Solvency Capital Requirement for German Unit-Linked Insurance Products

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Abstract

Innovative life insurance products such as unit-linked life insurance, hybrid life insurance, and variable annuities are rapidly gaining popularity and becoming a major part of new business in Germany. However, since traditional life insurance products still dominate the portfolios of life insurance companies, discussions about the standard formula for determining the solvency capital requirement have focused on this type of business. Any detailed discussion on how to calculate the solvency capital requirement for innovative life insurance products within the standard formula has yet to occur.

This paper brings to light some interesting facts about unit-linked business and Solvency II. In particular, it analyzes the impact of the transition from Solvency I to Solvency II on the solvency capital requirement of a German unit-linked insurance product with guaranteed death benefits. The modeling of lapses is another focus of research, reflecting the increased importance of lapse risks for innovative life insurance products. Since there are strong concerns about nonlinearities between the various risks, especially between market risk and lapse risk, the paper examines this problem as well. Finally, an alternative method for calculating the net solvency capital requirement, the so-called single equivalent scenario (also referred to as the killer-scenario), is presented.

Keywords: Unit-linked insurance, Solvency II, standard formula, single equivalent scenario, dynamic policyholder behavior

1 Introduction

Innovative life insurance products have been gaining in popularity during the last decade and now represent nearly a third of new business in Germany (see Helfenstein & Barnshaw (2003), Enz (2006) and Märten & Daalmann (2009)). However, and despite the importance of these products for the future of the insurance industry, most discussions about the Solvency II framework focus on traditional insurance products. The results of the last quantitative impact study, QIS4, indicate that most insurance companies do not calculate the solvency capital requirement for innovative life insurance products as systematically as they do for traditional products (see CEIOPS (2008a)).

In the academic literature of the last decade, fair valuation of life insurance products has been an emerging field. Traditional life insurance contracts with interest rate guarantees especially have been analyzed in detail (see Bauer *et al.* (2006), Bacinello (2001) and Steffensen (2002)). Common options of traditional policies such as the option to surrender also draw growing attention (see Grosen & Jorgensen (2000) and Steffensen (2002)). Furthermore, the recent low interest period, changing customer need and tax law led to increased new business of unit-linked life insurance, hybrid life insurance and variable annuities and therefore a development of pricing techniques (see Bauer *et al.* (2008)). However, innovative life insurance products still need to be examined in conjunction with recent regulatory changes in Europe (e.g. Solvency II).

The aim of this paper is to provide the first contribution to a discussion about the solvency capital requirement for innovative life insurance products. The paper brings together fair valuation, risk analysis and a detailed product design and should be of interest to academics as well as to practitioners.

Innovative life insurance products differ from traditional life insurance products in some fundamental aspects and therefore require an in-depth risk examination. For both insurers and policyholders, the value of an innovative life insurance product is expected to be somewhat volatile, since the capital is mostly invested in risky assets, compared to the fixed-income-oriented investment strategies of traditional life insurance products. Furthermore, innovative life insurance products are usually complex in their structure and contain a broad range of options and guarantees (see Gatzert (2009)). These insurance products also induce dynamic policyholder behavior. Their volatile value, complexity, dynamic policyholder behavior, options, and guarantees can all have an unexpected influence on the solvency capital requirement.

The contribution of this paper is to identify the main risks of a unit-linked life insurance product and to discuss two methods for calculating the solvency capital requirement, namely the standard formula and the single equivalent scenario. Furthermore, a way to implement dynamic policyholder behavior in the standard formula is presented. This paper also provides a methodology for calculating solvency capital requirement for other innovative life insurance products.

The paper is organized as follows: To establish a methodology, a German unit-linked insurance with guaranteed death benefits is examined based on the standard formula method (see CEIOPS (2008c)). A simplified version of the standard formula and information about the calculation of the solvency capital requirement is presented in chapter 2. The product design is illustrated in chapter 3 and includes fixed and variable costs, mortality, deterministic and dynamic lapses, kickbacks, a bonus system in accordance with German law, and a realistic set of parameters. The analysis is performed on products with single premiums as well as on products with regular premiums. Furthermore, the paper analyzes the impact of the transition from Solvency I (see Müller (1997) and Bundesministerium der Justiz (2009)) to Solvency II on the solvency capital requirement for the unit-linked insurance product. Chapter 4 provides information about the asset and the liability models as well as first results. The paper examines the linearity assumption by means of a sensitivity analysis in chapter 5. A method for modeling dynamic lapses is discussed in chapter 6. Different parameter sets are analyzed in chapter 7. The single equivalent scenario is presented in chapter 8.

2 Solvency capital requirement

The solvency capital of an insurance company guarantees the solvability of the latter during a financial distress. Regarding the importance of insurance to society, economy and public welfare, the insurance company should have enough capital to overcome almost every crisis. In Solvency I the solvency capital requirement is calculated through a factor-based framework. This framework is easy to understand and easy to use, and it requires only few balance sheet values and the corresponding risk factors. The Solvency I capital requirement for German unit-linked products, where the policyholder bears the investment risk, is defined as 1% of the investment fund value plus 0.3% of the risk premium[1]. The downside of a factor-based framework is that

[1]See Bundesministerium der Justiz (2009) and Müller (1997).

it does not depict the actual risks. Solvency II, as a risk based framework, will provide a more sophisticated view on the risk taking of an insurance company[2].
In the Solvency II framework, the amount of solvency capital an insurance company has to hold is in the broader sense defined as the amount of capital needed to survive a "one in two hundred years crisis".

2.1 SCR - the mathematical approach

Let X denote a risk, the solvency capital requirement of $SCR_\alpha(X)$ is defined as

$$SCR_\alpha(X) = VaR_\alpha(X) - \mathbb{E}[X]. \tag{1}$$

The following transformations lead to a mathematical definition of the SCR[3]:

$$\begin{aligned} SCR_\alpha(X) &= VaR_\alpha(X) - \mathbb{E}[X] \\ &= VaR_\alpha(X - \mathbb{E}[X]) \\ &= \underset{x}{\text{argmin}}\ \left(P\left[X - \mathbb{E}[X] \leq x\right] \geq \alpha\right) \\ &= \underset{x}{\text{argmin}}\ \left(1 - P\left[X - \mathbb{E}[X] > x\right] \geq \alpha\right) \\ &= \underset{x}{\text{argmin}}\ \left(P\left[X - \mathbb{E}[X] > x\right] \leq 1 - \alpha\right). \end{aligned}$$

Now consider a two-hundred-years crisis over a time horizon of one year. Denote the available capital at time t by AC_t[4]. Then $\mathbb{E}[X] = -AC_0$ is the negative available capital at time $t = 0$ and $X = -\frac{AC_1}{(1+i)}$ the discounted available capital at time $t = 0$. The solvency capital requirement SCR can then be expressed as[5]

$$SCR_\alpha = \underset{x}{\text{argmin}}\ \left(P\left[AC_0 - \frac{AC_1}{(1+i)} > x\right] \leq 1 - \alpha\right) \tag{2}$$

with $\alpha = 0.995$ and an interest rate i.

[2]See Doff (2008), Duverne & Le Douit (2009), Holzmüller (2009), Elderfield (2009) or Steffen (2008) for a comparison of different regulatory frameworks and general information about Solvency II.

[3]As introduced in Bauer *et al.* (2009). Bergmann's notion is used for practical applications. It is approximately equivalent to $P(AC_1 \geq 0 | AC_0 = x) \geq \alpha$, but avoids the implicit nature of the definition.

[4]The available capital can be expressed in terms of MCEV. See Bauer *et al.* (2009) for more information.

[5]Implicitly assuming that dividends have not been paid to shareholders yet at $t = 1$.

2.2 SCR - the Standard Formula of QIS4

Although the formula above perfectly defines the solvency capital requirement, it is not practical because of two reasons: it is very difficult to describe an insurance company as a whole with a stochastic model, and nested simulations are needed. In order to provide a more simple approach, especially for small insurance companies that do not use an internal model, CEIOPS introduced the Standard Formula. The main simplification is the definition of deterministic stress scenarios that should represent the one in two hundred years crisis. In addition, risks are supposed to be multivariate normally distributed. Let $X = -\Pi$ denote a random loss variable or the negative PVFP ("Present Value of Future Profits") Π, then the SCR can be simplified to[6]

$$\begin{aligned} SCR &= VaR(-\Pi) - \mathbb{E}[-\Pi] \\ &= (Liabilities - Assets)\,|_{stress} - (Liabilities - Assets) \\ &= (Assets - Liabilities) - (Assets|_{stress} - Liabilities|_{stress})\,. \end{aligned}$$

The stress scenarios are formulated for various risk modules (interest rates, equity, mortality, lapses and expenses) and are aggregated via a correlation matrix. Let X_i denote the loss variable exposed to a risk i defined in a risk module and $SCR\,(X_i)$ denote the solvency capital requirement calculated for the same risk module.

Then the aggregated solvency capital requirement $SCR\,(X)$ for the aggregated loss variable[7] $X = \sum_i X_i$ is defined as[8]:

$$\begin{aligned} SCR_\alpha(X) &= VaR_\alpha(X) - \mathbb{E}[X] \\ &= VaR_\alpha\left(\sum_i X_i\right) - \mathbb{E}[X] \\ &= \sqrt{\sum_{i,j} \rho_{i,j}\,(VaR_\alpha(X_i) - \mathbb{E}[X_i])\,(VaR_\alpha(X_j) - \mathbb{E}[X_j])} \\ &\qquad + \mathbb{E}\left[\sum_i X_i\right] - \mathbb{E}[X] \end{aligned}$$

[6] *Assets* and *Liabilities* denote the expected present value of all Assets and Liabilities as defined in QIS4 (Δ-NAV approach).

[7] With $\mathbb{E}[X] = \mathbb{E}\left[\sum_i X_i\right]$.

