Regulating balance sheet audit: A game theoretical analysis

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Regulating balance sheet audit:

A game theoretical analysis

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Summary: The decision situation implicit to balance sheet audit is modelled as a game theoretical model and examined for its behavioural implications**.** Possible types of decisions are identified as Nash equilibria. The basic game model is characterized by an equilibrium in mixed strategies. Its properties are discussed in detail with respect to alternative scenarios of statutory audit. The regulatory propositions of separation between audit and consulting, on the one hand, and the rotation of auditors, on the other, are analyzed in the context of the game. It turns out that these measures may have contradictory effects on the quality of the balance sheet audit and it depends on the specification of regulatory details and the reaction of the agents whether the one or the other effect dominates in the end. Another proposition to enhance the audit performance is the extension of the liability of statutory auditors. Prerequisites for the success of this measure are that either no professional liability insurance exists or that, if otherwise, deductibles are calculated as a fixed percentage of the loss. The examples refer to German legal rules, however, similar legal rules are valid for most of the developed market economies.

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1. Motivation

The recent accumulation of sensational balance sheet scandals, from Enron to WorldCom, from Flowtex over Comroad to Parmalat has kindled a public discussion about enterprise control, enterprise supervision, and other regulatory instruments with a focus on how to regain the confidence in the capital markets. The discussion emphasizes the following measures:

(a) separation of audit and consulting,[1]

(b) external auditors' rotation,[2] and

(c) expansion of professional liability of the auditor.

In this contribution we will examine the effects of these measures on the quality of the balance sheet audit. Obviously, a balance sheet audit could constitute a situation of conflicting interests. On the one hand, the bookkeeper may strive for aims that violate the generally accepted accounting principles and legal regulations. The reasons for such behaviour are manifold: pressures by capital market or owners, monetary or non-monetary self-interest, etc. On the other hand, an auditor can only give the unqualified audit opinion, if no objections were found upon completion of the audit and the financial statements present a true and fair view of the net worth, financial position and results of the company. This follows, e.g., from § 322 HGB (German Commercial Code).

Given this situation, the balance sheet audit represents a *situation of conflict*. The fact that in reality, the auditor must sometimes either qualify or disclaim his audit opinion is a strong indication for the conflict potential at the annual balance sheet audit.

In the next section, the decision situation implicit to balance sheet audit is modelled as a game: the so-called *audit game*. In section 3, this model will be examined for its behavioural implications. Possible patterns of decision-making are identified with the help of the Nash equilibrium concept. It will be demonstrated that the basic audit game has an equilibrium in mixed strategies. We will discuss its properties in detail. Alternatively, we will apply the maximin solution concept and compare the results with the Nash equilibrium. We will modify the game form and assume that the bookkeeper forms expectations over the care that the auditor will take over checking the balance

[1] See, e.g., Weiland (1996) for this proposition.
[2] See, e.g., Böcking and Löcke (1997) for this proposition.

sheet. Through shaping these expectations the auditors could induce the bookkeeper to forgo balance sheet manipulations.

In section 4, we apply the game theoretical model in order to analyze the effects of the proposed separation of audit and consulting on the quality of the balance sheet audit. Similarly, we discuss the external auditors' rotation that has been proposed as an alternative to enhance the quality of balance sheet auditing. It turns out that these measures can have opposing effects and the total effect depends especially on the individual conditions of the audited firm.

In section 5, the expansion of professional liability of the auditor will be examined with respect to its capacity to motivate the auditor to follow stricter auditing strategies. It is the prerequisite for the success of this measure that either no professional liability insurance exists or deductibles are calculated as a fixed percentage of the loss.

The examples in the following text refer to German laws and regulations, however, similar legal rules are valid for most of the developed market economies[3]. The discussion of how to improve the quality of the balance sheet seems rather universal and the proposed measures are of general interest.

[3]See Sarbanes-Oxley Act of 2002 for the United States and Commission Recommendation: Statutory auditors' independence in the EU (2002).

2. The model

The opportunities for bookkeepers and auditors to make *binding agreements* are restricted, particularly if the proposed actions or omissions contradict the legal prerequisites. As a consequence, a balance sheet audit represents a strategic decision situation that can be analyzed as *non-cooperative game* (which does however not exclude that players may co-ordinate their strategies to achieve a cooperative outcome).

In order to keep the corresponding game model as simple as possible, we abstract here from other persons involved or affected by a specific balance sheet audit. We focus on a single auditor, labelled player A, and the corresponding bookkeeper, named player B. References to the interest and reaction of others are included in the discussion of the outcomes decided by A and B. This applies especially to the social evaluation of the outcomes.

The game theoretical model assumes that the auditor, i.e. player A, has the following three audit strategies:

A_1: "He checks generously and superficially and therefore does not find any faults."

A_2: "He checks normally (in the context of the so-called risk-oriented audit approach) and half of the balance sheet manipulations are found."

A3: "He carries out a complete and cost-intensive audit and finds every balance sheetmanipulation."

With reference to the German legal system, it is assumed that the bookkeeper, i.e. player B, has the following three strategies when preparing the balance sheet:

B1: "Annual accounts are prepared in accordance with HGB (German Commercial Code) and GoB (Generally Accepted Accounting Principles)"

B2: "Annual accounts with small balance sheet manipulations (on the scale of 100,000 euros)"

B3: "Annual accounts with considerable balance sheet manipulations (on the scale of 500,000 euros)"

We assume that the balance sheet manipulation gives the following utilities to player B:

$$(U) \qquad U(M_n) = \frac{M_n}{1000},$$

Here, M_n represents the number of undetected faults due to manipulation.

Moreover, we assume that player B has a utility value of 200 for an unqualified audit opinion, a utility value of 0 for the qualified audit opinion and a utility value of -200 for a disclaimed audit opinion. Player A, the auditor, is assumed to qualify the audit opinion if faults up to 50,000 euros (materiality limit[4]) are found and disclaims the audit opinion if faults of more than 50,000 euros are uncovered.

