Sufficient Conditions for Expected Utility to Imply Drawdown-Based Performance Rankings

Frank Schuhmacher und Martin Eling

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Sufficient Conditions for Expected Utility to Imply Drawdown-Based Performance Rankings

Frank Schuhmacher[a,*], Martin Eling[b]

[a] *University of Leipzig, Grimmaische Straße 12, 04109 Leipzig, Germany*

[b] *Ulm University, Institute of Insurance Science, Helmholtzstraße 22, 89081 Ulm, Germany*

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Abstract

The least restrictive sufficient condition for expected utility to imply Sharpe ratio rankings is the location and scale (LS) condition (see Meyer, 1987). The LS condition includes the normal and many other (asymmetric and leptokurtic) distributions commonly used in finance. In this paper we argue that this condition is also sufficient for expected utility to imply drawdown-based performance measure rankings because for investment funds satisfying the LS condition, it does not matter whether funds are ranked with the Sharpe ratio or with a drawdown-based performance measure as the rankings are identical. Hence, the same conditions that provide an expected utility foundation for the Sharpe ratio also provide an expected utility foundation for drawdown-based performance measures. Theoretically drawdown-based performance measures are as good as the Sharpe ratio.

JEL classification: D81; G10; G11; G23; G29

Keywords: Asset management; Performance measurement; Sharpe ratio; Drawdown; Calmar ratio; Location and scale condition

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∗ Corresponding author: Tel.: +49 341 9733670; fax: +49 341 9733679.

E-mail addresses: schuhmacher@wifa.uni-leipzig.de (Frank Schuhmacher), martin.eling@uni-ulm.de (Martin Eling)

1. Introduction

The most widely known performance measure is the Sharpe ratio, which is a simple reward-to-risk ratio that measures the relationship between the mean and the standard deviation (MS) of excess returns generated by an investment fund. The Sharpe ratio is an adequate performance measure if the investor wishes to place all his or her risky assets in just one investment fund and either the investor's preferences or the investment fund returns satisfy certain conditions that ensure consistency between the MS and the expected-utility (EU) model.

The two most often cited of these conditions—quadratic utility functions and normally distributed returns—have theoretical defects and no empirical support. Fortunately, however, neither quadratic utility functions nor normally distributed returns are required to justify use of the Sharpe ratio for ranking investment funds. Sinn (1983) and Meyer (1987) show that if the investment fund returns are equal in distribution to one another except for location and scale (LS), a Sharpe ratio ranking can be derived from a ranking based on expected utility. Hence, contrary to popular belief, asymmetry and fat tails are not reasons to reject use of the Sharpe ratio. Many families of random variables described in statistics textbooks and used in finance to describe asset returns (see Levy and Duchin, 2004) are made up of members that differ from one another only by location and scale parameters.[1]

The last two decades have witnessed many authors proposing alternatives to the Sharpe ratio because they believe that the conditions that ensure consistency between the MS and the EU models are theoretically defective and have no empirical support. A common argument for using an alternative reward-to-risk ratio is the presence of asymmetry and fat tails in

[1] The normal, extreme value, and logistic families are two-parameter examples that satisfy the condition. The gamma, Student's t, and Weibull families are three-parameter examples that satisfy the condition if the shape parameter is fixed. The beta family is a four-parameter example that satisfies the condition if the two shape parameters are fixed. It is important to note that the LS condition applies to investment funds as a whole, specifying how the funds' returns must be related to each other, but it does not restrict individual return distributions.

financial market returns. For example, hedge funds frequently generate nonnormal returns and it is thus popularly believed that these cannot be evaluated using the Sharpe ratio (see Eling and Schuhmacher, 2007). Consideration of this issue has led to the development of many alternative reward-to-risk ratios.