[8] See GDV (2005, page 88-93).

$$= \sqrt{\sum_{i,j} \rho_{i,j} SCR_\alpha(X_i) SCR_\alpha(X_j)}.$$

Figure 1 shows a simplified modular view on the standard formula. Only relevant risks for a German unit-linked insurance product are considered.

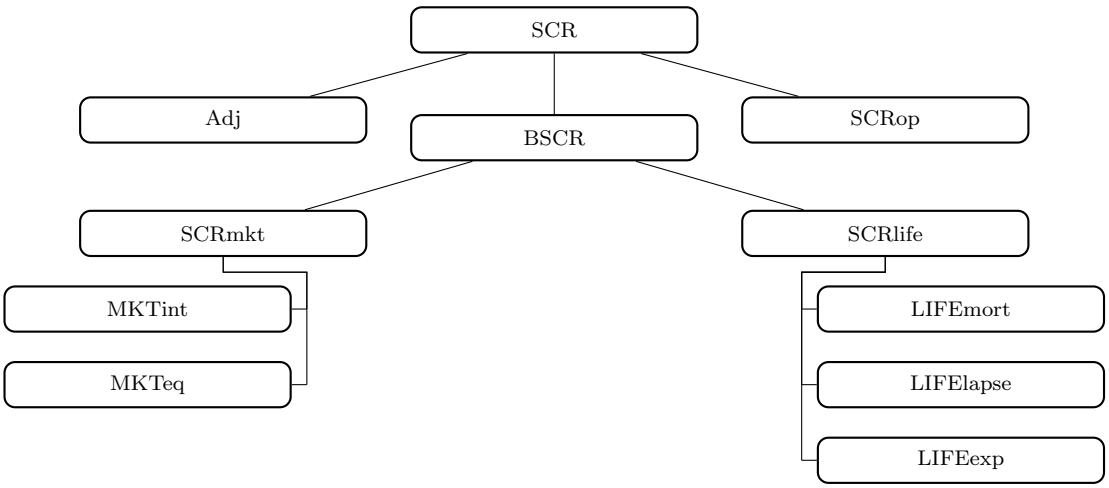

Figure 1: Modular structure of the SCR

The solvency capital requirement can be expressed with the following formulae[9]:

$$SCR = \sqrt{SCR_{mkt}^2 + 2 \cdot \rho_{mkt,life} \cdot SCR_{mkt} SCR_{life} + SCR_{life}^2}$$

$$SCR_{mkt} = \sqrt{SCR_{int}^2 + 2 \cdot \rho_{int,eq} \cdot SCR_{int} SCR_{eq} + SCR_{eq}^2}$$

$$SCR_{life} = \sqrt{SCR_{mort}^2 + SCR_{lapse}^2 + SCR_{exp}^2 + 2 \cdot \rho_{mort,lapse} \cdot SCR_{mort} SCR_{lapse}} \\ \overline{+2 \cdot \rho_{mort,exp} \cdot SCR_{mort} SCR_{exp} + 2 \cdot \rho_{lapse,exp} \cdot SCR_{lapse} SCR_{exp}}.$$

The corresponding correlation factors can be obtained from table 1.

According to the principles of Solvency II, a "best estimate is equal to the probability-weighted average of future cash-flows, taking account of the time value of money, using the relevant risk-free interest rate term structure. The calculation of best estimate should be based upon current and credible information and realistic assumptions and be performed using adequate actuarial methods and statistical techniques."[10]. In this case, the best estimate

[9] See chapter 2.5 for details on the relevant risk modules.

[10] See CEIOPS (2008c, page 13-14).

of technical provisions equals the best estimate of liabilities. In order to simplify the task, a risk margin will not be calculated. The valuation of assets is performed with a mark to model procedure. This framework ensures a market-consistent valuation of all assets and liabilities.

CorrSCR=	SCRmkt	SCRlife
SCRmkt	1	0.25
SCRlife	0.25	1

CorrMkt=	SCRint	SCReq
SCRint	1	0
SCReq	0	1

CorrLife=	SCRmort	SCRlapse	SCRexp
SCRmort	1	0	0.25
SCRlapse	0	1	0.5
SCRexp	0.25	0.5	1

Table 1: Correlation matrices

Let $\Pi = Assets - Liabilities$ denote the value of an insurance policy. Then, the solvency capital requirement for the particular risk modules as described in chapter 2.5 is defined as

$$\begin{aligned}
SCR_{int-up} &= \Pi - \Pi|_{up-shock} \\
SCR_{int-down} &= \Pi - \Pi|_{down-shock} \\
SCR_{int} &= \max\left(SCR_{int-up}, SCR_{int-down}; 0\right)
\end{aligned}$$

$$SCR_{eq} = \max\left(\Pi - \Pi|_{eqshock}; 0\right)$$

$$SCR_{mort} = \max\left(\Pi - \Pi|_{mortshock}; 0\right)$$

$$\begin{aligned}
SCR_{lapse-up} &= \Pi - \Pi|_{up-shock} \\
SCR_{lapse-down} &= \Pi - \Pi|_{down-shock} \\
SCR_{lapse-mass} &= \Pi - \Pi|_{mass-shock} \\
SCR_{lapse} &= \max\left(SCR_{lapse-up}; SCR_{lapse-down}; SCR_{lapse-mass}; 0\right)
\end{aligned}$$

$$SCR_{exp} = \max\left(\Pi - \Pi|_{expshock}; 0\right).$$

The solvency capital requirement for operational risk for unit-linked insurance is calculated with a factor-based formula. The risk factor $rf_{op} = 0.25$ is set by CEIOPS and is multiplied with the best estimate annual expenses (without aquisition costs) Exp_{ul} of the unit-linked policy portfolio:

$$SCR_{op} = 0.25 \cdot Exp_{ul}.$$

2.3 Methodology – developing a partial internal model

Partial internal models are an interesting approach especially for midsized or small insurers. Using a partial internal model, the difficult task of modeling the insurance company as a whole can be avoided. With a full internal model, the insurer is also required to model the correlations between all risks in order to obtain an empirical distribution function of the economic balance sheet and its quantiles. Instead, one can use the Solvency II standard formula as a starting point. Further aspects of the nature of the business can be modeled within the standard formula in the relevant risk modules while the correlation matrices and most of the remaining modules remain unchanged. The following partial internal model for calculating the SCR of unit-linked insurance products is designed to mirror the high dependency on financial markets of such products and probable dynamic policyholder behavior.

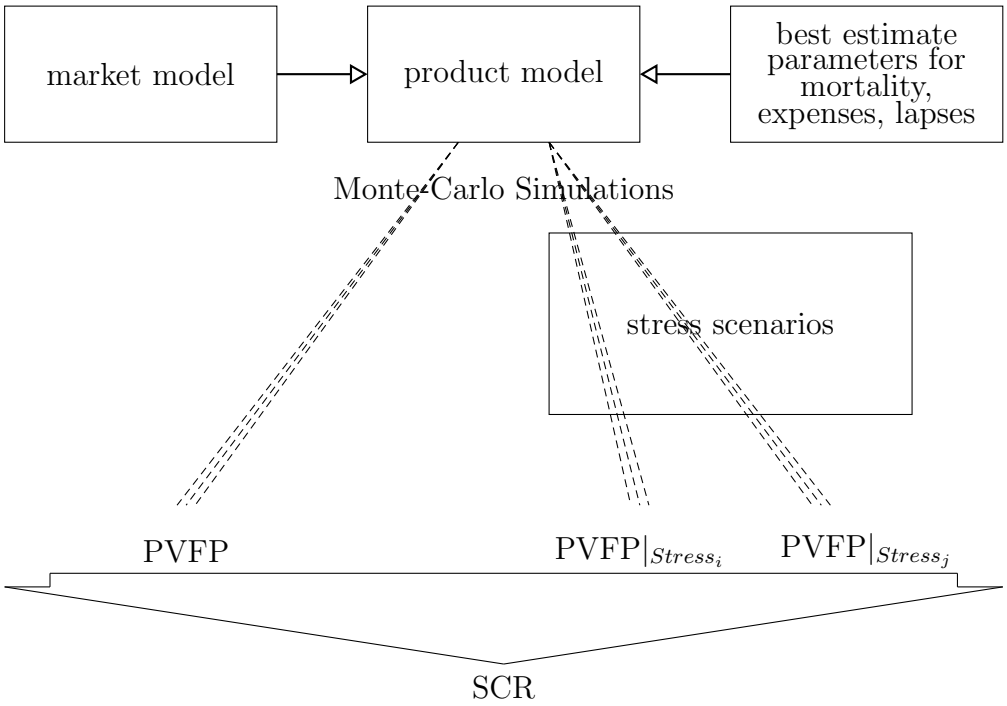

Figure 2: Methodology

For its deterministic parts such as mortality, expenses or deterministic lapses, the partial internal model requires the same best estimate parameters as the standard formula. The financial market model consists of stochastic models for assets and interest rates. The main part of the partial internal model is the product model. It contains all relevant parameters of the unit-linked policy, information about the insurers portfolio, management rules and

a model for dynamic lapses. With most of the cash flows being stochastic now, Monte-Carlo simulations are used to determine the expected discounted value of the insurance portfolio (denoted by PVFP).
In order to obtain the solvency capital requirement, stress scenarios of the standard formula are implemented. They affect either the best estimate assumptions or parameters of the market model. Again, Monte-Carlo simulations are used to determine the value of the insurance portfolio, now under the assumption that a stress occurs. Applying this procedure to every stress scenario of every relevant risk module, the outcomes can be aggregated to the resulting SCR the same way as in the standard formula.