Table 1 summarizes the payoffs[5] of player B for the alternative strategy choices B_1, B_2, and B_3 depending on the chosen strategy of player A. For example, take the strategy pair (A_2, B_2): the bookkeeper, i.e. player B, carries out balance sheet manipulations in the amount of 100,000 euros. In a normal audit, the auditor (player A) uncovers half of these faults, so that balance sheet manipulations in the amount of 50,000 euros remain undiscovered. In accordance with the utility function (1), this implies a utility value of 50 for player B. Since the annual accounts contain faults, but these faults do not exceed the materiality limit of 50,000 euros, the auditor qualifies the audit opinion. It has been assumed that a qualified audit opinion gives player B a utility value of 0. Therefore, the bookkeeper gains a payoff of 50 at the strategy combination (A_2, B_2).

[4] The materiality limit depends on the size of the company. The materiality limit of 50.000 euros means that balance faults of more than 50.000 euros are too high for this specific company, so that the auditor must disclaim the audit opinion.

[5] Pay-offs are utilities that satisfy the expected utility hypothesis. This implies that u[a,p;b,(1-p)] = pu(a) + (1-p)b where u[a,p;b,(1-p)] is the utility of the lottery [a,p;b,(1-p)] with a and b representing events and p representing the probability of event a.

		B_1	B_2	B_3
A_1	Utility value from not found manipulations	0	100	500
	Utility value from audit opinion	200	200	200
	Total utility	**200**	**300**	**700**
A_2	Utility value from not found manipulations	0	50	250
	Utility value from audit opinion	200	0	-200
	Total utility	**200**	**50**	**50**
A_3	Utility value from not found manipulations	0	0	0
	Utility value from audit opinion	200	-200	-200
	Total utility	**200**	**-200**	**-200**

Table 1 : Payoffs for the bookkeeper (player B)

For player A, the auditor, we assume that the utility value consists of three different components namely the saved audit costs, the liability risk from the failure to uncovered manipulations and the advantage from the corresponding audit opinion (unqualified, qualified or disclaimed). If A carries out a generous and superficial audit (strategy A_1), then he derives a utility value of 50 from the saved costs. In an audit of a normal size (strategy A_2) he receives a utility of 0. An extensive audit (strategy A_3) causes additional costs so that a negative utility value of -50 is attached to A's strategy in this case. The liability risk of the auditor reflects the payoffs of the bookkeeper from not uncovered balance sheet manipulations inasmuch as they are represented by player B's utility values with a negative sign.

Player A gains a utility of 100 from an unqualified audit opinion and is certain to be reappointed. If the player qualifies his audit opinion, his chance of reappointing becomes uncertain so that a utility value of 0 was assumed in this case. With disclaimed audit opinion it is for sure that the auditor will not be appointed again. Moreover, he will not have the possibility to serve as a consultant either. In this case, player A will gain a utility of -100. Note that we assumed that A qualifies the audit opinion if he finds faults amounting to 50,000 euros and disclaims the audit opinion if he uncovers

substantial faults. He will give an unqualified audit opinion if he finds no fault. Of course, whether or not A finds a fault depends on the strategy of the bookkeeper B and a foolproof audit of B's Balance sheet by A.

The payoffs of player A, i.e. the auditor, are summarized in table 2. For example, given the strategy combination (A_1, B_1), A carries out a superficial audit so that he gains a utility of 50 from saved auditing costs. However, as player B does not manipulate the balance sheet, A's liability risk would be 0. Since A gives an unqualified audit opinion, he can be sure to be reappointed for the next year's audit so that he gains a utility value of 100 from this component. Totally, the auditor receives a utility value of 150 for this strategy combination.

		B_1	B_2	B_3
A_1	Utility value from cost saving	50	50	50
	Liability risk	0	-100	-500
	Utility value from audit opinion	100	100	100
	Total utility	**150**	**50**	**-350**
A_2	Utility value from cost saving	0	0	0
	Liability risk	0	-50	-250
	Utility value from audit opinion	100	0	-100
	Total utility	**100**	**-50**	**-350**
A_3	Utility value from cost saving	-50	-50	-50
	Liability risk	0	0	0
	Utility value from audit opinion	100	-100	-100
	Total utility	**50**	**-150**	**-150**

Table 2: Payoffs for the auditor (player A)

From tables 1 and 2 we can derive the payoff matrix of the audit game in table 3. The matrix makes the strategic interdependence of the decisions of A and B transparent: Neither A nor B can determine the result independent of the decision of the other player. The interests of the players are largely antagonistic, however, the sum of the payoffs of A and B in a cell of the matrix is not constant sum, i.e., table 3 describes a variable-sum game.

	B_1	B_2	B_3
A_1	(150, 200)	(50, 300)	(-350, 700)
A_2	(100, 200)	(-50, 50)	(-350, 50)
A_3	(50, 200)	(-150, -200)	(-150, -200)

Table 3 : Payoff matrix of the audit game.

The following analysis assumes that both the players know this matrix, i.e., information is *complete*, however, it is assumed that a player cannot observe (or does not otherwise know) the strategy choice of the other player, i.e., the information of the players is *not perfect.*

3. Optimal Strategies

After having specified the payoffs of the decision makers we should ask about the public interest in the problem under discussion. It seems obvious that an efficient allocation of capital presupposes correct accounting reports. Legal regulations of auditing should be directed towards this aim. A priori a socially efficient solution should operate with a minimum of auditing as careful auditing is costly and not productive in a direct way. However, it seems that the possibility of careful auditing is necessary to induce correct accounting. But how much of auditing is necessary and can it be achieved?

