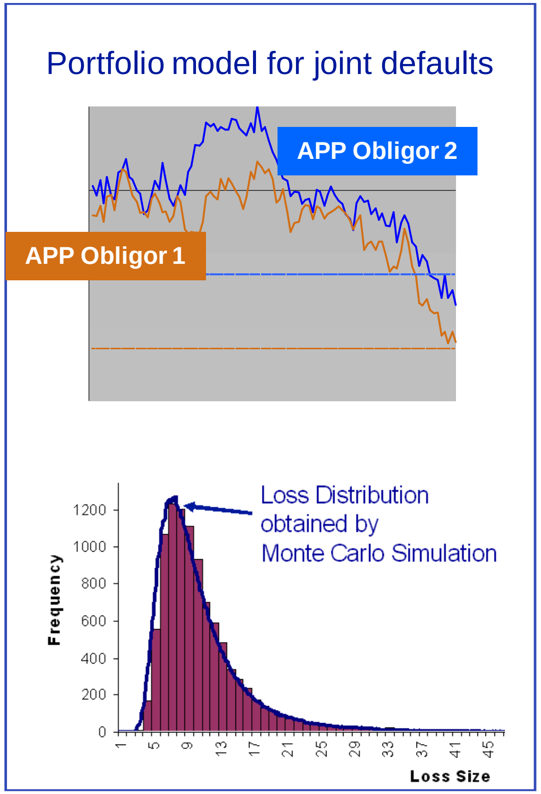
Deutsche Bank's Credit Portfolio Model

Inputs



Aggregation (Monte-Carlo simulation)



Definitions

Default events & loss variables

$$\mathrm{PD}_j = \mathcal{P}(D_j)$$

$$L_j = \mathrm{EAD}_j \cdot \mathrm{LGD}_j \cdot \mathbf{1}(D_j)$$

Risk factors & asset variables

$$D_j = \{A_j < c_j\}$$

$$A_j = \sqrt{R_j^2} \sum_{i=1}^{m} w_{ji} \Psi_i + \sqrt{1 - R_j^2}\, \varepsilon_j$$