2.4 The risk absorbing effect of future profit sharing

Future bonuses paid out to the policyholders will change while calculating the PVFP under a stress scenario when stochastic profit sharing rules are used. The solvency capital calculated with adjusted bonuses is referred to as the net solvency capital requirement ($nSCR$). The solvency capital calculated with constant bonuses through a stress is referred to as the basic solvency capital requirement ($BSCR$). The value of the future discretionary bonuses (FDB) can be defined as

$$FDB = \Pi|_{\text{no profit sharing}} - \Pi|_{\text{profit sharing}}.$$

The adjustment for the risk absorbing effect of future profit sharing to the BSCR is then defined as

$$Adj_{FDB} = \min\left(BSCR - nSCR, FDB\right)$$

and the overall SCR or net basic solvency capital requirement $nBSCR$ is defined as

$$SCR = nBSCR = BSCR - Adj_{FDB} + SCR_{op}.$$

The calculation of the $BSCR$ is performed with "constant" bonuses throughout all stress scenarios. There are several interpretations what "constant" means. One interpretation is that the $BSCR$ "should be calculated under the condition that the absolute amount of future discretionary benefits cash flows per policy and year remain unchanged before and after the shock being tested"[11]. This direct calculation of the $BSCR$ requires storage of bonuses for every simulation step and every simulation path. In order to avoid a huge

[11] See CEIOPS (2009b).

computational capacity requirement and improve the practicability, the problem can be simplified using an alternative interpretation:
The calculation of the $BSCR$ is performed with a "constant value" of bonuses. Therefore, the $BSCR$ is "calculated under the condition that the value of future discretionary benefits remains unchanged before and after the shock being tested"[12]. Let $Liabilities = Bonuses + otherLiabilities$ be a decomposition of the liabilities, then Π and $BSRC$ can be defined as

$$\begin{aligned}\Pi &= Assets - Bonuses - otherLiabilities \\ BSCR &= (Assets - Bonuses - otherLiabilities) \\ &\quad - (Assets|_{stress} - Bonuses|_{stress} - otherLiabilities|_{stress})\,.\end{aligned}$$

Since the bonuses should be constant in order to calculate the $BSCR$ and therefore $Bonuses = Bonuses|_{stress}$, the above equation can be simplified to

$$\begin{aligned}BSCR &= (Assets - otherLiabilities) \\ &\quad - (Assets|_{stress} - otherLiabilities|_{stress})\,.\end{aligned}$$

The above $BSCR$ corresponds to the $nSCR$ calculated without any profit sharing. Therefore, in order to calculate the $BSCR$ the profit sharing parameters "risk profit participation rate" and "expense profit participation rate" are set to zero. In general, with participation rates other than zero, the $nSCR$ is defined as:

$$\begin{aligned}nSCR &= (Assets - Bonuses - otherLiabilities) \\ &\quad - (Assets|_{stress} - Bonuses|_{stress} - otherLiabilities|_{stress})\end{aligned}$$

with

$$Bonuses \neq Bonuses|_{stress}.$$

Figures 3 and 4 show solvency balance sheets for both $BSCR$ and $nSCR$.

2.5 QIS4 stress scenarios

The solvency capital requirement is defined as the difference of the best estimate net asset value (PVFP) and the net asset value under stress. The stress scenarios defined in this chapter originate from QIS4. The design of the regarded insurance product requires the consideration of the following risks: in the market risk module, the interest rate risk and the equity risk are relevant. Mortality risk, lapse risk and expense risk are the relevant risks in the life underwriting risk module.

[12] See CEIOPS (2009b).

interest rate risk [13] – The interest rate risk module includes two stress scenarios: up-shift of the interest rate curve (zero coupon bond rate) and down-shift of the interest rate curve. The exact magnitude of the shifts can be found in the QIS4 tables.

equity risk [14] – The equity risk module contains an immediate loss of 32% of the risky assets[15].

mortality risk [16] – The mortality stress is defined as an increase of the mortality rates amounting to 10%.

lapse risk [17] – The lapse risk includes three stress scenarios: a long-term increase of the lapse rates (50%), a long-term decrease of the lapse rates (50%) and a massive immediate lapse of 30% of the policyholders.

expense risk [18] – The expense risk stress scenario is defined as an increase of 10% in future expenses and an increased expenses inflation (+1% per annum).

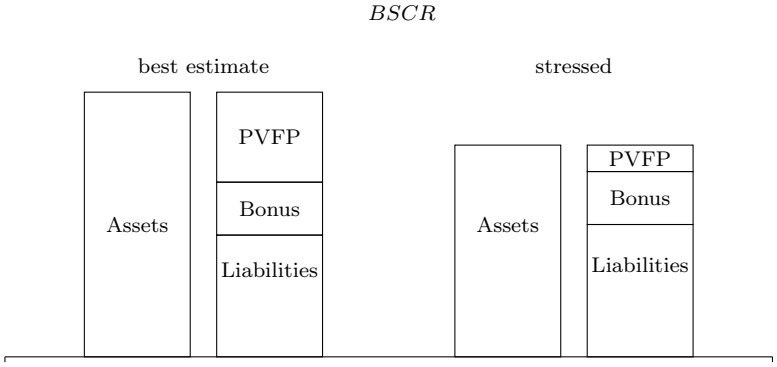

Figure 3: Risk absorbing effect of future profit sharing I

[13] See CEIOPS (2008c, pages 134-137).
[14] See CEIOPS (2008c, pages 137-143).
[15] The risky assets are assumed to belong to the asset category "Global".
[16] See CEIOPS (2008c, pages 162-164).
[17] See CEIOPS (2008c, pages 167-169).
[18] See CEIOPS (2008c, pages 169-170).

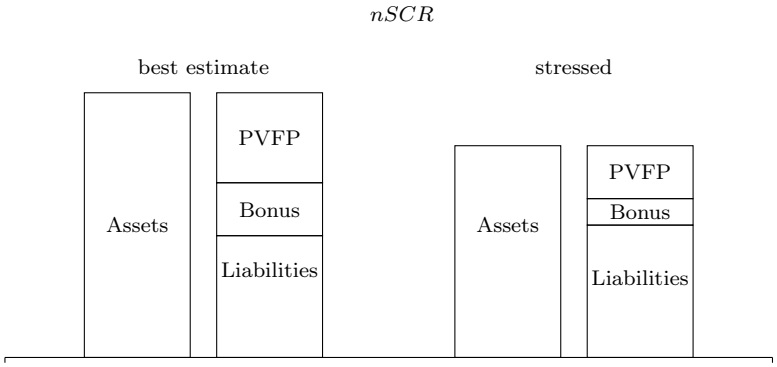

Figure 4: Risk absorbing effect of future profit sharing II

3 Product design and parameter assumptions

Premiums
In this paper two forms of the product are considered: single premium contracts and regular premium contracts. With a single premium contract, the policyholder has to pay only a lump-sum at the beginning of the contract period. Concluding a regular premium contract, the policyholder commits to pay a premium at the beginning of every month until the end of the contract period, death of the policyholder or lapse of the policy. The premium income is immediately used to buy shares of the investment fund after deduction of acquisition charges. Let T denote the policy term in years, then $t = 0, \ldots, 12T$ is counting the time steps (months). A premium payment at time t is denoted by P_t.

Expenses
Three kinds of expenses can be identified regarding a standard unit-linked insurance product: acquisition expenses, fixed monthly expenses, and variable monthly expenses. In order to refinance, the insurer deducts charges from the investment fund. These charges represent the prudent projected expenses. The prudent projected expenses consist of the expected expenses plus a risk margin. The acquisition charges for regular premium policies are calculated with expected interest rates but without any mortality or lapse assumptions. The fixed monthly charges are considered to be deterministic and constant for all t, while the variable charges are driven by the current investment fund value. The acquisition charges are immediately deducted from the premiums; in the single premium case, they are deducted from the single premium at once, in the regular premium case, the acquisition charges are decomposed into small payments and deducted from the premiums (for a

maximum of five years). The incurred monthly (fixed and variable) expenses are paid at the end of every month. In order to finance the incurred expenses, the insurer withdraws an amount equal to the prudent projected expenses from the investment fund at the beginning of every month and deposits it on a bank account earning the risk-free interest rate.

Mortality
German DAV 2008 T mortality tables are used for prudent mortality assumptions. Uniform distribution of deaths is used as an assumption for fractional ages.

Death benefits
Death benefits are paid at the end of the month. A set of different kinds of policies varying by their guaranteed death benefits structures were considered in the study. The paper highlights one typical death benefit scheme, denoted by "policy A". This guarantee refers to the current investment fund and total premiums (P^{tot}). Then, the death benefits DB_t at time t are defined as:

policy A: $DB_t = \max(1.1 \cdot FV_t, P^{tot})$.

Like the charges, death benefits are also financed by withdrawing an amount from the investment fund at the beginning of the month. The amount withdrawn from the investment fund is referred to as the "risk premium" and denotes the prudent estimated excess of the death benefits over the investment fund value. Let q_x be the probability of an x-year-old dying the ongoing year and let RP_t denote the risk premium at time t, then the following equation holds:

$$RP_t = (DB_t - FV_t)\,\frac{q_x}{12 - q_x}.$$

The risk premium is withdrawn from the investment fund at the beginning of the month and deposited on a bank account earning the risk free interest rate. Therefore, in case the death benefit paid to the policyholder is larger than the value of the amount of shares of the investment funds associated with the policy, the risk premium is used to close the gap. It is worth noting that the risk premium is calculated at the beginning of the month with respect to the fund value at the beginning of the month while the death benefit is calculated with respect to the fund value at the end of the month. An unfavorable development of the investment funds during the month can lead to insufficient funds and therefore to a negative impact on the PVFP for the insurer.

Assumptions
Best estimate assumptions are formulated for mortality, seperate assumptions are used for expenses.

- The best estimate mortality is assumed to be 60% of the prudent mortality.
- The fixed monthly expenses are considered to be deterministic but monthly increasing with a expenses inflation factor.
- The variable expenses are assumed to be zero.
- The aquisition charges equal the aquisition expenses.

The insurer uses prudent and best estimate assumptions for mortality and different assumptions for charges and expenses. Therefore, in the long run, the insurer will make profits out of the assumption of parameters. According to German law, these profits have to be shared with the policyholders. Two kinds of profits can be identified: mortality profits are profits generated by mortality risk taking and expense profits are profits generated by expenses risk taking. Expense profits also include profits from lapse fees and kickbacks[19]. Profits are generated every month and deposited on a bank account earning the risk free interest rate. At the end of the year the insurer credits at least 75% of the mortality profits and at least 50% of the expense profits to the policyholders investment fund. The rest of the profits are profits of the insurance company and denote the value of the policy (discounted at time $t = 0$) to the insurer.