For the understanding the complexity of the balance sheet audit and a discussion of possible improvements of the legal framework we have to examine the question which strategy combination is a likely candidate to describe the outcome of the decisions of the bookkeeper and the auditor. If a strategy pair describes an outcome such that no player can improve his payoff by choosing an

alternative strategy, given the strategy of the other player, then this strategy pair seems to be a *fair description* for a possible outcome. Needless to say that such a pair of strategies defines a Nash equilibrium of the audit game.

It is obvious from the table 3 that player A, the auditor, gets the maximum utility value of 150 at the strategy combination (A_1, B_1). If A, however, decides for the strategy A_1, the superficial audit, B will choose the strategy B_3, i.e., considerable balance sheet manipulations, to maximize his utility value. Therefore strategy combination (A_1, B_1) does not look to be a promising candidate for describing the strategy combination that represents the result of the audit game.

On the other hand, player B, the bookkeeper, will not always decide in favour of the strategy B_3 either. If A experiences a series of repeated serious balance sheet fakes, A is likely to change his audit strategy and choose A_2 or A_3. If, for example, B assumes that A chooses A_2 and carries out an audit of a normal size, then B's best strategy is to choose B_1 and to prepare correct annual accounts with the maximum utility of 200. However, A_2 is not the "best reply" to strategy B_1.

These considerations illustrate that a strategic uncertainty about the behaviour of the other co-players is an essential feature in the game situations. The outcome of the game depends fundamentally, which *expectations* the players have about the strategy choice of their opponents. In modern game theory the forming of these expectations is sometimes discussed as an epistemic game (see Holler, 2002). However, standard game theory applies solution concepts such as the Nash equilibrium or maximin solution to operationalize (or circumvent) the expectation problem.

Of course, there is no need to form expectations if the players have strictly dominant strategies in the audit game, i.e., strategies which give larger payoffs than other strategies, irrespective of which strategy the other player is choosing. This, for instance, is the case in the well-known prisoner's dilemma game. Rational players always choose strictly dominant strategies if they exist. However, the audit game in table 3 does not contain such strategies and we have to look for other possibilities to describe the outcome of the game.

3.1 The Nash equilibrium

Standard game theory proposes the Nash equilibrium as solution concept for non-cooperative games. Nash (1951) proved that every finite game[6] has at least one equilibrium, i.e., there exists a vector of strategies that are mutually best strategies to each other exists. If (A*,B*) is an equilibrium in the audit game of table 3, then there is no strategy $A \neq A^*$ such that player A gains a higher payoff for A than for A*, given the strategy B*. The corresponding argument holds for player B and strategy B*.

If we apply the Nash equilibrium concept to the audit game in table 3, then we see that there is no pair of pure strategies which satisfy the stability requirements just defined. However, as the game is finite, Nash's proof tells us that there is at least one equilibrium in mixed strategies. A mixed strategy for player A implies that he will choose his strategies A_1, A_2 and A_3 with probabilities p_1, p_2 and p_3 such that $p_1 + p_2 + p_3 = 1$ holds. However, if $p_1 > 0$, then the expected utility of strategy B_3 is greater than the expected utility of the weakly dominated strategy B_2. Player B will not choose B_2 if he expects $p_1 > 0$.

Correspondingly, player A will never choose A_2 if A expects B to choose strategy B_3 with probability $q_3 < 1$. In this case, the expected utility gained from A_1 is always greater than the expected utility gained from A_2. Generally, if player i expects player j to choose a completely mixed strategy, i.e. with probabilities strictly between 0 and 1, then the payoffs related to a weakly dominated strategy s are always strictly smaller then the payoffs of the weakly dominating strategy s'. Therefore s will not be chosen. If we apply this result to the audit game in table 3, we can conclude that strategies A_2 and B_2 will never be chosen and can be erased as possible alternatives. Table 4 describes the reduced payoff matrix of the audit game.

	B_1	B_3
A_1	(150, 200)	(-350, 700)
A_3	(50, 200)	(-150, -200)

Table 4: Reduced payoff matrix of the audit game.

[6]A game is finite, if the number of players and their pure strategies is finite.

Of course, the reduced audit game in table 4 does not have a Nash equilibrium in pure strategies either. Let us look for the Nash equilibrium in mixed strategies. We assume that p and q are the probabilities that A chooses strategy A_1 and B chooses strategy B_1, respectively. Then 1-p and 1-q are the probabilities for playing strategies A_3 and B_3. We derive the equilibrium in mixed strategies (p*, q*) from the following experiment (see Frey and Holler, 1998). Let's assume that A chooses p* so that B is indifferent between B_1 and B_3. This implies that B derives the same payoff by either playing strategy B_1 or B_3. This is the case if

(1) $\quad p^*200 + (1-p^*)200 = p^*700 + (1-p^*)(-200)$

Correspondingly, let us assume that B chooses q* so that A is indifferent between playing A_1 and A_3. Then the payoff related to A_1 is equal to the payoff related to A_3 and therefore

(2) $\quad q^*150 + (1-q^*)(-350) = q^*50 + (1-q^*)(-150)$

If equations (1) and (2) are simultaneously satisfied, there is no possibility for a player to achieve a higher payoff, given the strategy of the other player: (p*, q*) is a Nash equilibrium. Note that if B_1 and B_3 imply identical payoffs for player B, then any randomization over B_1 and B_3 implies the same payoff for B. The corresponding result holds for q* and the payoffs for player A.

If we apply (1) and (2) to the values in Table 4 and calculate p* and q* we get p* = 4/9 and q* = 2/3. Substituting these values in (1) and (2), we get $u_A^* = -50/3$ and $u_B^* = 200$ as equilibrium payoffs for players A and B, respectively.