A special class of these alternative reward-to-risk ratios consists of drawdown-based measures. These measures are unique in that they are completely ignored in theory (i.e., by top-tier finance journals) but widely used in practice. The most frequently used drawdown-based measure is the Calmar ratio, which is the ratio of mean (excess) return in relation to the maximum drawdown. There are numerous other variations of drawdown measures, including the Sterling ratio, the Burke ratio, the Martin ratio, and the Pain ratio (see Bacon, 2008). One reason for the large gap between theory and practice when it comes to drawdown-based performance measurement is that drawdowns are easy to interpret but difficult to analyze—at least formally. Hence, almost nothing is known about the properties of drawdown-based performance measures, not to mention whether they have a solid theoretical foundation.

The aim of this paper is to fill this void by providing a theoretical foundation for drawdown-based performance measurement. We numerically show, using Monte Carlo simulations, that the ranking of investment funds that satisfy the LS condition does not depend on whether the Sharpe ratio or an adequately defined drawdown-based measure is used. This means that all rankings are identical given that each investment fund is equal in distribution to the others except for location and scale and given that drawdowns are adequately defined. "Adequately defined" means that drawdowns are defined by cumulated uncompunded excess returns and not by cumulated compounded returns as is often done in practice.

This finding has important implications for drawdown-based performance measurement. The use of adequately defined drawdown-based measures is theoretically sound under the same conditions as is the Sharpe ratio, i.e., for random investment alternatives that satisfy the LS condition. Hence, from a theoretical point of view, drawdown-based performance measures are as good as the Sharpe ratio if drawdowns are adequately defined. More importantly from a practitioner point of view, we argue that drawdown-based measures as used in practice approximate adequately defined drawdown-based performance measures and, hence, lead to almost identical rankings as the Sharpe ratio. This conclusion is supported by Eling and Schuhmacher (2007), who indeed find high rank correlations between the Sharpe ratio and the drawdown-based performance measures as used in practice.

The rest of the paper is organized as follows. Section 2 provides definitions for drawdown-based performance measures. Section 3 states the LS condition and shows which commonly used distributions satisfy this condition and which do not. In Section 4, we show with Monte Carlo simulations that for return distributions that satisfy the LS condition, drawdown-based performance measures are strictly increasing functions of the Sharpe ratio, and that for return distributions that do not satisfy the LS condition, drawdown-based performance measures are not strictly increasing functions of the Sharpe ratio. We conclude in Section 5.

2. Drawdown-based performance measurement

Drawdown-based performance measures are widely used in practice (e.g., by commodity trading advisors) but almost completely ignored in theory. All these measures were developed by practitioners who, understandably, did not base their work on theoretical considerations. The literature dealing with drawdown-based performance measures is mostly found in nonfinance journals,[2] nonrefereed finance journals,[3] and third-tier finance journals primarily aimed at the investment community.[4] The measures are also discussed in many

[2] See Dacorogna et al. (2001) and Petroni/Rotundu (2008).

[3] See Young (1991), Burke (1994), and Kestner (1996).

[4] See Ammann/Moerth (2008), Cogneau/Hübner (2009), Magdon-Ismail/Atiya (2004), Pedersen/Rudholm-Alfvin (2003), Sharma (2004), Chekhlov et al. (2005), Hameling/Hoesli (2004), and Câmara Leal/vaz de Melo Mendes (2005). The last three papers use drawdowns as a risk measure and do not explictely analyze performance measures. There are also a few articles in higher-class finance journals (as measured by the

finance textbooks.[5] A systematic analysis of this literature leads to five drawdown-based performance measures that can be broadly defined as follows (all these measures are mentioned in Bacon, 2008):

- Calmar ratio: average excess return divided by the maximum drawdown.
- Sterling ratio: average excess return divided by the average of the *n* most significant drawdowns.
- Burke ratio: average excess return divided by the square root of the sum of the squares of the *n* most significant drawdowns.
- Martin ratio*:* average excess return divided by the square root of the mean of the squared monthly percentage drops from the previous peak.
- Pain ratio: average excess return divided by the average of the monthly percentage drops from the previous peak.