Lapses
Evaluating the value of the policies in its portfolio, the insurance company must take into account that the insured might use their option to surrender, withdraw, or lapse his policy[20]. There are several factors that influence the number of lapses: the remaining policy term, the performance of the policy compared to other products, the age of the policyholder, unemployment rates, growth of the GDP, the rating of the insurance company, marketing and marketing channels as well as personal reasons[21]. Lapses triggered by

[19]The investment fund management pays kickbacks to the insurer. Kickbacks are seen as an allowance on management fees due to a high transaction volume.

[20]The three expressions are used synonymously in literature.

[21]For more information about lapse, see Anzilli & De Cesare (2007), Cerchiara *et al.* (2008), Kuo *et al.* (2003), Mauer & Holden (2007), Bacinello (2003), Cox & Lin (2006), Outreville (1990) and Prestele (2006).

these factors are not incorporated in this model in particular but combined and defined as deterministic lapses and modeled by deterministic lapse rates. Dynamic lapses are triggered by the value of the policy to the policyholder, more precisely, the surrender value of the policy. Dynamic lapses are also often referred to as dynamic policyholder behavior, since they cannot be modeled with deterministic assumptions. In literature, dynamic lapses are frequently used in connection with the valuation of a surrender option and therefore lapses are assumed to occur at any time the surrender value is larger than the value of the policy. Note that this definition of lapses, also denoted as rational lapses, differs from the lapses as presented in this paper. Dynamic policyholder behavior should be carefully managed by the insurance company because changes might be excessive and lead to huge financial losses[22]. This paper also examines dynamic lapses (dynamic lapse functions)[23].

Deterministic lapses are assumed to evolve with a deterministic monotonically decreasing lapse rate lr_t^{det}. Deterministic lapse rates start at 10% and decrease by 1% yearly until they reach 2%. They remain constant at 2% for the rest of the policy term.

Parameter	Value	Description	Category
T	30	policy term in years	general
$gender$	male	gender of the policyholders	
x	30	age of the policyholders at $t = 0$	
NP_0	10000	number of policyholders at $t = 0$	
P_0	100000	single premium in Euro	
$P_{t=0,\ldots,12\cdot T}$	305	regular premium in Euro	
$acharges^{rate}$	6%	acquisition charges in per cent of P^{tot}	charges
cpu	4	fixed charges per policy per month in Euro	
$vcharges^{rate}$	0%	variable charges per month in per cent of FV_t (single premium case)	
$vcharges^{rate}$	0.15%	variable charges per month in per cent of FV_t (regular premium case)	
$aexpenses^{rate\prime}$	6%	acquisition expenses in per cent of P^{tot}	expenses
cpu'	4	fixed expenses per policy per month in Euro	
$cinf$	2%	fixed expenses inflation per annum	
$vexpenses^{rate\prime}$	0%	variable expenses per month in per cent of FV_t	
rb^{rate}	75%	risk profit participation rate	bonus system
cb^{rate}	50%	expense profit participation rate	

Table 2: Parameter assumptions

The insurance company is allowed to deduct a lapse fee from the surrender value of the policy in most European countries[24]. The main reasons for lapse

[22]See CEIOPS (2009a) for more information about deterministic and dynamic lapses.

[23]See chapter 6.

[24]E.g. in Germany (see VVG §169(5)), but not in France (see Helfenstein & Barnshaw (2003)), Norway (see Nordahl (2008)).

fees are adverse selection, administration expenses, acquisition expenses and solvency[25].

Lapse fees are set to have a deterministic and monotonically decreasing lapse fee rate. In case a policyholder decides to surrender his policy, he receives the investment fund value less the lapse fee. The lapse fee rate starts at 5% and decreases by 0.5% yearly until it reaches 0%.

The parameters set in table 2 represent the standard setting unless otherwise noted.

4 Financial market model and Simulations

The financial market model consists of one risky asset (e.g. a share) and a riskfree investment possibility (e.g. a state bond). The risky asset is modeled by using the standard Black-Scholes-Merton model, while the interest rates are modeled with the Cox-Ingersoll-Ross model.

Investment fund
The investment fund contains only risky assets and is modeled with respect to investment fund fees and kickbacks to the insurance company.
Let S_t denote the value of one share of the risky asset with a constant volatility σ, the risk-free short-rate r_t[26] and a Brownian motion W_t under the risk-neutral measure[27] at time $t \in [0, T]$, then S_t solves the following SDE:

$$dS_t = r_t S_t dt + \sigma S_t dW_t.$$

The explicit analytical solution is given by:

$$S_t = S_{t-1}\ exp\left(\int_{t-1}^{t}\left(r_s - \frac{\sigma^2}{2}\right)ds + \int_{t-1}^{t}\sigma\ dW_s\right).$$

Now let $FundFee$ denote a constant rate of fees, which will be retained by the investment fund management and let A_t denote the value of one share of the investment fund, then

$$dA_t = r_t A_t dt + \sigma A_t dW_t + \ln\left(1 - FundFee\right) A_t dt$$

[25]See DAV-Arbeitsgruppe Stornoabzüge (2007) and Gatzert (2009) for further information.
[26]Assuming an adapted interest rate process r_t, see Shreve (2000, page 215).
[27]See Shreve (2000, page 214-217).

describes the evolution of the investment fund. The investment fund is modeled as a dividend paying share[28]. The analytical solution of this SDE can be written as

$$\begin{aligned} A_t &= A_{t-1}\ exp\left(\int_{t-1}^{t}\left(r_s - \frac{\sigma^2}{2}\ + \ln\left(1 - FundFee\right)\right)ds + \int_{t-1}^{t}\sigma\ dW_s\right) \\ &= A_{t-1}\ \frac{S_t}{S_{t-1}}\left(1 - FundFee\right). \end{aligned}$$

Kickbacks are paid by the investment fund management to the insurer and are financed by the investment fund management fees[29]. The amount of kickbacks per investment fund share is

$$\text{Kickbacks (per share)} = A_t \cdot \text{kickbackrate}.$$

Interest rates

The Cox-Ingersoll-Ross model is used to model the short rate[30]. Unfortunately, this model has no closed-form solution. Despite this drawback compared to e.g. the Vasiceck model, the interest rates stay always positive. Let lm denote the constant long run short rate, mrs the constant mean reversion speed, σ_r the volatility of the interest rates and W_t^r a Brownian motion (uncorrelated to W_t), then the model for the short rate process r_t is

$$dr_t = mrs\left(lm - r_t\right)dt + \sigma_r\sqrt{r_t}dW_t^r.$$

Parameter assumptions

The parameters set in table 3 represent the standard setting and are used unless otherwise noted.

Parameter	Value	Description	Category
n	200000	number of simulations	general
ρ	0	correlation between the Brownian motions	
S_0	100	starting value of the risky asset	risky asset
σ	20%	volatility per annum	
r_0	4%	starting value	interest rate
mrs	0.3	mean reversion speed	
lm	4.5%	long run short rate	
σ_r	2.5%	volatility per annum	
$aFundFee$	1.5%	investment fund fee per annum	investment fund
$akickbackrate$	0.5%	kickback rate per annum	

Table 3: Financial market model parameter assumptions

[28]See Shreve (2000, page 234-240).

[29]Therefore, the rate of kickbacks should be chosen smaller than the rate of investment fund management fees.

[30]See Shreve (2000, page 151-153).

Figure 5 shows the simulation steps in a schematic manner.

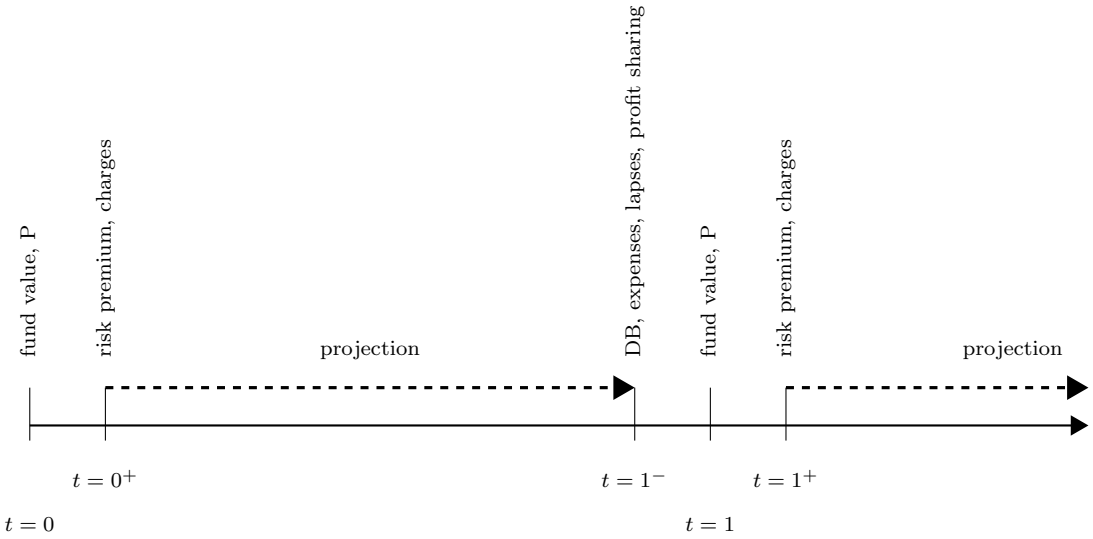

Figure 5: Timeline

Numerical results

Policy type	P type	SCR	PVFP	Solvency ratio	BSCR	SCR	SES-BSCR	SES-SCR
A	sin	€ 14,98 mln	€ 39,69 mln	264,99%	€ 28,64 mln	€ 14,86 mln	-4,51%	-4,79%
A	reg	€ 14,82 mln	€ 40,00 mln	269,92%	€ 27,37 mln	€ 14,70 mln	-6,41%	-6,28%

Table 4: Numerical results I

Table 4 presents the simulated SCRs and the insurer's PVFP for policies with both premium types ("sin" for single premium and "reg" for regular premium). Furthermore, it presents an important financial ratio, the solvency ratio[31]. Figures 6 and 7 show the composition of the BCSR and the nSCR before diversification.