Portfolio loss

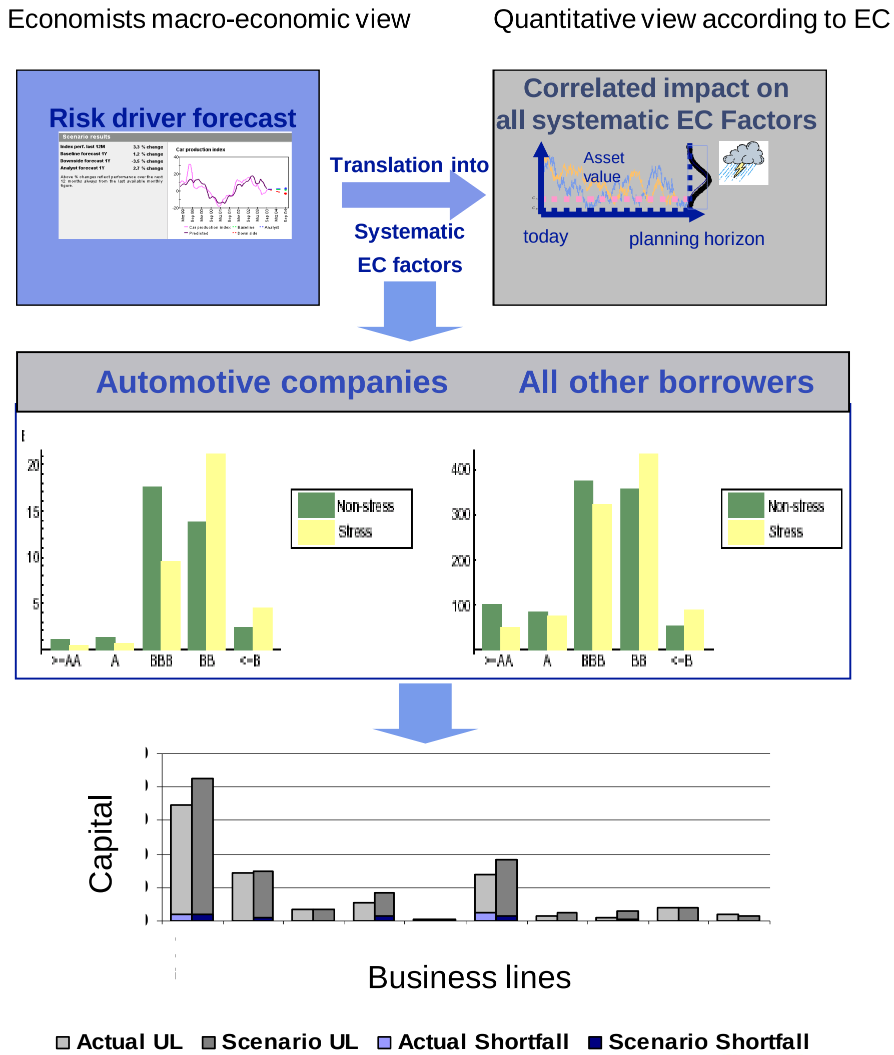
$$L = \sum_{j=1}^{n} L_j$$

Credit Portfolio Stress Testing

Overview

- <u>Step 1:</u> a specific economic stress scenario is defined, e.g.
  - decline of 20% in the automotive production index
  - increase of oil price to $120
  - decline of 15% in local equity index
- <u>Step 2</u>: the stress scenario is translated into constraints on the risk factors of the credit portfolio model
- <u>Step 3:</u> risk and regulatory capital implications for all relevant portfolios are calculated for
  - Expected Loss/Economic Capital framework
  - Basel II minimum requirements

  at any granularity level down to individual facilities



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Credit Portfolio Stress Testing
Ensuring consistency with model correlation structure

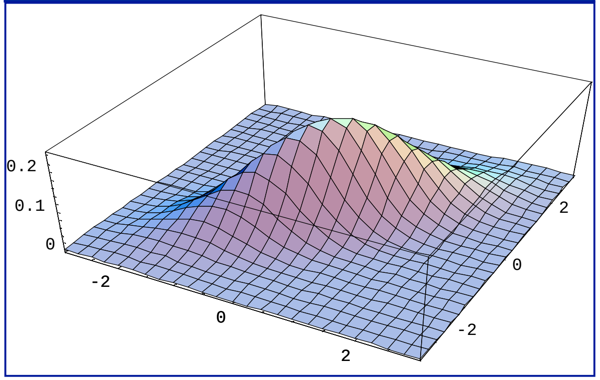
The distribution of the stressed factor is truncated

The response of the unstressed risk factor is specified by the dependence structure of the model