Given the variety of possibilities for calculating drawdowns, great care needs to be taken in defining these performance measures. In the remainder of this paper, the following definitions are used. All performance measures are calculated over the same time period, using the same frequency of data. Let p_t be the price of a risky portfolios at the end of each month t = 0, 1, …, T. To be able to compare the drawdown-based performance measures to the Sharpe ratio, simple rates of returns are calculated using the formula $r_t = p_t/p_{t-1} - 1$, with t = 1, …, T. Furthermore excess rates of return $R_t = r_t - r_{f,t}$ over the risk-free rate $r_{f,t}$ are used in all following formulas.

The average excess rate of return is $R = (R_1 + \ldots + R_T)/T$. It is common practice to calculate drawdowns as compounded cumulative rates of return. However, for our purpose, it is necessary to calculate drawdowns as uncompounded cumulative excess rates of return. The

impact factor), such as Tashman/Frey (2009), Eling (2008), Eling/Schuhmacher (2007), Alexander/Baptista (2006), and Grossman/Zhou (1993). The last two papers use drawdowns as a risk measure and do not explictely analyze performance measures.

5 See Jaeger (2003), Phillips/Surz (2003), Gregoriou et al. (2004), Lhabitant (2004), Ridley (2006), Gregoriou (2006), Chincarini/Kim (2006), Bacon (2008), Siddiqi/Hrubri (2008), and Narang (2009).

uncompounded cumulative excess rate of return from buying in t = i and selling in t = j with j > i is $R_{i,j} = R_{i+1} + \ldots + R_j$.

The maximum drawdown (MDD) is defined as the largest negative uncompounded cumulative excess rate of return, i.e., MDD = $\max_{i,j>i} (-R_{i,j})$. Continuous drawdowns are defined as uncompounded cumulative excess rates of return that are not interrupted by a positive monthly excess rate of return. The index n runs from 1 to N, where CDD_1 is the largest continuous drawdown, CDD_2 the second largest continuous drawdown, and CDD_N the smallest continuous drawdown. Drawdowns from a previous peak, DDP_t, are calculated at the end of each month t = 1, 2, …, T, and are defined as uncompounded cumulative excess rates of return from the previous peak, i.e., $DDP_t = \max_{i \leq t} (-R_{i,t})$.

Using these definitions, the performance measures are calculated as follows:

$$\text{Calmar ratio} = \frac{R}{MDD}$$

$$\text{Sterling ratio} = \frac{R}{\frac{1}{N}\sum_{n=1}^{N} CDD_n}$$

$$\text{Burke ratio} = \frac{R}{\sqrt[2]{\sum_{n=1}^{N} CDD_n^2}}$$

$$\text{Martin ratio} = \frac{R}{\sqrt[2]{\frac{1}{T} \cdot \sum_{t=1}^{T} DDP_t^2}}$$

$$\text{Pain ratio} = \frac{R}{\frac{1}{T}\sum_{t=1}^{T} DDP_t}$$

It is important to calculate drawdowns with uncompounded cumulative excess rates of return because using compounded cumulative rates of return creates the problem that the values of these measures can easily be increased (or decreased) by moving along the capital allocation line (i.e., combinations of the risk-free asset and risky portfolio). A simple example will illustrate this problem: consider 60 monthly returns and assume that the risk-free rate is 1% every month. An investment fund realizes a rate of return of 2% in all but the last two months. In the last two months, the investment fund realizes –2% each month. The investment fund's average excess rate of return is approximately 1% and the maximum drawdown with compounded cumulative rates of return is approximately 4%, meaning that the Calmar ratio is approximately 0.25.[6] Now consider a portfolio that invests 50% of initial wealth in the investment fund and 50% in the risk-free instrument. This portfolio realizes a rate of return of 1.5% in all but the last two months. In the last two months, the portfolio realizes –0.5% each month. The portfolio's average excess rate of return is approximately 0.5% and the maximum drawdown with compounded cumulative rates of return is approximately 1%, meaning that the Calmar ratio is approximately 0.5.[7] Hence the investment fund can easily increase its Calmar ratio with compounded cumulative rates of return from 0.25 to 0.5 by moving along the capital allocation line. Since the value of a performance measures should not change when moving along the capital allocation line, drawdown-based performance measures should not be calculated with compounded cumulative rates of return. Calculating the maximum drawdown with uncompounded cumulative excess rates of return has the effect that both the Calmar ratios's enumerator and denominator are weighted sums of excess rates of returns. Moving along the capital allocation line thus affects the enumerator and the denominator in the same way such and the quotient is unaffected.[8]