The first observation is that market risks and lapse risk dominate the risk structure of the respective product. Throughout all simulation runs the long-term increase of the lapse rates proved to be the relevant stress scenario. Expense risk and mortality risk are both almost negligible. Therefore, the type of death benefits has also only little impact on the solvency capital requirement.

[31]With solvency ratio = $\frac{\Pi}{SCR}$.

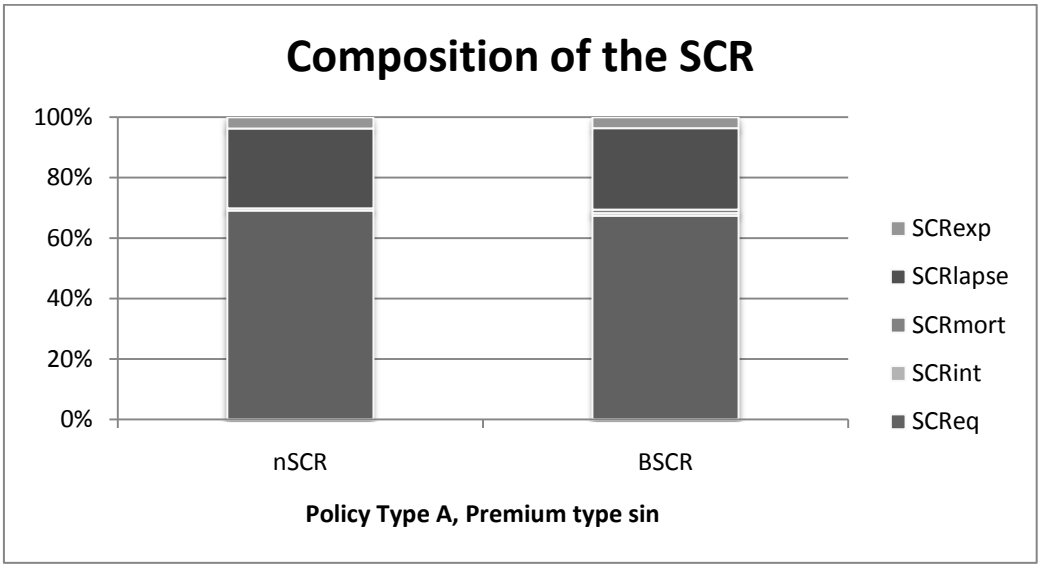

Figure 6: Composition of the SCR – single premium

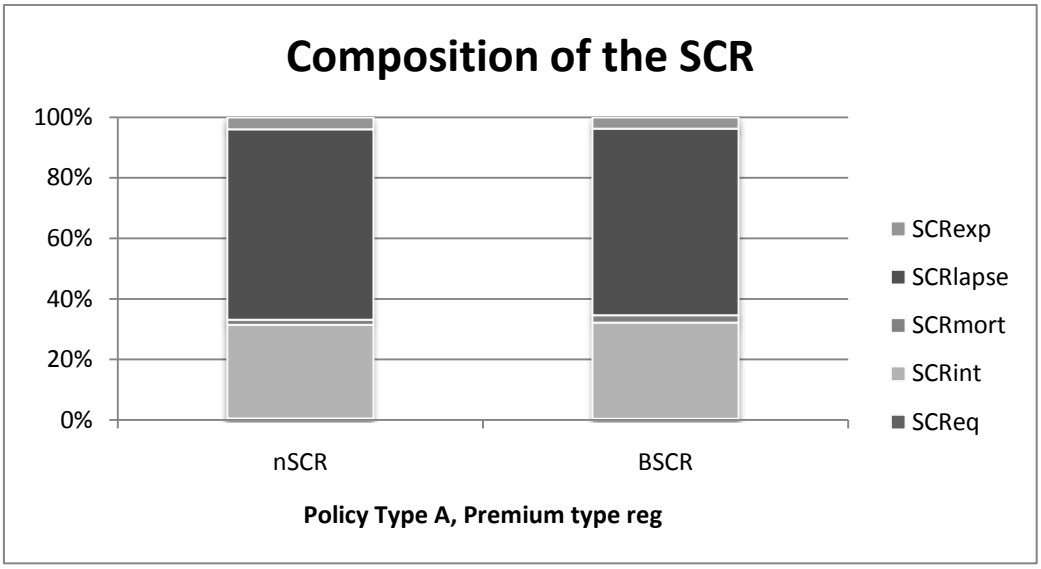

Figure 7: Composition of the SCR – regular premium

Secondly, comparing the results of the simulations, the premium type of the policy proves to be very important for the policies' risk structure. Although the regular premium policy is just insignificantly more risky than the single premium policy (by comparing the solvency ratio), interest rate risk accounts for market risks almost completely. This fact is not surprising, since, with a regular premium policy, the fund value is small at the beginning. On the other hand, the market risk of a single premium policy is dominated by the equity risk. A shock of interest rates does not have a significant impact

on the PVFP. Since a change of interest rates does affect the discounting of future profits as well as the trend of the risky assets and since the profits are mostly generated or triggered by the investment fund value, both effects seem to offset each other.

Thirdly, the solvency capital requirement calculated with the standard formula of the Solvency II framework, seems to be much higher than the solvency capital requirement calculated according to the Solvency I framework (which is about 1% of the investment fund value, the risk premium is almost negligible). For the regular premium policy, Solvency I requires only little solvency capital at the beginning of the policy term and the largest amount of solvency capital at the end of the policy term although this is illogical since the risk obviously decreases by the end of the policy term in general.

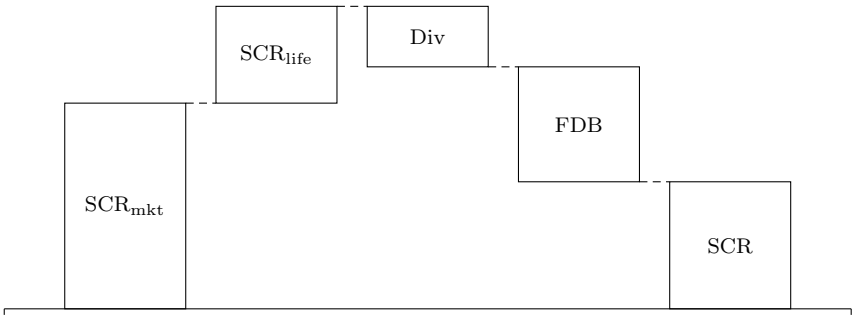

Figure 8: SCR - Structure - single premium

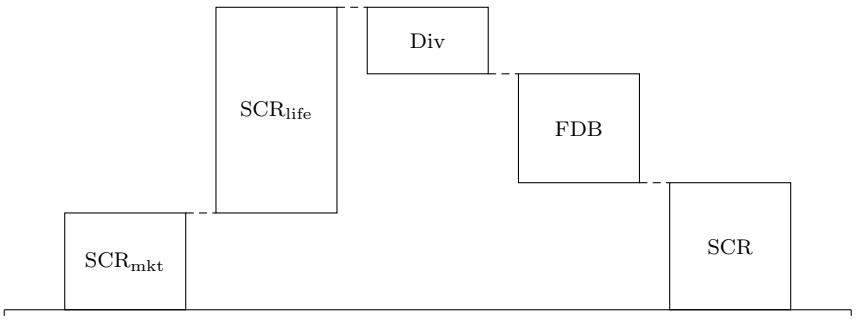

Figure 9: SCR - Structure - regular premium

PVFP distribution	with profit sharing	Policy Type A	Premium Type sin		
	total	from risk	from expenses		
			total	from lapse	from kickbacks
PVFP	€ 39,69 mln	€ 0,59 mln	€ 39,10 mln	€ 7,37 mln	€ 32,26 mln
PVFP after mort-shock	€ 39,56 mln	€ 0,50 mln	€ 39,06 mln	€ 7,37 mln	€ 32,22 mln
PVFP after exp-shock	€ 39,01 mln	€ 0,59 mln	€ 38,42 mln	€ 7,37 mln	€ 32,25 mln
PVFP after lapse-shock up	€ 34,84 mln	€ 0,41 mln	€ 34,43 mln	€ 10,03 mln	€ 24,78 mln
PVFP after eq-shock	€ 27,03 mln	€ 0,72 mln	€ 26,31 mln	€ 5,00 mln	€ 21,83 mln
PVFP after int-shock up	€ 39,72 mln	€ 0,44 mln	€ 39,28 mln	€ 7,37 mln	€ 32,31 mln

PVFP distribution	without profit sharing	Policy Type A	Premium Type sin		
	total	from risk	from expenses		
			total	from lapse	from kickbacks
PVFP	€ 78,02 mln	€ 2,39 mln	€ 75,63 mln	€ 14,61 mln	€ 62,08 mln
PVFP after mort-shock	€ 77,59 mln	€ 2,03 mln	€ 75,56 mln	€ 14,60 mln	€ 62,01 mln
PVFP after exp-shock	€ 76,70 mln	€ 2,39 mln	€ 74,31 mln	€ 14,61 mln	€ 62,08 mln
PVFP after lapse-shock up	€ 68,37 mln	€ 1,66 mln	€ 66,71 mln	€ 19,86 mln	€ 47,60 mln
PVFP after eq-shock	€ 53,81 mln	€ 2,99 mln	€ 50,82 mln	€ 9,92 mln	€ 41,96 mln
PVFP after int-shock up	€ 77,74 mln	€ 1,75 mln	€ 75,99 mln	€ 14,61 mln	€ 62,18 mln