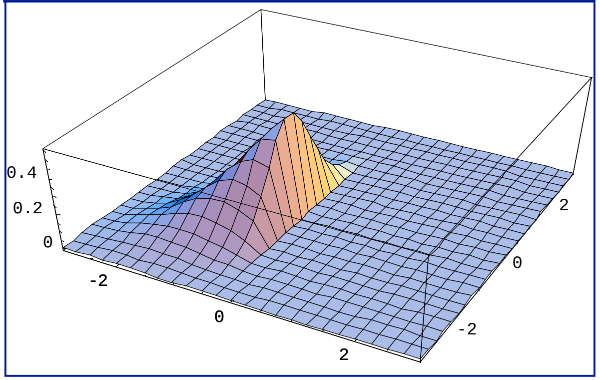
Stress has an impact on

- marginal distributions
- correlations

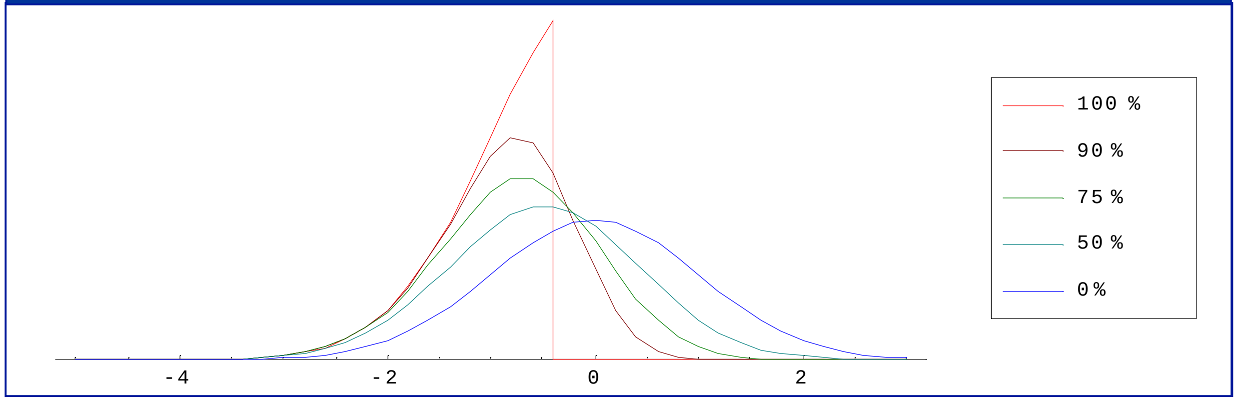
Joint distribution of two systematic factors with correlation 0.75



Joint distribution if factor 1 is capped (here: cap at 40%-quantile)



Marginal distributions of factor 2 for different correlation assumptions



Agenda

1.	Stress Testing in DB's Credit Portfolio Model
2.	**Analysis of Stressed Model Correlations**
3.	Empirical Examples

Kalkbrener, M., and Packham, N. (2010).
Correlation under stress in normal variance mixture models. Working paper, Deutsche Bank and Frankfurt School of Finance & Management.

Kalkbrener, M., and Overbeck, L. (2008).
Stressed testing in credit portfolio models. Preprint, Deutsche Bank.

What is the Impact of Stress on Asset Correlations?

Setup

- Assume that a stress test is specified by truncating the first systematic factor
- Only MC scenarios are considered where the value of the first factor is below a threshold *C*

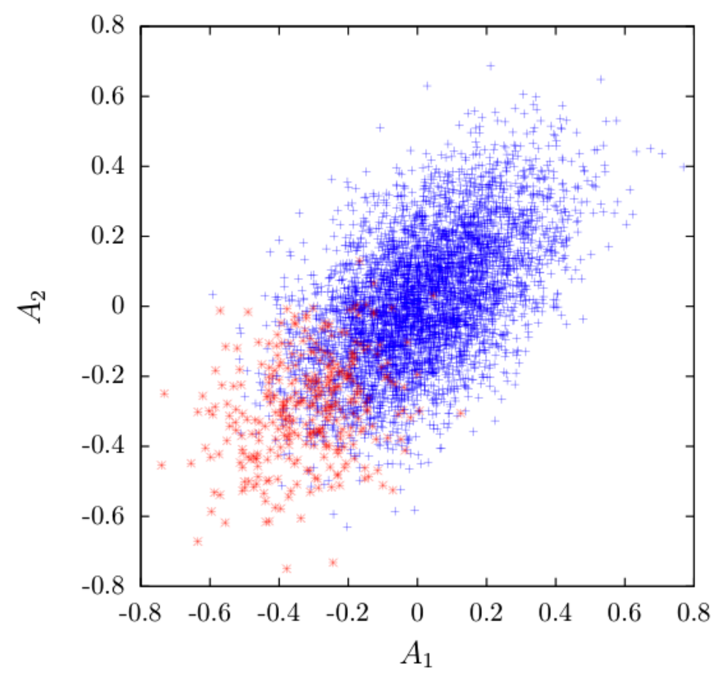
Analysis

- For asset (returns) A_1 and A_2 we analyse the impact of the stress scenario on their correlation, i.e., we compare