From this we can conclude that in 2/3 of the cases the bookkeeper will prepare a correct annual account. Moreover, there is a probability of 8/27 for the social efficient outcome of a correct account and a minimum of audit work, strategies pair (A_1,B_1). There is a probability of 10/27 for wasted audit work: the bookkeeper prepares a correct account and the auditor "carries out a complete and cost-intensive audit". This waste is *necessary*, and cannot be reduced for the given payoff matrix, since otherwise the probability of correct balance sheet and of the corresponding social benefits will be zero[7]. This seems quite a convincing outcome, however, how plausible is the mixed-strategy equilibrium as a possible description of real-world behaviour?

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[7] If $p < 4/9$ then B will not be indifferent between the two strategies B_1 and B_3 any more. In this case the strategy B_3 is the optimal choice for him.

3.2 Interpretation of the mixed-strategy equilibrium

If auditor A and bookkeeper B behave in accordance to the mixed-strategy Nash equilibrium (p*,q*) and the corresponding random mechanism selects A_1 and B_1 with probabilities 4/9 and 2/3, respectively, then each pair of strategies in the reduced audit game of table 4 is a possible outcome. If we observe that the auditor A selected A_1 and the bookkeeper B selected B_1 we can neither conclude that the players played mixed strategies nor can we conclude that they decided to follow pure strategies. A strategy is a plan and a mixed strategy will always be realized by means of pure strategies, however, these pure strategies are not proper strategies, but are *moves* in the game. We observe moves, but not strategies.

If there is a sequence of observations such that the relative frequency of (A_1, B_1) approximates $(4/9)\cdot(2/3) = 8/27$ then we have an indication that the players follow the mixed strategy pattern described by p* and q*. Then, however, tables 3 and 4 should be analyzed as a repeated game with the possibility that the strategic implications are totally different of the one-shot setting that we assumed so far (despite the fact that we considered that the auditor might suffer from not being re-appointed because he disclaimed the audited opinion).

Obviously, there is a problem of observation with the mixed-strategy Nash equilibrium. However, there are still other problems waiting: for example, (p*,q*) implies a serious motivation problem. Like any equilibrium in mixed strategies, it constitutes a weak equilibrium: If A selects a $p \neq p^*$, then A's payoffs will not change, given that B mixes his pure strategies in accordance with q*. The corresponding result applies for B, if A randomizes in accordance to p*: given p*, player B will get the same payoffs for any q. Why should A choose p* if B chooses q*, and why should B choose q* if A chooses p*? There is no obvious answer to this question.

Closely related to the motivation problem there is a problem of plausibility for the mixed-strategy equilibrium. On the one hand, equation (1) demonstrates that the equilibrium strategy of the auditor A, p*, exclusively depends on the payoffs of the bookkeeper while, on the other hand, equation (2) indicates that the equilibrium strategy of the bookkeeper B is determined by the payoffs of A, only. This holds for all payoffs such that p* and q* are in the interval [0,1]. An implication of this result is that equilibrium strategy of A, p*, will change only if the payoffs of the B change, but not when A's payoffs vary. We will come back to this result below.

3.3 The maximin solution

Because of the problems related to the mixed-strategy equilibrium it seems straightforward to look for an alternative solution concept. A possible candidate is the maximin solution. This concept has been proposed for constant-sum games by von Neumann and Morgenstern, 1947[1944]) in "Theory of Games and Economic Behaviour" As already noted the audit game is a variable-sum game, however, it expresses conflicting interests. It therefore seems reasonable to characterize the payoff which a player can guarantee himself, irrespective of which strategy the other player chooses. The maximin solution gives an answer to this problem.

	B_1	B_2	B_3	Min(A)
A_1	(150, 200)	(50, 300)	(-350, 700)	-350
A_2	(100, 200)	(-50, 50)	(-350, 50)	-350
A_3	(50, 200)	(-150, -200)	(-150, -200)	**-150**
Min(B)	**200**	-200	-200	

Table 5: Maximin solution of the audit game

Table 5 shows the minimum payoffs related to the pure strategies of A and B. The bold numbers represent the maxima of the minima. Herewith the two strategies are identified which describe the maximin solution: A_3 and B_1. If the players choose these strategies, the outcome is (50, 200). However, note that the maximin payoffs are -150 and 200 for A and B, respectively. A surprising result is that the player A choosing his maximin strategy receives a much higher payoff than the amount that he expects. The reason for this phenomenon is that the negative expectation of A implied in playing maximin strategy is not confirmed if B chooses a maximin strategy, too. In general, B's maximin strategy does not minimize the payoff of A.

It could be interesting to note that in the case of the maximin solution (A_3,B_1) auditor A achieves with the payoff of 50 a better outcome than in case of the Nash equilibrium (p*,q*) with the payoff of -50/3 whereas bookkeeper B achieves in both cases a payoff of 200. This result demonstrates that the Nash equilibrium outcome is not efficient. Moreover, it further questions its plausibility. If B

chooses his maximin strategy B_1 then he can *guarantee* himself a value of 200 while in the case of the Nash equilibrium the expected value of 200 depends on whether A chooses his equilibrium strategy p*[8]. We have seen that the motivation to choose equilibrium strategies p* and q* are "small".

If, however, auditor A assumes that bookkeeper B will choose B_1 then A_1 and not A_3 is A's best reply. As already discussed (A_3,B_1) is not a Nash equilibrium and therefore does not imply mutual best replies. However, the same applies to (A_1,B_1) which offers B an incentive to choose a different strategy. Somewhat paradoxically, the maximin strategy B_1 makes sense only if A does not expect that B will choose B_1. Otherwise we are back to the circular reasoning which, in the end, selects the equilibrium in mixed strategies as the only consistent strategy pair, despite the fact of its deficiencies in motivation and plausibility.[9]

From a social point of view we have to note that the maximin solution (A_3,B_1) implies a higher level of auditing than what is necessary to guarantee correct accounting. This amounts to an excessive waste, i.e., a waste which is larger than necessary given the strategic decision situation and imperfect information if we accept the waste in the mixed-strategy Nash equilibrium as standard. However, maximin players are not likely to trust the logic of the mixed-strategy Nash equilibrium: that is why they rely on maximin and spend more efforts on auditing than appropriate from the point of view of social efficiency.