PVFP distribution	with profit sharing	Policy Type A	Premium Type reg		
	total	from risk	from expenses		
			total	from lapse	from kickbacks
PVFP	€ 40,00 mln	€ 1,11 mln	€ 38,90 mln	€ 0,37 mln	€ 8,48 mln
PVFP after mort-shock	€ 39,70 mln	€ 0,94 mln	€ 38,76 mln	€ 0,37 mln	€ 8,45 mln
PVFP after exp-shock	€ 39,27 mln	€ 1,11 mln	€ 38,16 mln	€ 0,37 mln	€ 8,47 mln
PVFP after lapse-shock up	€ 28,25 mln	€ 0,84 mln	€ 27,41 mln	€ 0,47 mln	€ 5,93 mln
PVFP after eq-shock	€ 39,92 mln	€ 1,11 mln	€ 38,81 mln	€ 0,37 mln	€ 8,47 mln
PVFP after int-shock up	€ 34,21 mln	€ 0,86 mln	€ 33,36 mln	€ 0,35 mln	€ 7,26 mln

PVFP distribution	without profit sharing	Policy Type A	Premium Type reg		
	total	from risk	from expenses		
			total	from lapse	from kickbacks
PVFP	€ 74,57 mln	€ 4,68 mln	€ 69,89 mln	€ 0,72 mln	€ 15,26 mln
PVFP after mort-shock	€ 73,74 mln	€ 3,98 mln	€ 69,76 mln	€ 0,72 mln	€ 15,23 mln
PVFP after exp-shock	€ 73,25 mln	€ 4,68 mln	€ 68,56 mln	€ 0,72 mln	€ 15,26 mln
PVFP after lapse-shock up	€ 52,93 mln	€ 3,53 mln	€ 49,40 mln	€ 0,92 mln	€ 10,69 mln
PVFP after eq-shock	€ 74,43 mln	€ 4,69 mln	€ 69,74 mln	€ 0,71 mln	€ 15,23 mln
PVFP after int-shock up	€ 63,43 mln	€ 3,59 mln	€ 59,83 mln	€ 0,68 mln	€ 13,02 mln

Table 5: Composition of the PVFP

Table 5 presents the impact of the stress-scenarios on the PVFP. Furthermore, the composition of the PVFP is shown. Note that PVFP from lapse fees and kickbacks are part of the PVFP from expenses. Table 5 also displays the impact of profit sharing on PVFP and its risk absorbing effect. Table 6 presents the effect of profit sharing in more detail. The insurer is able to mitigate the risk almost identical to the profit participation rates.

Policy Type A	Premium Type sin		
	with profit sharing	without profit sharing	risk mitigation
PVFP	€ 39,69 mln	€ 78,02 mln	49%
SCR	€ 14,86 mln	€ 28,64 mln	48%
SCReq	€ 12,66 mln	€ 24,21 mln	48%
SCRint	€ 0,00 mln	€ 0,28 mln	100%
SCRmort	€ 0,13 mln	€ 0,43 mln	70%
SCRlapse	€ 4,85 mln	€ 9,66 mln	50%
SCRexp	€ 0,68 mln	€ 1,32 mln	49%

Policy Type A	Premium Type reg		
	with profit sharing	without profit sharing	risk mitigation
PVFP	€ 40,00 mln	€ 74,57 mln	46%
SCR	€ 14,70 mln	€ 27,37 mln	46%
SCReq	€ 0,08 mln	€ 0,14 mln	43%
SCRint	€ 5,79 mln	€ 11,15 mln	48%
SCRmort	€ 0,30 mln	€ 0,84 mln	64%
SCRlapse	€ 11,75 mln	€ 21,64 mln	46%
SCRexp	€ 0,74 mln	€ 1,32 mln	44%

Table 6: Risk absorbing effect of future profit sharing

5 Linearities

The Solvency II standard formula is based on the assumption of linearity. Two types of linearity can be identified: Linearity within a risk and linear relations between risks. Linearity within a risk ensures that the solvency capital requirement of a single risk module increases linearly with the risk factor. Following equation holds:

$$kSCR(X_i) = SCR(kX_i)$$

for any positive k and every risk i. The linearity between risks guarantees that the separately calculated diversified solvency capital requirement of several risk modules equals the solvency capital requirement of a simultaneous shock with adjusted risk factors:

$$SCR^{k \cdot SES}(X) = \sqrt{\sum_{i,j} \rho_{i,j} SCR(kX_i) SCR(kX_j)}$$

with $X = \sum_i X_i$ and the single equivalent scenario SES[32].

Non-linearities can compromise the accuracy of the solvency capital requirement calculated with the standard formula. Excessive non-linearities nearing the defined stress scenarios can lead to significant changes of the solvency capital requirement. More crucially, non-linearities between risks can not be evaluated with the standard formula. It is possible that an insurance company facing unfavorable developments in several risk modules is in need of much more or much less capital than aggregated with the standard formula. Furthermore, the single equivalent scenario method requires both, intra and inter risk linearities.

[32]See chapter 8 for a thorough investigation on SES.

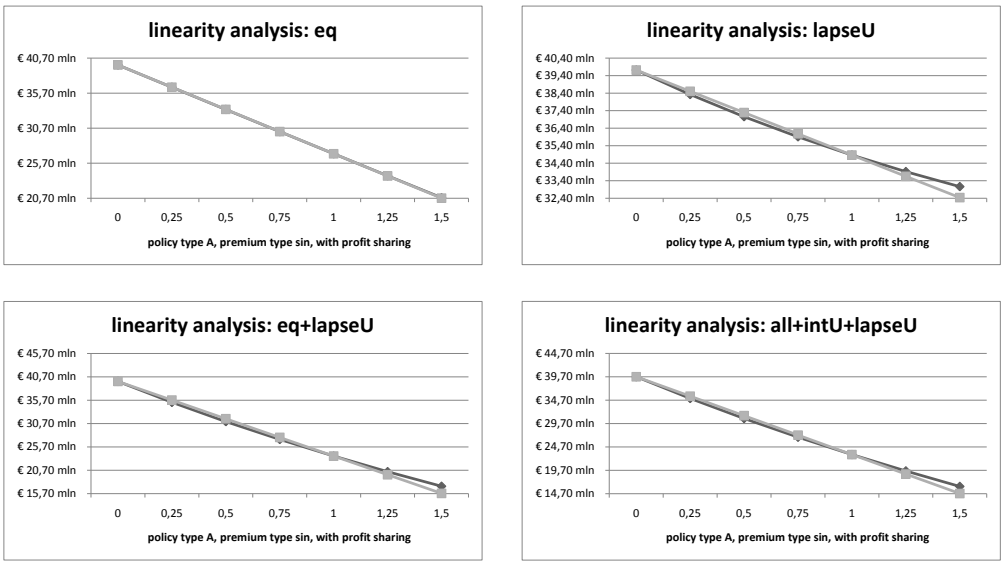

Figure 10: Numerical results - Linearities - single premium

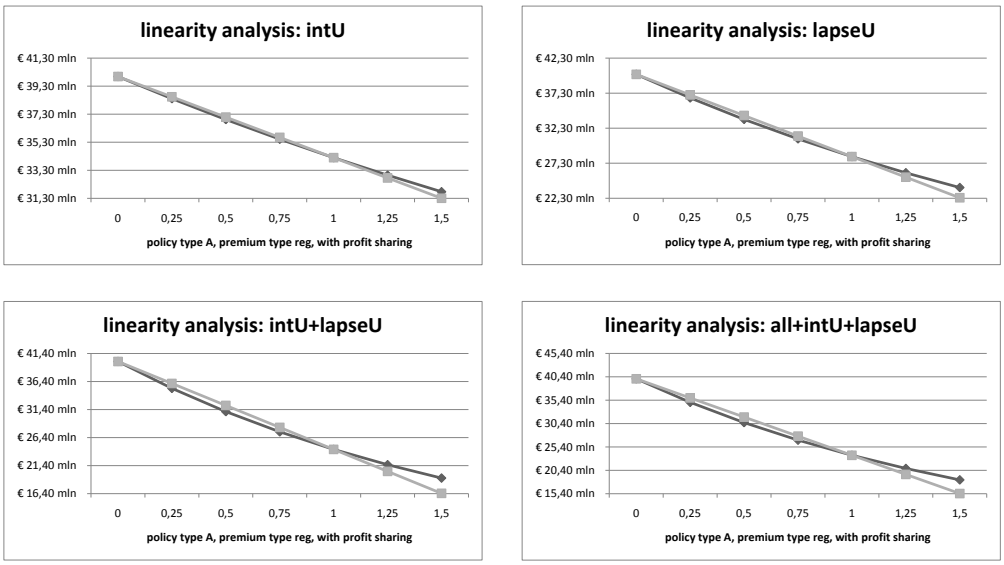

Figure 11: Numerical results - Linearities - regular premium

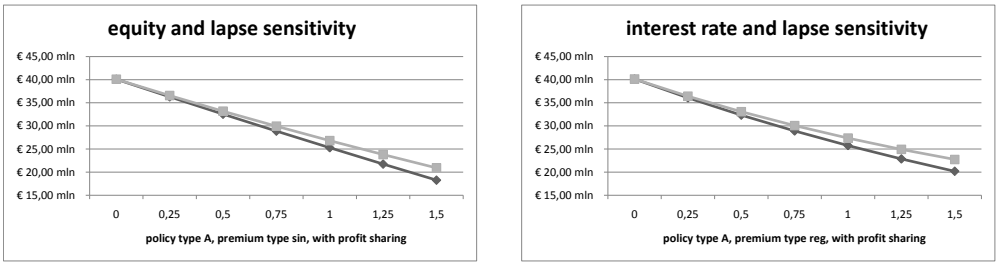

Figure 12: Numerical results - Non-linearities between risks

Figures 10 and 11 show sensitivity graphs of the relevant risks, equity and lapse (up-shock) for a single premium policy (type A). For a regular premium policy (type A), sensitivity graphs of the interest rate (up-shock) and lapse (up-shock) are presented. The values on the x-axis denote the reduction factor for the risk from zero ("no stress") to one (full QIS4 stress-scenario")[33]. The grey curves represent the impact on the PVFP while the black curves represent an assumed linear impact. Most graphs indicate almost perfect linearity. However, a nonlinear behavior can be found within the lapse risk. There is also non-linearity between market risks and lapse risk as seen in figure 12. Here, the grey curves represent the impact on the PVFP of simultaneous stress-scenarios with adjusted risk factors, the black curves represent the total impact on the PVFP of separately calculated stress-scenarios including diversification. This result is important for the single equivalent scenario.