$\mathrm{Corr}(A_1, A_2)$	unconditional correlation
$\mathrm{Corr}^C(A_1, A_2)$	conditional (or stressed) correlation

Correlated asset values

Conditional: red Unconditional: red + blue



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Stressed Correlations in Gaussian Models

Assume that the risk factors follow a multivariate normal distribution and define

$$\rho_i := \mathrm{Corr}(\Psi_1, A_i), \qquad \rho_{ij} := \mathrm{Corr}(A_i, A_j)$$

Stressed asset correlations in a Gaussian model (for *C*<0)

$$\mathrm{Corr}^C(A_i, A_j) = \frac{\rho_i\,\rho_j\,\mathrm{Var}^C(\Psi_1) + \rho_{ij} - \rho_i\,\rho_j}{\sqrt{(\rho_i^2\,\mathrm{Var}^C(\Psi_1) + (1-\rho_i^2))\,(\rho_j^2\,\mathrm{Var}^C(\Psi_1) + (1-\rho_j^2))}},$$

where

$$\mathrm{Var}^C(\Psi_1) = 1 - \frac{C\,\phi(C)}{\mathrm{N}(C)} - \frac{\phi(C)^2}{\mathrm{N}(C)^2}$$

Limit

$$\lim_{C\to-\infty} \mathrm{Var}^C(\Psi_1) = 0 \quad \Rightarrow \quad \lim_{C\to-\infty} \mathrm{Corr}^C(A_i, A_j) = \frac{\rho_{ij} - \rho_i\,\rho_j}{\sqrt{(1-\rho_i^2)(1-\rho_j^2)}}$$

Numerical Example
Stressed asset correlations in Gaussian models

Formulas for stressed asset correlations

$$\mathrm{Corr}^C(A_i, A_j) = \frac{\rho_i\,\rho_j\,\mathrm{Var}^C(\Psi_1) + \rho_{ij} - \rho_i\,\rho_j}{\sqrt{(\rho_i^2\,\mathrm{Var}^C(\Psi_1) + (1-\rho_i^2))\,(\rho_j^2\,\mathrm{Var}^C(\Psi_1) + (1-\rho_j^2))}},$$

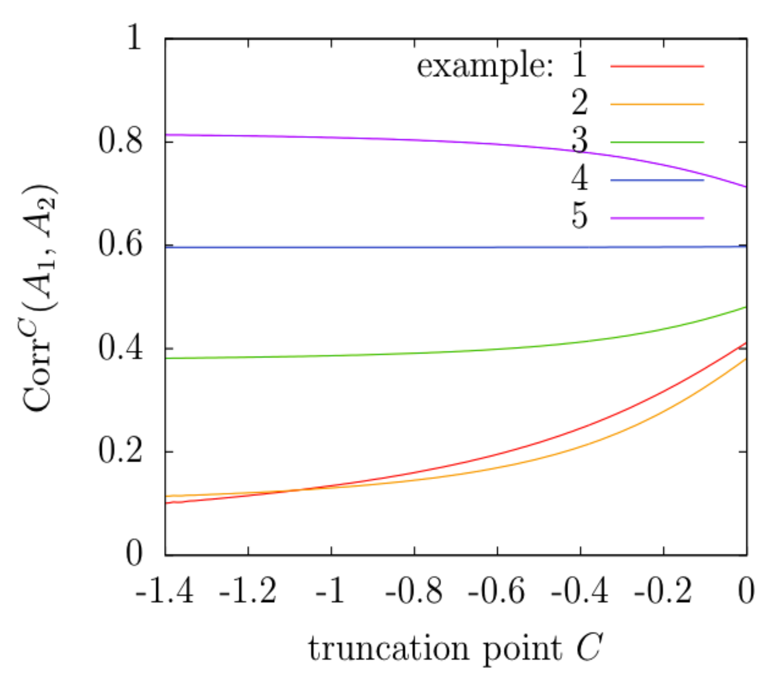
$$\lim_{C \to -\infty} \mathrm{Corr}^C(A_i, A_j) = \frac{\rho_{ij} - \rho_i\,\rho_j}{\sqrt{(1-\rho_i^2)(1-\rho_j^2)}}$$