[6] More exactly, the average excess rate of return is 0.87% and the maximum drawdown with compounded cumulative rates of return is 3.96% = – ((1 – 0.02) (1 – 0.02) – 1). The Calmar ratio with compounded cumulative rates of return is therefore 0.22.

[7] More exactly, the average excess rate of return is 0.43% and the maximum drawdown with compounded cumulative rates of return is 0.9975% = – ((1 – 0.005) (1 – 0.005) – 1), meaning that the Calmar ratio with compounded cumulative rates of return is 0.43.

[8] Note that the portfolio's average excess rate of return is half of the investment fund's average excess rate of return and the maximum drawdown calculated with uncompounded cumulative excess rates of return is also

3. Return distributions that satisfy the LS condition

Our hypothesis is that when ranking investment funds that satisfy the LS condition, it does not matter whether it is the Sharpe ratio or a drawdown-based measure, as defined in Section 2, that is used. To analyze and explain which distributions satisfy and which distributions do not satisfy the LS condition, Table 1 describes nine distributions commonly used in finance to describe asset returns (see Levy and Duchin, 2004; Kassberger and Kiesel, 2006) and one other distribution, namely, the unform distribution, that is not a good asset return model but does satisfy the LS condition.

According to Meyer (1987), the LS condition requires that the random alternatives differ from one another only by location and scale parameters. Two density functions, f and g, differ only by location parameter a and scale parameter b if f(x) = (1/b) g[(x–a)/b].[9] The normal, logistic, and student's t distribution with fixed degrees of freedom are elliptical distributions and as such satisfy the LS condition. According to Meyer and Rasche (1992), the uniform distribution satisfies the LS condition. Furthermore, it is easy to see from the probability density functions set out in Table 1 that the beta, the gamma, and the Weibull distribution with fixed shape parameters, as well as the extreme value distribution, also satisfy the LS condition (all these distributions contain the general form (x–location parameter)/scale parameter, which is then again divided by the scale parameter). The lognormal and the normal inverse Gaussian do not satisfy the LS condition.

half of the investment fund's maximum drawdown calculated with uncompounded cumulative excess rates of return. Hence, the fund's and the portfolio's Calmar ratios are identical.

9 The location parameter indicates the shift of a probability distribution, while the scale parameter determines the dispersion of a probability distribution. See Meyer (1987) for another definition of the LS condition.