6 Dynamic policyholder behavior

Dynamic policyholder behavior is a major concern to actuaries. The lack of statistical data and the amount of factors that may influence the policyholder's behavior have to be taken into account and make it difficult to model or project the policyholder's actions. The challenge is even bigger considering a situation of a new product launch and therefore only little experience. On the other hand, it is common sense among actuaries that dynamic policyholder behavior, especially dynamic lapses, can be a major risk. Throughout the literature, there are indicators that suggest a more distinct dynamic be-

[33]A value of 0.5 for the equity risk would denote an immediate loss of 16% of the risky assets.

havior for unit-linked products[34] caused by a higher volatility in the "value" of options, guarantees or the fund value. CEIOPS addresses the existence of options and guarantees as well as the financial markets as reasons for possible dynamic policyholder behavior[35].

In this chapter, since it is rather unlikely that policyholders would tie their lapse behavior to the guaranteed death benefits of a simple German unit-linked insurance, dynamic lapses are triggered by the fund value. A very simple lapse function is used, more sophisticated lapse functions can be found in literature[36].

The impact of dynamic policyholder behavior on the solvency capital requirement is measured with the following approach: the output of a lapse function, denoted as the dynamic lapse multiplier, adjusts the deterministic lapse rates. The lapse rates therefore reflect a combination of deterministic lapse bahavior and dynamic lapse behavior. This setup ensures that, when the lapse stress scenario is performed, only the deterministic lapse rates are affected directly while the risk from dynamic lapses is taken into account in the sub-module of the trigger (here: the market risk at most)[37]. In the second step, the solvency capital requirement is recalculated with the average annual lapse rates of the first step just as if the insurer would experience lapses without the assumption of dynamic policyholder behavior. The impact of dynamic lapse rates is then the ratio of the SCR's obtained.

Using a simple step function and assuming that bad fund performance leads to higher lapse rates while good fund performance reduces lapses, the dynamic lapse multiplier can be defined as follows:

$$lr_t = lr_t^{det} \cdot dlm_t \qquad \text{with}$$

$$dlm_t = \begin{cases} dlm_{min}, & \text{for } \frac{A_t}{A_{\max\{0,t-d\}}} > adj_a \\ dlm_{max}, & \text{for } \frac{A_t}{A_{\max\{0,t-d\}}} < adj_b \\ 1, & \text{else} \end{cases}$$

where dlm_{max} and dlm_{min} denote the maximum and the minimum value of

[34]See Helfenstein & Barnshaw (2003, page 20), Hochreiter *et al.* (2007, page 8), Edwards (2008), Cerchiara *et al.* (2008) and Milliman (2009).

[35]See TS.II.D.11-15 CEIOPS (2008c, page 34).

[36]See Kolkiewicz & Tan (2006), Smink (2001), Zenios (1999), De Giovanni (2010) and Kochanski (2009).

[37]This approach is presented in CEIOPS (2009a, page 20-24).

the dynamic lapse multiplier, while adj_a and adj_b set the fund value performance that triggers dynamic lapse behavior and d denotes the number of months the policyholder monitors the fund value until he makes a decision.

Parameter	Value	Description	Category
dlm_{max}	1.5	maximum value of the dynamic lapse multiplier	Dynamic lapse multiplier triggered by the fund value
dlm_{min}	0.5	minimum value of the dynamic lapse multiplier	
adj_a	1.5	fund value performance triggering lower lapses	
adj_b	0.9	fund value performance triggering higher lapses	
d	12	monitoring period in months	

Table 7: Dynamic lapse model parameter assumptions

Policy Type	P type	lapse function	SCR	PVFP	Solvency ratio	SCRmkt	SCRmort	SCRlapse	SCRexp
A	sin	2	-3,20%	-1,04%	2,23%	-3,99%	-0,47%	-0,10%	-0,06%
A	reg	2	0,52%	-0,84%	-1,35%	2,32%	-0,20%	0,07%	-0,13%

Table 8: Numerical results - dynamic lapses triggered by the fund value

Table 8 shows relevant results from simulations with dynamic lapses triggered by the fund value (lapse function 2). Dynamic lapses lead mainly to changes of the SCR_{mkt}. For single premium policies, the SCR_{mkt} has a bigger influence on the SCR than for regular premium policies. The PVFP decreases in both cases. Overall, the use of dynamic lapses triggered by the fund value defined as in the model improves the solvency ratio for single premium policies and worsens the solvency ratio for regular premium policies. The changes of the deterministic lapses (run 1 – original deterministic lapses and run 2 – average overall lapses from run 1) are presented in table 14[38].

The impact of dynamic lapse behavior as modeled in this paper on the solvency capital requirement of a German unit-linked insurance with guaranteed death benefits is not alarming. However, this may not be the case with unit-linked products that contain strong guarantees and options.

7 Parameter analysis

In order to gain stability for the results achieved so far, simulations should also be performed with other values of the crucial parameters policy term and deterministic lapse vector. The solvency capital requirement for unit-linked products is calculated for policy terms of 10, 20, 40 and 50 years (parameter 0.5 to 1.5) and for several multiples of the original deterministic lapse vector (0.5, 0.75, 1.25, 1.5).

[38]See Appendices.

parameter	SCR	PVFP	solvency ratio	SCReq	SCRint	SCRmort	SCRlapse	SCRexp
0,5	€ 9,53 mln	€ 28,27 mln	296,71%	€ 9,01 mln	€ 0,00 mln	€ 0,04 mln	€ 1,05 mln	€ 0,34 mln
0,75	€ 12,27 mln	€ 34,29 mln	279,54%	€ 10,94 mln	€ 0,00 mln	€ 0,07 mln	€ 2,92 mln	€ 0,51 mln
1	€ 14,98 mln	€ 39,69 mln	264,99%	€ 12,66 mln	€ 0,00 mln	€ 0,13 mln	€ 4,85 mln	€ 0,68 mln
1,25	€ 17,20 mln	€ 43,78 mln	254,53%	€ 13,94 mln	€ 0,00 mln	€ 0,23 mln	€ 6,53 mln	€ 0,83 mln
1,5	€ 18,78 mln	€ 46,51 mln	247,72%	€ 14,77 mln	€ 0,05 mln	€ 0,38 mln	€ 7,76 mln	€ 0,93 mln

Table 9: Parameter analysis – policy term – single premium

parameter	SCR	PVFP	solvency ratio	SCReq	SCRint	SCRmort	SCRlapse	SCRexp
0,5	€ 4,84 mln	€ 17,82 mln	367,80%	€ 0,08 mln	€ 1,60 mln	€ 0,05 mln	€ 3,87 mln	€ 0,36 mln
0,75	€ 9,34 mln	€ 28,89 mln	309,20%	€ 0,09 mln	€ 3,41 mln	€ 0,14 mln	€ 7,48 mln	€ 0,54 mln
1	€ 14,82 mln	€ 40,00 mln	269,92%	€ 0,08 mln	€ 5,79 mln	€ 0,30 mln	€ 11,75 mln	€ 0,74 mln
1,25	€ 19,81 mln	€ 48,33 mln	243,99%	€ 0,07 mln	€ 8,12 mln	€ 0,58 mln	€ 15,53 mln	€ 0,91 mln
1,5	€ 23,27 mln	€ 52,97 mln	227,67%	€ 0,06 mln	€ 9,88 mln	€ 0,98 mln	€ 18,04 mln	€ 1,03 mln

Table 10: Parameter analysis – policy term – regular premium

parameter	SCR	PVFP	solvency ratio	SCReq	SCRint	SCRmort	SCRlapse	SCRexp
0,5	€ 17,61 mln	€ 46,87 mln	266,13%	€ 14,95 mln	€ 0,00 mln	€ 0,19 mln	€ 5,55 mln	€ 0,95 mln
0,75	€ 15,88 mln	€ 42,93 mln	270,40%	€ 13,69 mln	€ 0,00 mln	€ 0,16 mln	€ 4,63 mln	€ 0,80 mln
1	€ 14,98 mln	€ 39,69 mln	264,99%	€ 12,66 mln	€ 0,00 mln	€ 0,13 mln	€ 4,85 mln	€ 0,68 mln
1,25	€ 14,10 mln	€ 37,03 mln	262,61%	€ 11,81 mln	€ 0,00 mln	€ 0,11 mln	€ 4,77 mln	€ 0,58 mln
1,5	€ 13,27 mln	€ 34,84 mln	262,61%	€ 11,12 mln	€ 0,00 mln	€ 0,09 mln	€ 4,50 mln	€ 0,50 mln

Table 11: Parameter analysis – lapse vector – single premium

parameter	SCR	PVFP	solvency ratio	SCReq	SCRint	SCRmort	SCRlapse	SCRexp
0,5	€ 19,52 mln	€ 57,16 mln	292,78%	€ 0,11 mln	€ 8,52 mln	€ 0,42 mln	€ 14,88 mln	€ 1,02 mln
0,75	€ 15,42 mln	€ 47,77 mln	309,74%	€ 0,09 mln	€ 7,02 mln	€ 0,36 mln	€ 11,49 mln	€ 0,87 mln
1	€ 14,82 mln	€ 40,00 mln	269,92%	€ 0,08 mln	€ 5,79 mln	€ 0,30 mln	€ 11,75 mln	€ 0,74 mln
1,25	€ 14,10 mln	€ 33,58 mln	238,20%	€ 0,07 mln	€ 4,78 mln	€ 0,26 mln	€ 11,66 mln	€ 0,63 mln
1,5	€ 13,11 mln	€ 28,25 mln	215,54%	€ 0,07 mln	€ 3,95 mln	€ 0,22 mln	€ 11,14 mln	€ 0,54 mln