Numerical example

Example	ρ_{12}	ρ_1	ρ_2	limit corr
1		1	0.6	0
2		0.8	0.7	0.093
3	0.6	0.6	0.6	0.375
4		0.1	0.1	0.596
5		0.7	0.02	0.821

Stressed asset correlations

for different correlation parameters and thresholds



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Extension to Normal Variance Mixture Models

Assume that the risk factors follow a normal variance mixture (NVM) distribution

There exist standardized normally distributed rv $X, Y_1, \ldots, Y_n$ and a real-valued rv $W \geq 0$ with $\mathrm{E}(W) > 0$, which is independent of $X, Y_1, \ldots, Y_n$, such that

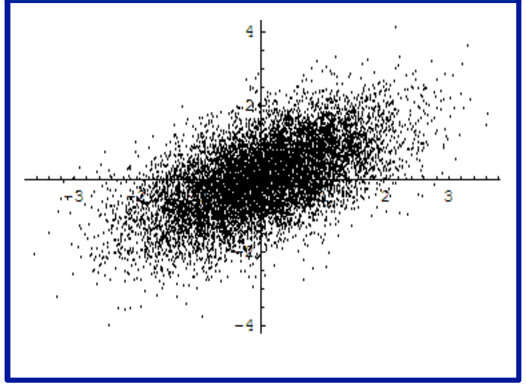
$$\Psi_1 = \sqrt{W} X, \qquad A_i = \sqrt{W} Y_i$$

Examples

- Multivariate normal if *W*=1
- Multivariate t
  if *W* is inverse gamma

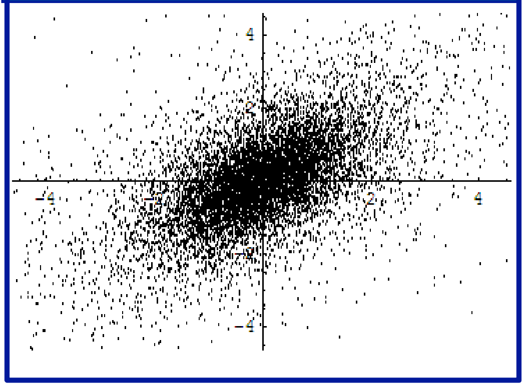
Bivariate normal distribution

corr = 0.6



Bivariate t distribution

corr = 0.6 df = 3



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Stressed Asset Correlations in NVM Models

Stressed asset correlations (for C<0)

$$\mathrm{Corr}^C(A_i, A_j) = \frac{\rho_i\,\rho_j\,\frac{\mathrm{Var}^C(\Psi_1)}{\mathrm{E}^C(W)} + (\rho_{ij} - \rho_i\,\rho_j)}{\sqrt{(\rho_i^2\,\frac{\mathrm{Var}^C(\Psi_1)}{\mathrm{E}^C(W)} + (1-\rho_i^2))\,(\rho_j^2\,\frac{\mathrm{Var}^C(\Psi_1)}{\mathrm{E}^C(W)} + (1-\rho_j^2))}}$$

Problem

Calculation of

$$\frac{\mathrm{Var}^C(\Psi_1)}{\mathrm{E}^C(W)} \quad \text{and} \quad \lim_{C\to-\infty}\frac{\mathrm{Var}^C(\Psi_1)}{\mathrm{E}^C(W)}$$