Table 1: Distributions, Parameters, and Probability Density Functions

Distribution	Parameter	Probability density function
Beta (Page 210)	Location: a Scale: b – a > 0 Shape: p, q > 0	$\frac{1}{B(p,q)\cdot(b-a)}\cdot\left(\frac{x-a}{b-a}\right)^{p-1}\cdot\left(1-\frac{x-a}{b-a}\right)^{q-1}$
Extreme value (Page 111)	Location: ξ Scale: θ > 0	$\frac{1}{\theta}\cdot e^{-\frac{x-\xi}{\theta}}\cdot exp\left[-e^{-\frac{x-\xi}{\theta}}\right]$
Gamma (Page 337)	Location: γ Scale: β > 0, Shape: α > 0	$\frac{1}{\beta\cdot\Gamma(\alpha)}\cdot\left(\frac{x-\gamma}{\beta}\right)^{\alpha-1}\cdot e^{-\left(\frac{x-\gamma}{\beta}\right)}$
Logistic (Page 116)	Location: α Scale: β > 0	$\frac{1}{\beta}\cdot\left[exp\left(\frac{x-\alpha}{\beta}\right)\right]\cdot\left[1+exp\left(\frac{x-\alpha}{\beta}\right)\right]^{-2}$
Normal (Page 80)	Location: μ Scale: σ > 0	$\frac{1}{\sigma\cdot\sqrt{2\cdot\pi}}\cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$
Student’s t (Page 26)	Location: m Scale: b, Shape: υ	$\frac{\Gamma\left(\frac{\upsilon+1}{2}\right)}{\mathrm{b}\cdot\sqrt{\upsilon\cdot\pi}\cdot\Gamma(\upsilon/2)}\cdot\left(1+\frac{\left(\frac{x-m}{b}\right)^2}{\upsilon}\right)^{-\frac{\upsilon+1}{2}}$
Uniform (Page 276)	Location: α Scale: β	$\frac{1}{\beta-\alpha}$
Weibull (Page 629)	Location: ξ Scale: $\alpha > 0$ Shape: c > 0	$\frac{c}{\alpha}\cdot\left(\frac{x-\xi}{\alpha}\right)^{c-1}\cdot exp\left[-\left(\frac{x-\xi}{\alpha}\right)^{c}\right]$
Lognormal (Page 208)	$\xi$and σ	$\frac{1}{x\cdot\sigma\cdot\sqrt{2\cdot\pi}}\cdot exp\left[-\frac{1}{2}\cdot\left(\frac{\ln(x)-\xi}{\sigma}\right)^2\right]$
Normal inverse Gaussian (Page 105)	α, β, μ, δ	$a\cdot q\left(\frac{x-\mu}{\delta}\right)^{-1}\cdot K_1\left(\delta\cdot\alpha\cdot q\left(\frac{x-\mu}{\delta}\right)\right)\cdot e^{\beta\cdot x}$ with $a=\pi^{-1}\cdot\alpha\cdot exp\left(\delta\cdot\sqrt{\alpha^2-\beta^2-\beta\cdot\mu}\right)$ and $q(y)=\sqrt{1+y^2}$

Note: The probability density function (pdf) of the normal inverse Gaussian distribution is taken from Barndorff-Nielsen and Prause (2001), the pdf of Student’s t distribution is from Fergusson and Platen (2006), and all other pdfs are from Johnson, Kotz, and Balakrishnan (1994, 1995).

4. Numerical calculations of drawdown-based performance measures

In this section, we numerically show, using Monte Carlo simulations, that when ranking investment funds that satisfy the LS condition, it does not matter whether it is the Sharpe ratio or an adequately defined drawdown-based measure that is used. To explain the Monte Carlo simulations, we use the beta distribution as an example. Since the beta distribution satisfies the LS condition with fixed shape parameters, the first step is to fix these parameters. We have fixed the shape parameters such that skewness and kurtosis of the resulting beta distribution are broadly consistent with empirical values for monthly rates of returns. The results, however, also hold for other fixed shape parameters.

For the beta distribution, we fixed the shape parameters at p = 5 and q = 1, which implies a skewness of –1.2 and a kurtosis of 4.2. This distribution is thus aymmetric and has fat tails, which is typical of time series in financial markets (e.g., for stocks or for hedge funds). Note that both skewnness and kurtosis are independent of the location and scale parameters. There is a one-to-one relationship between the location and scale parameter of the beta distribution on one hand, and the mean and standard deviation of that distribution on the other hand.[10] Hence, skewness and kurtosis of the beta distribution are also independent of the mean and the standard deviation of that distribution.