Table 12: Parameter analysis – lapse vector – regular premium

8 Single equivalent scenario

The single equivalent scenario was developed to avoid double-counting of the loss-absorbing capacity of future discretionary benefits and to detect non-linearities[39]. As opposed to perform single stress tests to determine the solvency capital requirement for every risk module and then using the SCR-formulas, only one stress scenario is performed but with all stresses at once

[39]See CEIOPS (2009b) for general description and CEIOPS (2008b) for implementation in the standard formula.

and therefore with reduced shocks. The calibration of the shocks should be performed on the $BSCR$, therefore the derivation starts with the standard formula for basic solvency capital requirement:

$$BSCR = \sqrt{\sum_{i,j} \rho_{i,j} SCR_i SCR_j}.$$

Since the stresses in the single equivalent scenario happen simultaneously, all correlation factors are changed to $\rho_{i,j} = 1,\ \forall i, j$[40]. The change of the correlation factors increases the level of significance. To ensure a constant level of significance, a change of correlation factors must be accompanied by an adjustment to the stress scenarios. Furthermore, linearity is assumed throughout the entire model, therefore, the solvency capital requirement for every sub-module can be adjusted by multiplying with a diversification reduction factor, which is also applied to the stress scenarios. Using the adjusted SCR_i and replacing the correlation factors, the $BSCR$ can be expressed as

$$\begin{aligned}
BSCR &= \sqrt{\left(\sum_i drf_i\ SCR_i\right)^2} \\
&= \sqrt{\left(SCR_{int}^{SES} + SCR_{eq}^{SES} + SCR_{mort}^{SES} + SCR_{lapse}^{SES} + SCR_{exp}^{SES}\right)^2} \\
&= SCR_{int}^{SES} + SCR_{eq}^{SES} + SCR_{mort}^{SES} + SCR_{lapse}^{SES} + SCR_{exp}^{SES} \\
&= BSCR^{SES} \\
&= \Pi - \Pi|_{SES} \qquad \text{(without profit sharing)}
\end{aligned}$$

where SCR_i^{SES} denotes the solvency capital requirement of the sub-module i resulting from an adjusted shock[41]. In QIS4, most shocks are expressed with factors to the relevant rates (e.g. mortality rates, lapse rates), reduced shocks as used in the single equivalent scenario are created with diversification reduction factors. The diversification reduction factors that adjust the shock rates are derived through the following approach: The diversified solvency capital requirement is allocated to every sub-module with the covariance principle[42]. Then, the diversification reduction factors that adjust the stress scenarios are defined as the proportion of the allocated diversified solvency capital requirement to the stand-alone solvency capital requirement.

[40] The single equivalent scenario method requires positive definite initial correlation matrices (see CEIOPS (2009a) for further information).

[41] $SCR_i^{SES} = drf_i\ SCR_i$ holds only with a linear model.

[42] See Albrecht & Koryciorz (2004).

Let C_{SCR}, C_{mkt} and C_{life} denote the correlation matrices of the overall SCR, the market risk and the underwriting risk modules. Let

$$U = \begin{pmatrix} SCR_{mkt} \\ SCR_{life} \end{pmatrix}, \qquad V = \begin{pmatrix} SCR_{int} \\ SCR_{eq} \end{pmatrix}, \qquad W = \begin{pmatrix} SCR_{mort} \\ SCR_{lapse} \\ SCR_{exp} \end{pmatrix},$$

then the 1st step diversification reduction factors f are defined by:

$$\begin{pmatrix} f_{mkt} \\ f_{life} \end{pmatrix} = \frac{1}{BSCR} C_{SCR} U,$$
$$\begin{pmatrix} f_{int} \\ f_{eq} \end{pmatrix} = \frac{1}{SCR_{mkt}} C_{mkt} V,$$
$$\begin{pmatrix} f_{mort} \\ f_{lapse} \\ f_{exp} \end{pmatrix} = \frac{1}{SCR_{life}} C_{life} W.$$

The 2nd step diversification reduction factors drf are obtained by multiplying the risk module 1st step diversification reduction factors with the overall 1st step diversification reduction factors, e.g. $drf_{mort} = f_{mort} f_{life}$ (the reduced mortality shock would be $10\% \cdot drf_{mort}$).

Now, the reduced shocks can be used to calculate the net solvency capital requirement via the single equivalent scenario. A significant difference between the $nSCR$ and the $nSCR^{SES}$ suggests a significant double counting of loss-absorbing capacity of future discretionary benefits.

The existence of non-linearities leads to significant difference between the $BSCR$ and the $BSCR^{SES}$. Therefore, the single equivalent scenario can also be used to detect non-linearities.

Table 13 shows the $BSCR$ and the SCR obtained by the standard formula and the percental deviation of the $BSCR$ and the SCR obtained by the single equivalent scenario method. There is no indication for double counting of loss-absorbing capacity of future discretionary benefits since the deviation is almost identical for the $BSCR$ and the SCR. The deviation does not change with different bonus participation rates[43]. The reason for the difference of the solvency capital requirements is non-linearity[44]. The

[43] Tested with higher and lower bonus participation rates and also without minimum participation rates.

[44] As shown in chapter 5, figure 12.

diversification reduction factors are presented in table 15, the composition of the PVFP is shown in table 16[45]. Nevertheless, requiring less computational capacities than the standard formula, the single equivalent scenario can be useful, once the diversification reduction factors are obtained. Unfortunately, the adjustment of the diversification reduction factors requires the calculation of the solvency capital requirement with the standard formula method. Therefore, the single equivalent scenario can not be used to replace the standard formula.

Policy type	P type	SCR	PVFP	Solvency ratio	BSCR	SCR	SES-BSCR	SES-SCR
A	sin	€ 14,98 mln	€ 39,69 mln	264,99%	€ 28,64 mln	€ 14,86 mln	-4,51%	-4,79%
A	reg	€ 14,82 mln	€ 40,00 mln	269,92%	€ 27,37 mln	€ 14,70 mln	-6,41%	-6,28%

Table 13: Numerical results - SES

9 Summary

The analysis reveals that market and lapse risk are in fact the main risks associated with a German unit-linked insurance product with guaranteed death benefits. Mortality and expense risks are negligible. The type of the death benefits has no impact on the solvency capital requirement. On the other hand, the premium type influences the type of market risks. The product is linear to the risk factors for the most part. Some non-linearity has been revealed attached to lapse risks. This matter of fact causes a lower solvency capital requirement calculated with the single equivalent method. There is no indication of double-counting of the loss-absorbing capacity of future discretionary benefits so far. The single equivalent scenario method also proves to be a tool to review main assumptions of the standard formula. Dynamic policyholder behavior does not have a large impact on the solvency capital requirement for this particular insurance product. Nevertheless, dynamic lapses have the potential to be a major risk and an analysis with other unit-linked products and other lapse functions is left for further research.

[45]See Appendices for both tables.

Appendices

policy year	det. lapse Run 1	det. lapse Run 2
1	10,00%	10,94%
2	9,00%	10,14%
3	8,00%	9,01%
4	7,00%	7,89%
5	6,00%	6,76%
6	5,00%	5,64%
7	4,00%	4,51%
8	3,00%	3,38%
9	2,00%	2,26%
10	2,00%	2,26%
11	2,00%	2,26%
12	2,00%	2,26%
13	2,00%	2,26%
14	2,00%	2,26%
15	2,00%	2,26%
16	2,00%	2,26%
17	2,00%	2,25%
18	2,00%	2,26%
19	2,00%	2,26%
20	2,00%	2,26%
21	2,00%	2,26%
22	2,00%	2,26%
23	2,00%	2,26%
24	2,00%	2,26%
25	2,00%	2,26%
26	2,00%	2,25%
27	2,00%	2,26%
28	2,00%	2,25%
29	2,00%	2,26%
30	2,00%	2,25%

Table 14: Change of deterministic lapses - dynamic lapses triggered by the fund value - single and regular premium policy type A

Policy Type A	
premium type	sin
drf_eq	0,936
drf_int	0,011
drf_mort	0,042
drf_lapse	0,570
drf_exp	0,346

Policy Type A	
premium type	reg
drf_eq	0,008
drf_int	0,612
drf_mort	0,048
drf_lapse	0,917
drf_exp	0,508

Table 15: Diversification reduction factors - SES

PVFP distribution – SES	Policy Type A	Premium Type sin			
	total	from risk	from expenses		
			total	from lapse	from kickbacks
with profit sharing					
PVFP	€ 39,69 mln	€ 0,59 mln	€ 39,10 mln	€ 7,37 mln	€ 32,26 mln
PVFP after combined shock	€ 25,55 mln	€ 0,57 mln	€ 24,97 mln	€ 6,27 mln	€ 19,32 mln
without profit sharing					
PVFP	€ 78,02 mln	€ 2,39 mln	€ 75,63 mln	€ 14,61 mln	€ 62,08 mln
PVFP after combined shock	€ 50,67 mln	€ 2,37 mln	€ 48,30 mln	€ 12,41 mln	€ 37,11 mln

PVFP distribution – SES	Policy Type A	Premium Type reg			
	total	from risk	from expenses		
			total	from lapse	from kickbacks
with profit sharing					
PVFP	€ 40,00 mln	€ 1,11 mln	€ 38,90 mln	€ 0,37 mln	€ 8,48 mln
PVFP after combined shock	€ 26,23 mln	€ 0,74 mln	€ 25,49 mln	€ 0,45 mln	€ 5,56 mln
without profit sharing					
PVFP	€ 74,57 mln	€ 4,68 mln	€ 69,89 mln	€ 0,72 mln	€ 15,26 mln
PVFP after combined shock	€ 48,96 mln	€ 3,08 mln	€ 45,88 mln	€ 0,88 mln	€ 10,01 mln

Table 16: Composition of the PVFP – SES

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