3.4 Stackelberg solution with mixed strategies

So far we assumed that the bookkeeper B cannot observe which strategy the auditor A will choose and vice versa. However, the bookkeeper B might have some information on the average zeal of the auditors to "carry out a complete and cost-intensive audit and find every balance sheet manipulation" or to choose one of his two less cost-intensive strategies. Given the reduced payoff matrix of table 4 it is immediate that B will react with a pure strategy if his expectation p° is different from p*. If

p° > p*, then B will choose B_3, i.e., produce "annual accounts with considerable balance sheet

[8] If the player is risk averse, he would prefer the maximin solution.

[9] There is no maximin solution in mixed strategies for the audit game as there is no probability p for A such that A can guarantee himself a payoff which is independent of whether B plays B_1 or B_3. The expected utility of A will always be lower if B chooses B_3 and therefore B_3 determines the maximin payoff of A..

manipulations (on the scale of 500,000 euros)." If $p° < p^*$, then B will choose B_1, i.e., prepare the annual accounts correctly. Obviously, the second alternative is more profitable to the auditor A as it always holds that $u_A(p°) = p°150 + (1-p°)\ 50 > p°\ (-350) + (1-p°)\ (-200)$. Moreover, since $u_A(p°)$ is decreasing with p°, the auditors should aim for a p° such that $p° + \varepsilon = p^*$ and $\varepsilon > 0$ is minimal but large enough so that the bookkeepers are aware of $p° < p^*$. If we assume an ε close to 0 and insert $p^* = 4/9$ for p° into $u_A(p°)$, then we get $u_A(p°) = 850/9$. This is clearly a larger utility value than $u_A^* = -50/3$ which A achieves in the mixed-strategy equilibrium. Again, by choosing B_1 the bookkeeper secures a utility value of 200 for himself. This is equal to the $u_B^* = 200$ which results from the mixed-strategy equilibrium if A chooses p*.

This solution, which derives from Andreozzi (2004), confirms that the Nash equilibrium is not efficient. Technically speaking, it assumes that the auditor A is a Stackelberg leader and the bookkeeper B is a follower. Moreover, it assumes that A randomizes in accordance to p° and p° is known to B. Note that A has to stick to his mixed strategy p° and is not allowed to revise his strategy to play the pure strategy A_1 after B has chosen B_1. Otherwise B will react with B_3 and we are back to the non-equilibrium cycle of pure strategy pairs.

Evolutionary theory might give a convincing argument for a mixed strategy p° which is binding for A. We assume a population of auditors: some prefer to play it tough and choose strategy A_3, others are more easy going in their audit and choose A_1. If the bookkeepers know the relative frequency but have no further information on individual auditors, then this relative frequency might be a good approximation for the probability p° which describes the expectation of a bookkeeper B that a specific auditor chooses A_1.

If there are too many easy-going auditors and $p° > p^*$, then the bookkeepers will choose B_3. Table 4 illustrates that in this case it is better to be hard working and some easy going will "die out"[10]. As a consequence we can expect that p° decreases to a $p° < p^*$. If, however, $p° < p^* - \varepsilon$, then it is profitable to choose A_1 more often. This invites easy-going auditors to enter business, or hard-working auditors to become more leisurely. It seems that $p° + \varepsilon = p^*$ implies an asymptotically stable evolutionary equilibrium. This makes the described Stackelberg equilibrium look rather plausible.

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[10] This sometimes happens in reality as we saw the disappearance of Arthur Andersen after the Enron balance sheet scandal.

However, what if the auditors also have an a priori expectation on bookkeepers and the degree of manipulation they hide in their balance sheets? An evolutionary underpinning of the Nash equilibrium could derive if both parties form their expectations in accordance to the relative frequencies that reflect the distributions within the two populations.

The one-sided Stackelberg equilibrium is efficient from a social point of view. It guarantees correct accounting and a minimum waste of auditing. This outcome weakly dominates the mixed-strategy Nash equlibrium result because the latter implies only marginally lower waste from auditing for A but a substantial probability of incorrect reporting by bookkeeper B. This makes the Stackelberg equilibrium look as a very interesting design for the audit game. However, we have to see that the above result depends on the specific payoffs assumed for the game in tables 3 and 4. Alternative payoffs could initiate a randomization of the auditor A such that the bookkeeper prefers B_3, i.e., to be lazy with correct accounting. Obviously, this presupposes a rather substantial shift of payoffs for player A: so far A strictly prefers B to choose B_1 and to do correct accounting. More likely, B's expectation p° of A being somewhat leisurely in looking into the balance sheet could be too high.

It is the duty of social designer to take care of the potential of the described defects. To some extent, the social designer can perhaps rely on the common interest of the auditor community for self-regulation. In Germany, peer auditing of auditors has been established: every third year a auditor gets audited by a colleague[11].

In the next section we will look at some regulatory measures which the social designer could apply to achieve socially appropriate results, especially if both self-control of the auditors and control of the bookkeepers by the firms' own supervisory boards do not work sufficiently.

[11]The auditing of an auditor is an ex-post check-up of previous work of the auditor to enhance the quality of audit and to reduce the percentage of easy going auditors. Peer review for the auditors can have a signalling effect to the bookkeepers in the way that their expectation about the probability p° will be lower.

4. Separating of audit and consulting and external auditor's rotation

In the course of the balance sheet scandals during the last few years[12] it has been often proposed to forbid the auditor any consulting activity at the audited enterprise. The aim of this regulation is to strengthen the independence of the auditor. In addition, an external rotation of the auditors has been proposed to support this measure.