Stressed Asset Correlations in t Distributed Models

Let alpha >2 denote the degree of freedom of a multivariate t distribution

$$\frac{\mathrm{Var}^C(\Psi_1)}{\mathrm{E}^C(W)} = \left(B\left(\frac{\alpha}{C^2+\alpha}; \frac{\alpha-2}{2}, \frac{3}{2}\right) - \frac{4\left(\frac{\alpha}{C^2+\alpha}\right)^{\alpha-1}}{(\alpha-1)^2 B\left(\frac{\alpha}{C^2+\alpha}; \frac{\alpha}{2}, \frac{1}{2}\right)} \right) \Bigg/ \left(\frac{B\left(\frac{1}{2}, \frac{\alpha}{2}\right)}{\alpha-2} - \frac{\left(B\left(\frac{\alpha-2}{2}, \frac{1}{2}\right) - B\left(\frac{\alpha}{\alpha+C^2}; \frac{\alpha-2}{2}, \frac{1}{2}\right)\right)}{\alpha-1} \right)$$

with beta functions $B(z;a,b) := \int_0^z t^{a-1}(1-t)^{b-1}\,\mathrm{d}\,t$ and $B(a,b) := B(1;a,b)$

Limit

$$\lim_{C\to-\infty} \frac{\mathrm{Var}^C(\Psi_1)}{\mathrm{E}^C(W)} = \frac{1}{\alpha-1} \quad \Rightarrow$$

$$\lim_{C\to-\infty} \mathrm{Corr}^C(A_i, A_j) = \frac{\rho_i\,\rho_j + (\rho_{ij} - \rho_i\,\rho_j)\,(\alpha-1)}{\sqrt{(\rho_i^2 + (1-\rho_i^2)\,(\alpha-1))\,(\rho_j^2 + (1-\rho_j^2)\,(\alpha-1))}}$$

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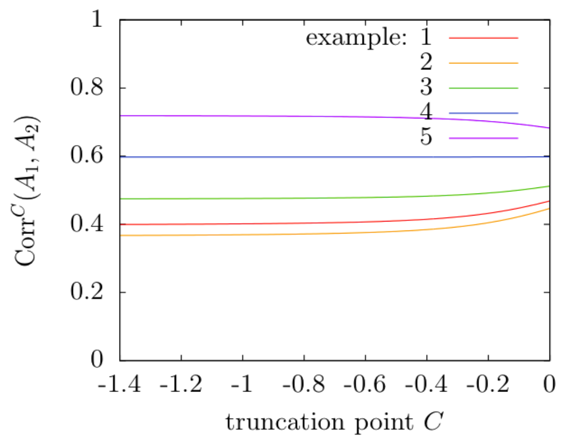
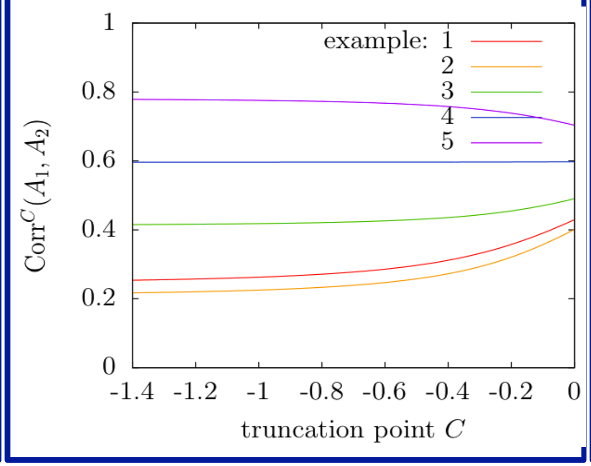
Numerical Example
Stressed asset correlations in t distributed models

Exa	ρ_{12}	ρ_1	ρ_2	lim corr: $\alpha = 4$	lim corr: $\alpha = 10$	lim corr: Gaussian
1		1	0.6	0.397	0.243	0
2		0.8	0.7	0.365	0.207	0.093
3	0.6	0.6	0.6	0.474	0.412	0.375
4		0.1	0.1	0.597	0.596	0.596
5		0.7	0.02	0.720	0.782	0.821