The first result, which is a necessary precondition to the main result, is:

Lemma: For the beta distributrion and all other distributions that satisfy the LS condition, the values of the drawdown-based performance measures depend only on the Sharpe ratio.

This lemma is proved numerically with Monte Carlo simulations and to explain the proof we again use the beta distribution as an example. The lemma means that for two beta distributions with identical shape parameters and identical Sharpe ratios, but different means and standard deviation we numerically calculate the same values for all drawdown-based performance measures. As an illustration, assume that two funds have excess rates of returns

[10] For given location and scale parameters, the mean is (a p+b q)/(p+q) and the variance is (p q) (b-a)2/((p+q)2 (p+q+1)). For given mean and variance, the location parameter is (mean (a+b) – n a)/b and the scale parameter is ((b variance^{0,5} (a+b) (a+b+1)^{0,5})/((a*b)^{0,5}) + mean*(a+b))/(b+a).

that follow the beta distribution with identical shape parameters p = 5 and q = 1, where fund 1 has a standard deviation of 10% and a mean of 5% and fund 2 has a standard deviation of 8% and a mean of 4%. Because both funds have identical Sharpe ratios, it can be numerically shown that both funds have identical values for all the drawdown-based performance measures described in Section 2. To avoid any misunderstanding, we again stress that this result does *not* depend on the specific choice of any parameter. Given the above lemma, we can numerically show our main result.

Proposition: For the beta distribution and all other distributions that satisfy the LS condition, the values of the drawdown performance measures are increasing in the Sharpe ratio.

The numerical proof of this proposition is clearly illustrated by Figure 1, which shows, for different distributional assumptions, the values of all the drawdown-based performance measures under consideration here that we have calculated with Monte Carlo simulations as explained below. To elucidate these figures, we discuss the Calmar ratio as an example.

The x-axis shows the values of the Sharpe ratio that we fixed (from 0.01 to 1.00) in our simulations. The y-axis shows the results for the Calmar ratio for different distributional assumptions. The left part of Figure 1 shows the results for all distributions (from our set) that fulfill the LS condition; the right part shows the results for all distributions (from our set) that do not fulfill the LS condition. The normal distribution is accentuated in the figure by the fat line. Due to the high number of distributions in the left part of the figure, it is not possible to identify single distributions, but there are two important results worth emphasizing. First, all distributions shown in the left part of the figure are strictly increasing functions of the Sharpe ratio, while all distributions in the right part are not strictly increasing. The second notable finding is that the results for various distributions are very comparable to the results for the normal distribution.