Here is a list of arguments that support the separation of audit and consulting and an external rotation of the auditors[13].

- In accordance with § 319 Abs.1 Nr.5 HGB, the auditor is not allowed to participate in the keeping of the accounting records or in the preparation of the financial statements of the company to be audited.
- A consulting ban increases the credibility of the auditor in public (see Böcking and Löcke, 1997, p.464).
- The risk of the personal and financial dependence of the auditor increases with simultaneous audit and consulting.
- A legally specified rotation of the auditors reduces the risk of the operational blindness and alliance with the bookkeeper (see Kleekämper et al., 1999, p.594).

Prominent arguments against a consulting ban or the external rotation are as follows (see Marten et al., 2003, p.157):

- A simultaneous audit and consulting leads to positive synergy effects and cost reduction possibilities.
- The audit quality is increased by a better insight into the enterprise processes.
- The frequent auditor change increases the transaction costs (search costs, training costs, rearrangement costs).

[12]For a discussion of the Enron case, see Frankfurter (2002) and Matzner (2002).
[13]For a detailed overview, see Mößle (2003), p. 218 and Ballwieser (2001, p. 102).

A comparison of the two lists shows quite controversial arguments. At this stage it is not obvious that which of the arguments are more valid than others. It seems that there are substantial tradeoffs between the various causes and effects proposed in the above list. We hope that the game theoretical model developed above could help to clarify some of the relationships underlying these arguments.

4.1 Strategic implications of the separation of audit and consulting

In order to keep the analysis of the consulting ban or the external rotation with the help of the game theoretical approach, we assume that the payoffs of the bookkeeper (player B) do not change due to consulting ban or external auditor's rotation. We focus on the payoff of the auditor to discuss the effects of alternative regulations.

		B_1	B_2	B_3
A_1	Utility value from cost saving	50	50	50
	Liability risk	0	-100	-500
	Utility value from audit opinion	200	200	200
	Total utility	**250**	**150**	**-250**
A_2	Utility value from cost saving	0	0	0
	Liability risk	0	-50	-250
	Utility value from audit opinion	200	0	-200
	Total utility	**200**	**-50**	**-450**
A_3	Utility value from cost saving	-50	-50	-50
	Liability risk	0	0	0
	Utility value from audit opinion	200	-200	-200
	Total utility	**150**	**-250**	**-250**

Table 6 : Modified payoffs of the auditor (player A)

First, we assume that the auditor will get additional consulting fees by giving an unqualified audit opinion. We assume that his utility value of an unqualified audit opinion is augmented to 200. However, with a qualified audit opinion re-appointment and additional consulting are uncertain. We assume that the utility value is 0 as before. If the auditor disclaims the audit opinion, he can be sure that he will not be re-appointed and excluded from additional consulting arrangements. We assume that this gives a negative utility value of - 200 to the auditor.[14] Table 6 summarizes the modified payoffs of the auditor (player A) taking fees for consulting work, etc. into consideration.

If we consider the modified payoffs for the auditor (player A) we get the revised payoff matrix in table 7. We can easily observe that, in this game, strategy A_2 of auditor A is strictly dominated by strategy A_1: It is always better for the auditor to choose the strategy A_1 instead of A_2, no matter which strategy the bookkeeper B pursues. Therefore, we do not have to take this strategy into consideration in what follows.

	B_1	B_2	B_3
A_1	(250, 200)	(150, 300)	(-250, 700)
A_2	(200, 200)	(-50, 50)	(-450, 50)
A_3	(150, 200)	(-250, -200)	(-250, -200)

Table 7 : Payoff matrix of the modified auditor game

Moreover, we can see that strategy A_3 is weakly dominated by A_1. If A assumes that B_3 will not be played for sure, i.e. with probability 1, then it is always better for him to pursue the strategy A_1. Neither does the strategy combination (A_3, B_3) imply a Nash equilibrium nor is it justified by the maximin solution. So why should A choose the strategy A_3?

[14]Of course, one might argue that in the case that he loses the potential for a consulting, the auditor should have the same payoff as in table 2 when consulting was not taken into consideration. However, in terms of utility a loss could be evaluated more severe then a lack of possibility despite the fact that the final state is the same. This is confirmed by many empirical tests. See Kahneman and Tversky (1979) for a seminal paper.

Even if A assumes that B chooses B_3 with a high probability, possibly tempted by its maximum payoff 700, it is never better for A to pursue the strategy A_3 instead of A_1. In other words: The auditor will probably choose the strategy A_1 and carry out an audit of "small size" in the modified game. Given this strategy he will not uncover any balance sheet manipulation and assigns an unqualified audit opinion even when there are serious balance sheet manipulations.

If the bookkeeper B takes this reasoning of the auditor A into account, B will maximize his payoff by choosing the strategy B_3 which possibly implies serious balance sheet fake. With the strategy combination (A_1, B_3) as a likely outcome of the modified game, which allows for consulting work as a co-product of auditing, the way is paved for the next balance sheet scandal.

The higher the utility advantage that the auditor derives from an unqualified audit opinion the higher the probability that the audit strategy A_1 is a dominant strategy for the auditor. Additional consulting appointments that are awarded to the auditor A in case of an unqualified audit opinion increase the likelihood of the auditor examining only superficially. As a consequence the balance sheet audit will deteriorate to an alibi ritual to be abused for arranging well-paid consulting services (see, e.g., Schmidt, 1997, p. 235).

A consulting ban or the compulsion to the external auditor rotation would reduce the utility advantage, which the auditors can get out of the unqualified audit opinion. In our model, this implies a payoff of 100 instead of 200. This would lead to the initial game situation in which the auditor possibly chooses A_3, i.e., "he carries out a complete and cost-intensive audit and find every balance sheet manipulation." This possibility proposes to the bookkeeper to choose the strategy B_1 and present correct annual accounts which is considered a prerequisite for socially efficient allocation of capital. Again, there could be social waste related to choosing A_3 too often.