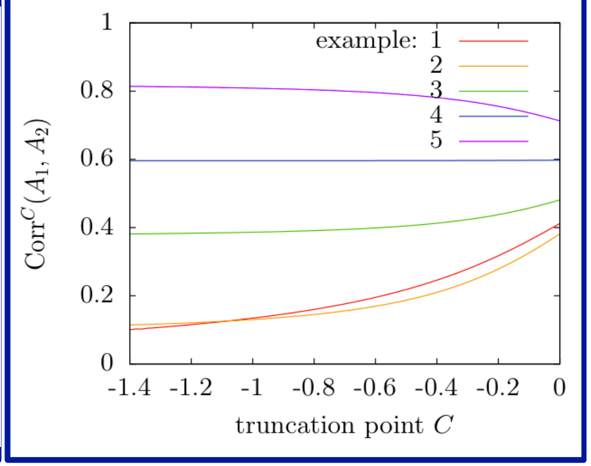
alpha = 4



alpha = 10



Gaussian



Asymptotic Limit of Stressed Asset Correlations
General formulas for Normal Variance Mixture Models

- Heavy-tailed NVM models
  - Risk factors and asset returns are in the Maximum Domain of Attraction of a Frechet distribution
  - Example: multivariate t distribution
  - Assymptotic limit of asset correlations (tail index alpha specifies the decay of the tail function):

$$\lim_{C \to -\infty} \mathrm{Corr}^C(A_i, A_j) = \frac{\rho_i\,\rho_j + (\rho_{ij} - \rho_i\,\rho_j)\,(\alpha - 1)}{\sqrt{(\rho_i^2 + (1-\rho_i^2)\,(\alpha-1))\,(\rho_j^2 + (1-\rho_j^2)\,(\alpha-1))}}$$

- Light- or medium-tailed NVM models
  - Risk factors and asset returns are in the Maximum Domain of Attraction of a Gumbel distribution
  - Example: multivariate normal distribution
  - Asymptotic limit of asset correlations:

$$\lim_{C \to -\infty} \mathrm{Corr}^C(A_i, A_j) = \frac{\rho_{ij} - \rho_i\,\rho_j}{\sqrt{(1-\rho_i^2)(1-\rho_j^2)}}$$

Stressed Asset Correlations in NVM Models
Summary

- Our analysis is not restricted to credit portfolio models but can be applied to all models where risk factors and asset (returns) follow a NVM distribution
- Correlations in heavy-tailed NVM models are less sensitive to stress, i.e., conditional correlations do not differ significantly from unconditional correlations
- In the light- or medium-tailed NVM models, the impact of stress on the conditional correlations is usually much stronger:
  - The asymptotic conditional correlation may be either greater or smaller than the unconditional asset correlation depending largely on the correlations between the risk factor and the respective asset returns
  - In particular, when the assets in question are sufficiently correlated with the risk factor, the conditional correlation is smaller than the unconditional correlation

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Empirical Stressed Correlations
Example: correlations conditional on negative DAX returns

Definition of stressed correlations

- The behaviour of correlations under stress depends heavily on the specific definition of the scenario, e.g. whether stressed correlations are calculated
  - for periods of high default rates, falling equity prices, etc
  - conditional on high asset volatility
  - conditional on large negative asset returns

Example

- We consider daily log-returns of the DAX and 5 of its components from Jan 1999 to Nov 2009
- Correlations of the companies are calculated conditional on negative DAX returns, e.g. only days are considered with DAX returns below -1%

Empirical (unconditional) correlation of asset pairs

Assets A_i, A_j	ρ_i	ρ_j	ρ_{ij}
BASF, Merck	0.73	0.36	0.30
Daimler, Telekom	0.76	0.68	0.43
Daimler, Kali & Salz	0.76	0.41	0.37