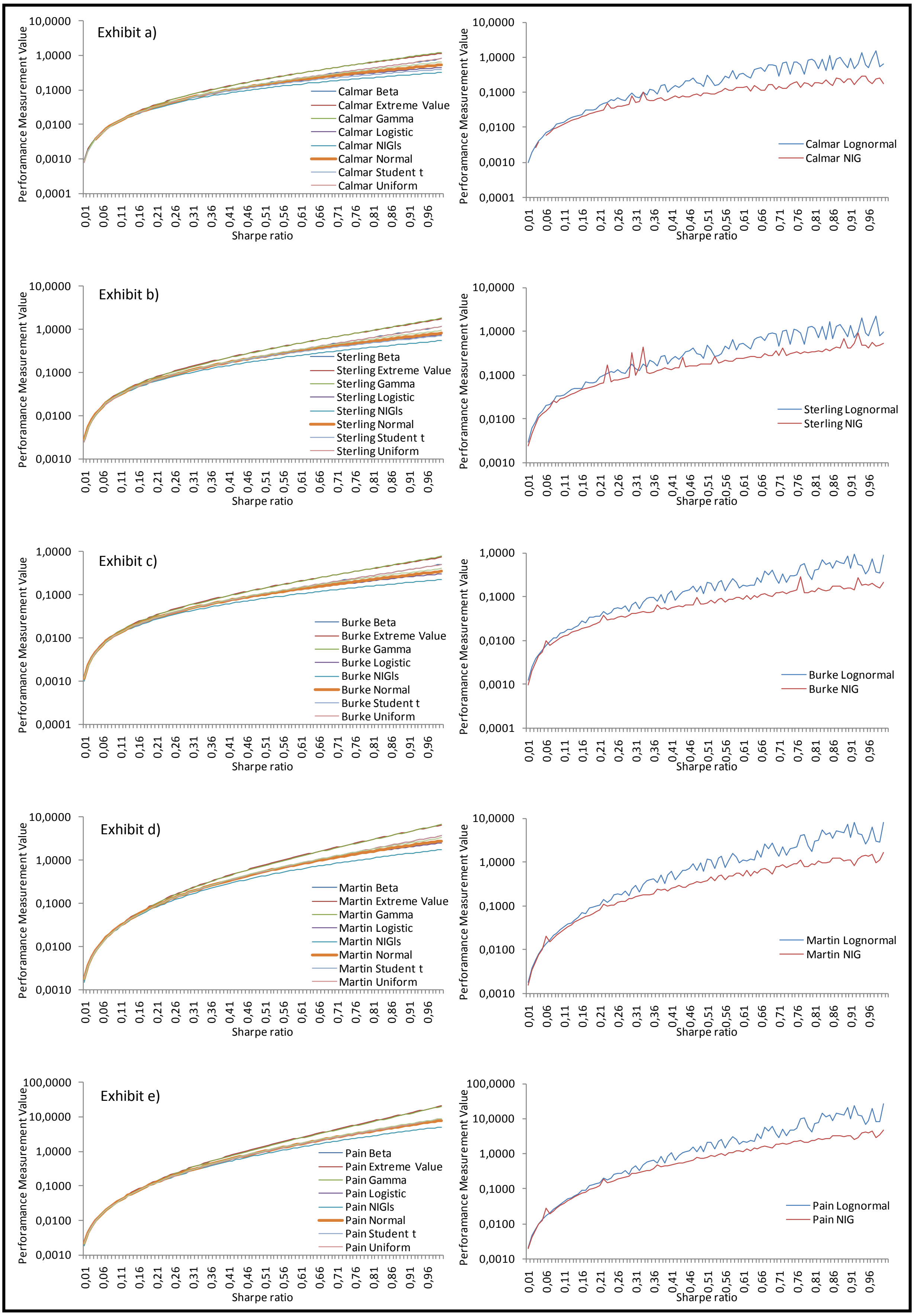
Figure 1 demonstrates that for any distribution satisfing the LS condition, all drawdown-based performance measures are increasing functions in the Sharpe ratio, i.e., the higher the Sharpe ratio, the higher all the other performance measures. More precisely, this means that fund A's Sharpe ratio is greater than fund B's Sharpe ratio if and only if fund A's drawdown-based performance measure is greater than fund B's drawdown-based performance measure.

Note that while our results do not depend on the specific choice of parameter, these functions do depend on this specific choice. For example, for the beta distribution and other shape parameters, the function would be different but would still be an increasing function of the Sharpe ratio. All drawdown-based performance measures are calculated with Monte Carlo simulations from 100,000 samples, each sample consisting of 120 returns.[11]

Further simulations show that the drawdown-based performance measures result in the same ranking as the given by the Sharpe ratio when the fund returns satisfy a generalization of the location and scale condition described in Meyer and Rasche (1992). This is an important extension of the result presented in Figure 1 because it shows that funds with different skewness and kurtosis values (vs. idential skewness and kurtosis for all funds under the standard LS condition) are also ranked appropriately by the drawdown-based measures.