Of course, we have seen that the strategy combination (A_3, B_1) is not an equilibrium of the original game situation, described in tables 3 and 5, but only a possible outcome. Nevertheless we can conclude that, so far, the game theoretical analysis suggests that it would be appropriate to introduce a strict consulting ban in order to increase the quality of the annual accounts. Similar arguments can be derived from our model to support an external auditor rotation.

4.2 Synergy effects and transaction costs

The above analysis is somewhat biased as so far we have not considered the loss of synergy effects and corresponding higher transaction costs due to consulting ban and external auditor rotation. These cost components can, however, be modelled by adjusting the entries of "utility value from cost saving" in table 2 which represent the payoffs of auditor A. To exemplify the corresponding effects we assume that A's utility value from cost saving related to strategy A_1 amounts to 50 as, in this case, the auditor examines only superficially costs are independent of synergy effects, etc. We assume that audit costs will increase for strategies A2 and A3 by 200 each if synergy effects do not prevail because of consulting ban and external auditor rotation. Table 8 summarizes the corresponding payoffs of the auditor.

		B_1	B_2	B_3
A_1	Utility value from cost saving	50	50	50
	Liability risk	0	-100	-500
	Utility value from audit opinion	100	100	100
	Total utility	**150**	**50**	**-350**
A_2	Utility value from cost saving	-200	-200	-200
	Liability risk	0	-50	-250
	Utility value from audit opinion	100	0	-100
	Total utility	**-100**	**-250**	**-550**
A_3	Utility value from cost saving	-250	-250	-250
	Liability risk	0	0	0
	Utility value from audit opinion	100	-100	-100
	Total utility	**-150**	**-350**	**-350**

Table 8: Payoffs of auditor A without synergy effects

If we take the payoffs in table 8 into account, we get the modified payoff matrix in table 9. This matrix reflects the strategic decision situation of auditor A and bookkeeper B when we assume a possible loss of synergy effects and corresponding increased transaction costs due to consulting ban and related regulatory measures.

	B_1	B_2	B_3
A_1	(250, 200)	(150, 300)	(-250, 700)
A_2	(200, 200)	(-50, 50)	(-450, 50)
A_3	(150, 200)	(-250, -200)	(-250, -200)

Table 9: Payoff matrix at increased audit costs

Obviously, in the game of Table 9, strategy A_1 strictly dominates the strategy A_2. A rational auditor will therefore never choose A_2. Furthermore we see that the strategy A_1 dominates the strategy A_3 weakly: only if the bookkeeper B decides in favour of the strategy B_3 and prepares annual accounts "with considerable balance sheet manipulations", then the auditor A is indifferent between the choice of A_1 and A_3. In all other cases A prefers the strategy A_1.

If B identifies a clear preference of the player A for the strategy A_1, then he himself will choose B_3. The result is an insufficient audit and considerable balance sheet manipulations. Of course, in the case of B3, A could choose A_3 without any utility losses (or gains), however, there is no obvious motivation why to do this and therefore why B should choose a strategy different from B3. The result of an insufficient audit, on the one hand, and considerable balance sheet manipulations, on the other, seems quite convincing, although the equilibrium (A1,B3) is weak.

From this we can conclude that the rise of the audit costs by the consulting ban or the compulsion to the external auditor rotation counteracts the intended purpose of these measures. Which of these two opposite effects predominates in the end cannot be judged without a detailed examination of the individual case.

It is not surprising that the strategic problems of the games in tables 7 and 9 are very much alike. This reflects the discussion about the pros and cons of consulting ban and external auditor rotation.

To be clearer, the decision situation must be specified in detail. This consequence concurs with the corresponding literature. For example, Ballwieser (2001, p.105) argues that the arguments pros and cons on consulting ban cannot be judged without considering the specific form of consulting work on which the amount of efficiency increase and cost saving depends. To some extent, the game theoretical analysis confirms this problem.

5. Augmented liability

In Germany, the professional liability of the auditor is primarily established in § 323 HGB (German Commercial Code). In accordance with § 323 Abs.1 HGB, auditors, his assistants and the participating legal representatives of an audit firm are obliged to audit conscientiously and impartially and to exercise discretion. If the auditor neglects these duties deliberately or negligently, he has to compensate for the damage which happens to the audited firm.

The German law says that the auditor is unlimitedly liable for a deliberate behaviour. For negligent breaches of duty the liability sum is restricted. With the law for the control and transparency in companies (KonTraG) of April 27th, 1998 the maximum liability sum of the auditor for damages caused negligently was increased from half a million DM to two million DM for unlisted firms. With the beginning of the year 2002, the corresponding liability sum is one million Euro. It is four million Euros for listed firms. § 323 HGB regulates merely the liability of the auditor with respect to the audited firm.

In the German law the third party liability of the auditor concerning, e.g., investors, lenders, employees and other agents who are not party of the audit contract, but suffer from losses because of a wrong audit opinion, derives from § § 823 BGB pp. (German Civil Code). The current jurisdiction shows a clear tendency toward the expansion of the third party liability of the auditor (see Mößle, 2003, p. 262 pp.).

An increase of the maximum of the liability sum in the context of the KonTraG and an increase of third party liability can oblige the auditor to pay compensation for damages which may exceed a multiple of the size of the balance sheet manipulations. We model this fact in our balance sheet audit game by defining the liability risk of the auditor as four times sum of the balance sheet manipulations that remain uncovered. Table 10 shows the corresponding payoffs for the auditor.