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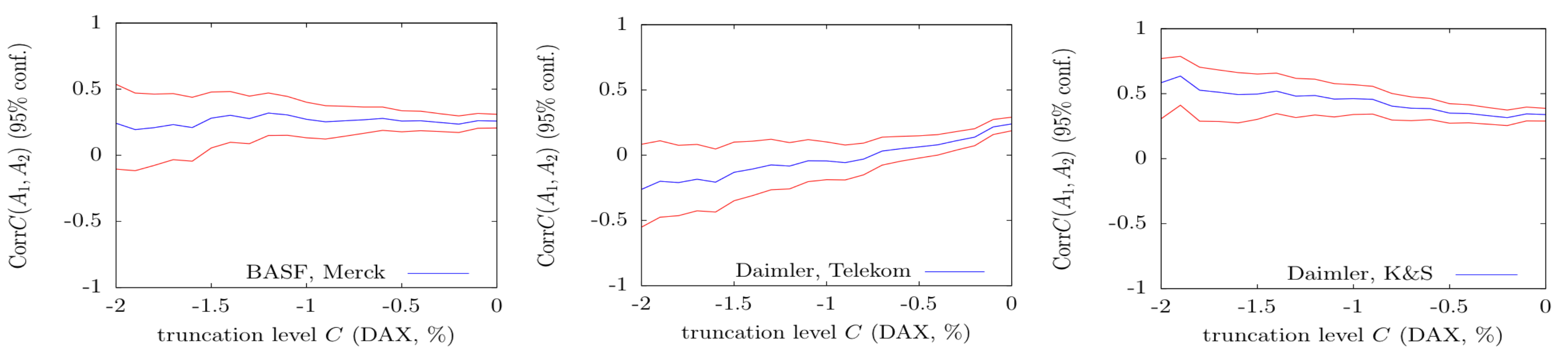
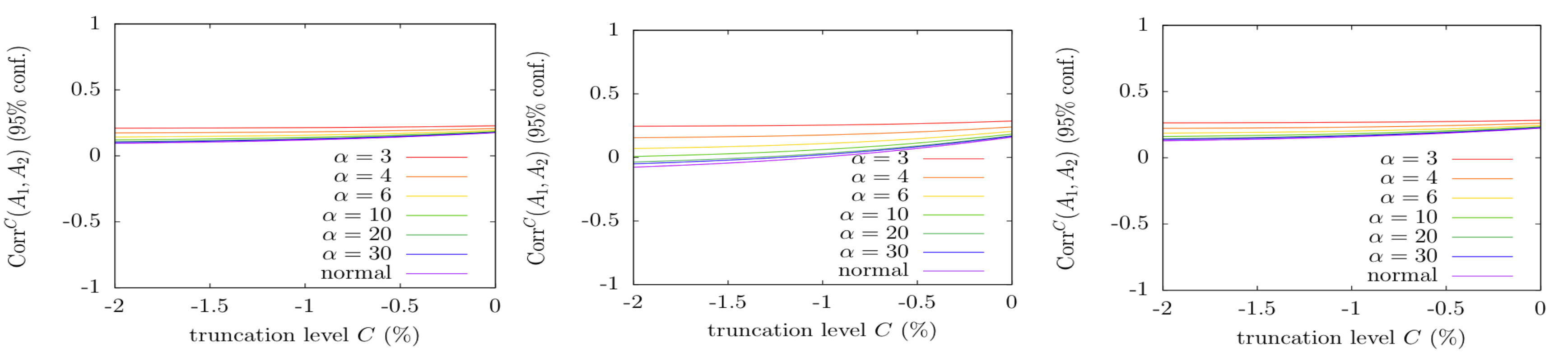
Empirical Stressed Correlations
Conditional correlations behave in different ways

Empirical: conditional correlations of asset pairs together with 95% confidence levels



Model: conditional correlations assuming t distributed and normally distributed asset returns



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