[11] We thus calculate the mean, standard deviation, and the performance measures across 120 monthly returns and then calculate their means across 100,000 simulations. The question may arise as to why we did not choose fewer samples with many more returns for each sample so as to achieve more accurate estimates. The answer to this is found in the nature of the drawdown-based risk measures: these risk measures increase with the number of returns in each sample. For example, the expected maximum drawdown for 120 returns is smaller than the expected maximum drawdown for 1,200 returns. Since our returns are monthly returns, we chose 120 returns, which is equal to 10 years with monthly returns. The programming was conducted in Matlab; the code is available under http://www.uni-ulm.de/mawi/ivw/team/meling.html.

Figure 1: Calmar (a), Sterling (b), Burke (c), Martin (d), and Pain (e) Ratios



4. Conclusions

Drawdown-based performance measures such as the Calmar and Sterling ratios are increasingly popular in the investment community but are almost completely ignored in theory. The aim of this paper was to bridge this gap between research and practice. It is already well established that use of the Sharpe ratio is justified for return distributions, such as the normal, student's t, logistic, and the like, that satisfy certain location and scale parameter conditions. Using Monte Carlo simulations, we show that for these distributions drawdown-based performance measures are strictly increasing functions of the Sharpe ratio. This means that for these distributions, performance ranking will be the same, regardless of whether it is conducted via a drawdown-based measure or via the Sharpe ratio. This finding has a basic but important implication in regard to the use of drawdown-based measures because it means that using drawdown-based measures is theoretically justified under the same conditions as is the Sharpe ratio. In other words, the same conditions that provide a decision theoretic foundation for the Sharpe ratio also provide a decision theoretic foundation for drawdown-based performance measures.

It is important to note, however, that the LS condition provides a decision theoretic foundation only if drawdowns are calculated as uncompounded cumulative excess rates of return. In practice, drawdowns are typically calculated as compounded cumulative rates of return. Hence the question arises as to how big the difference is between these two methods. To answer this question, we considered the CISDM database used in Eling and Schuhmacher (2007) and found that the rank correlation between the Calmar ratio with uncompounded cumulative excess rates of return and the Calmar ratio with compounded cumulative rates of return is 0.995. Hence, the difference appears to be rather small. This result confirms the findings of Eling and Schuhmacher (2007) and Eling (2008), who find that rankings based on drawdown-based performance measures with compounded cumulative rates of return are nearly identical to those determined via the Sharpe ratio.

Given the finding that rankings provided by the Sharpe ratio and drawdown-based measures are nearly identical and given that the Sharpe ratio is both simpler to calculate and easier to explain, among other advantages (e.g., significance tests), the next question is why drawdown-based performance measures are used at all. Seeing as our motivation for writing this paper was to discover a theoretical foundation for these measures, not to discover why they are so popular, we cannot answer this question. Rather, we view our work here as a first contribution toward a more detailed discussion of the advantages and disadvantages of drawdown-based performance measures.

However, discovering why these measure are so popular might also answer the question as to whether we should use them. We suspect that one reason for their popularity is that they capture features not (directly) revealed by the Sharpe ratio. Drawdown-based performance measures focus on worst-case scenarios[12] and this highlights an interesting difference between them and the Sharpe ratio: with the Sharpe ratio, the order of the returns is irrelevant, but for drawdown-based measures, autocorrelation is an important issue. It thus could be that drawdown-based measures capture time series characteristics of returns that cannot be captured by the Sharpe ratio and that are important, e.g., with regard to the performance evaluation of commodity trading advisors. It might also be that drawdown-based measures are more difficult to manipulate than other performance ratios since the effect of performance manipulation strategies (see Goetzman et al., 2007) on drawdown-based measures has not yet been analyzed. Thus, there is room for a great deal more research into these measures, including, for example, with regard to autocorrelation of returns and how easily (or not) they can be manipulated.

[12] Chekhlov et al. (2005) describe how drawdown regulations are imposed in trading strategies of commodity trading advisors in order to avoid such worst case scenarios, e.g. in that a trading account might be shut down if a 20% drawdown is breached.

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