		B_1	B_2	B_3
A_1	Utility value from cost saving	50	50	50
	Liability risk	0	-400	-2,000
	Utility value from audit opinion	100	100	100
	Total utility	**150**	**-250**	**-1,850**
A_2	Utility value from cost saving	0	0	0
	Liability risk	0	-200	-1,000
	Utility value from audit opinion	100	0	-100
	Total utility	**100**	**-200**	**-1,100**
A_3	Utility value from cost saving	-50	-50	-50
	Liability risk	0	0	0
	Utility value from audit opinion	100	-100	-100
	Total utility	**50**	**-150**	**-150**

Table 10: Payoffs of auditor A at increased liability

In principle, the increase of liability and corresponding compensation can have a positive effect on the payoffs of the bookkeeper B. However, if the liability is due to the fact that B manipulated the balance sheet, it is not likely that this very B will be allowed to profit from liability payments of A. If we assume that the payoffs of the bookkeeper (player B) are independent of the increased liability of the auditor, then we get the game represented in the payoff matrix in table 11.

	B_1	B_2	B_3	Average payoffs of player A
A_1	(150, 200)	(-250, 300)	(-1.850, 700)	**-650**
A_2	(100, 200)	(-200, 50)	(-1.100, 50)	**-400**
A_3	(50, 200)	(-150, -200)	(-150, -200)	**-67**

Table 11: Payoff matrix with increased liability for A

In the decision situation described in table 11, neither of the two players has a strictly dominant strategy. The strategy B_3 dominates B_2 weakly, though. But even if we eliminate B_2, the decision situation of the players does not simplify fundamentally. Note that we cannot eliminate a strategy of the auditor A. The reduced game does not show any dominant strategies and a Nash equilibrium in pure strategies does not exist. Of course, an equilibrium exists in mixed strategies as the game is finite (see section 3.1). However, we have already seen that a mixed strategy equilibrium represents a very high degree of abstraction in such a game. It is difficult to interpret it as an instruction for action or as an instrument for forecasting actions. As it auditor A has to decide over a set of three strategies, the game in table 11 is, at least at the first glance, even more complex than the game in table 4. Perhaps it seems more realistic to assume that A cannot derive the equilibrium and applies the Laplace rule (the "principle of insufficient reason") instead and assumes a probability of 1/3 for each of the three strategies of the bookkeeper B. Table 11 shows the average payoffs for this case. This suggests that auditor A selects A_3. Note that the decision for A3 will not change if the auditor A assumes that the bookkeeper B will not choose his weakly dominated strategy B_2.

If auditor A succeeds to convince bookkeeper B that he will play A_3, then B will choose B_3 and offer correct balance sheets. The increase of the liability results that (A_3,B_1) becomes likely and balance sheets will not be manipulated. Thus an increase in the liability will not only be an appropriate instrument to increase the intensity of auditing but also to achieve correct balance sheets. Of course, the costs of intensive auditing can be considered as waste, given that the bookkeeper does not manipulate his report, however, without this costs it cannot be assured that the balance sheets will be correct.

Of course, the strategy combination (A_3,B_1) is not a Nash equilibrium. We can only expect this result either as an outcome of randomization or as a consequence of bounded rationality of the auditor A as A does not expect that bookkeeper B will choose B_1 "for sure". Moreover, we can only expect (A_3,B_1) if the auditor has to pay the liability himself. However, according to § 54 Abs. 1 WPO (German auditors law) an auditor is obliged to have a professional liability insurance to cover losses by carrying out his profession. If an auditor can refer to such an insurance without deductibles, an increase of maximal liability or an extension of the liability for third party losses results only in an increase of premium payments. This has been observed during the last ten years. These costs are relevant for all three strategies of the auditor A so that A's preference for A_3 is no longer obvious. Thus, if a high intensity of auditing should be achieved then the insurance contracts should be qualified by substantial deductibles. Moreover these deductibles should be a percentage of the liability, i.e., it should not be a constant sum for each case of liability.

By expanding the third party liability of the auditor it should be distinguished between a gross and simple negligence (see Schäfer, 2004, p. 21), depending on whether the balance sheet audit predominantly has an effect on the primary or secondary capital market. If the audit opinion is relevant to the primary capital market (e.g., leaflet audit at an initial going public, when the audited enterprise gets fresh capital), a simple negligence could already imply a liability reason. A false judgement on the part of the auditor can trigger a misallocation of resources and thus reduce the overall welfare. Given the specific social responsibility of the auditor in this case, simple negligence should be sufficient to trigger third party liability.

If the balance sheet audit is effective only for the secondary capital market, then a false judgement has distributional effects only: If the capital value is set too highly, then the sellers will win what the buyers will lose through the wrong audit opinion. In these cases the third party liability should take effect only at a gross negligence of the auditor.

6. Conclusion

In this essay, we have looked at the annual balance sheet audit from the game theoretical view. The game theoretical perspective makes the strategic decisions between auditor and bookkeeper more transparent. It demonstrates the complexity of the decision problems involved in “balance sheet audit”. The essay develops new approaches to analyze these complexities.

The game theoretical approach allows us to discuss the effects of the ban of simultaneous audit and consulting on the quality of the audit and the reliability of the audit opinion. It can be seen that the separation of audit and consulting and the compulsion to the external auditor rotation can change the structure of the audit game such that the auditor rather tends to apply a stricter audit strategy and the bookkeeper is motivated to prepare a correct balance sheet. However, these measures have the side-effect that the additional costs caused by the consulting ban and the compulsion to the auditor rotation can work against the initial effect so that the total effect of these measures remains uncertain.

Measures to rise the maximum liability sum and to expand third party liability seems more promising from the game theoretical view. As a rule, these measures force the auditor to a stricter audit strategy without suffering from undesirable side-effects. This result only stands up for case that no liability policy for the audit profession exists. If, however, a vocational liability insurance exist, deductibles should be applied as a percentage of the loss sum to achieve the desired effect described above.

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