Solvency capital requirement for hybrid products

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Preprint Series: 2011-08

Fakultät für Mathematik und Wirtschaftswissenschaften

UNIVERSITÄT ULM

Preliminary Version manuscript No.
(will be inserted by the editor)

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Received: date / Accepted: date

Abstract Static and dynamic hybrid products have been recently gaining in popularity in Germany. While offering a high degree of security to the policyholders, they are attractive because of the potential of higher returns than traditional insurance products. The premiums are invested in a conventional premium reserve stock as well as in risky assets such as equity or guarantee funds. For static hybrid products the allocation of premiums to the conventional premium reserve stock is set at the beginning and remains unchanged, whereas dynamic hybrid products are rebalanced monthly in order to meet the guarantees. Due to the recent popularity of these products, the task of analyzing the risks of hybrid products and developing a method to assess the solvency capital requirement becomes more and more imminent. In our paper, we explain the characteristics of static and dynamic hybrid products and present a partial internal model to assess the corresponding solvency capital requirement. We also present and interpret qualitative and quantitative results from several simulation studies.

Keywords Solvency II · static hybrid products · dynamic hybrid products · solvency capital requirement · partial internal model

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1 Introduction

Innovative life insurance products have been gaining in popularity during the last decade and now represent a majority of new business in Germany.[1] However, and despite the importance of these products to the future of the life insurance industry, most discussions about the Solvency II framework focus on traditional insurance products. The results of the last quantitative impact studies QIS4 and QIS5 indicate that most insurance companies do not calculate the solvency capital requirement for innovative life insurance products as systematically as for traditional products.[2] This paper is a contribution to the discussion about the solvency capital requirement for innovative life insurance products. Innovative life insurance products differ from traditional life insurance products in some fundamental aspects and therefore require an in-depth risk examination. For both insurers and policyholders, the value of an innovative life insurance product is expected to be rather volatile, since the capital is mostly invested in risky assets, as opposed to the fixed-income-oriented investment strategies of traditional life insurance products. Furthermore, innovative life insurance products are usually complex in their structure and contain a broader range of options and guarantees. Their volatile value, their complexity, and the inherent options and guarantees can all have an unexpected influence on the solvency capital requirement.

Among all innovative life insurance products, static and dynamic hybrid products attract the most attention. While offering a high degree of security to the policyholders, they have the potential of higher returns than traditional insurance products. From the policyholder's point of view, hybrid products are a reasonable choice in terms of a private pension plan. The accumulation benefits of hybrid products are fully or partly guaranteed by investing a portion of the premiums in a conventional premium reserve stock. The remainder of the premiums is invested in risky assets such as equity or guarantee funds. Many insurers provide their policyholders with the option to choose among several funds or to switch funds free of charge at certain reference dates. For static hybrid products the allocation of premiums to the conventional premium reserve stock is set at the beginning and remains unchanged, whereas dynamic hybrid products are rebalanced monthly in order to meet the guarantees. Due to the recent popularity of these products, the task of analyzing the risks of hybrid products and developing a method to assess the solvency capital requirement becomes more and more imminent. The current standard formula to calculate the solvency capital requirement for traditional German insurance products appears to be inaccurate for hybrid products because of the high-volume investments in risky assets, the complex design and the rebalancing algorithms. In our paper, we present a partial internal model to assess the solvency capital requirement for hybrid products.

[1] See Daalmann and Märten (2009), Daalmann and Märten (2010) and Daalmann and Bause (2011).

[2] See CEIOPS (2008) and CEIOPS (2011).

The European Commission and CEIOPS[3] have published numerous documents that characterize the standard formula. Since the Solvency II process is in full swing the definition of the standard formula is still fluid and partly still lacks detailed specification. Nevertheless, the Solvency II directive (The European Parliament (2009)), the technical specifications of the latest Quantitative Impact Study (QIS5, CEIOPS (2010)) and the amount of more than 80 consultation papers (CEIOPS (2004-2010)) provide a reasonable understanding of how the final standard formula will take shape. Of course, the Solvency II framework and the standard formula have already been discussed in academic literature. Eling et al (2007), Doff (2008), Duverne and Le Douit (2009), Elderfield (2009), Steffen (2008) and Van Hulle (2011) discuss general aspects of the Solvency II framework. An elaborate comparison of regulatory systems is done by Holzmüller (2009). The standard formula and its shortcomings are discussed thoroughly e.g. by Pfeiffer and Strassburger (2008), Ronkainen et al (2007), Sandström (2007) and Devineau and Loisel (2010). Bauer et al (2009) and Bauer et al (2010) presented an approach of calculating the solvency capital requirement using nested simulations. Christiansen et al (2010) and Börger (2010) suggested improvements of particular risk modules such as the mortality and longevity risk modules.

The amount of literature on the valuation of insurance products and their embedded options and guarantees has increased significantly during the last decade. Since the work of Brennan and Schwartz (1976), most of the popular life insurance products have been reviewed. Participating life insurance contracts have been discussed by Grosen and Jorgensen (2000), Bauer et al (2006), Zaglauer and Bauer (2008), Kling et al (2007), Herr and Kreer (1999) and Tanskanen and Lukkarinen (2003). Also, variable annuities were analyzed by Bauer et al (2008). Hybrid products have not been of academic interest so far. Literature is restricted to product descriptions as done by Zwiesler (2007), Deichl (2008), Fix and Käfer (2008) or Hammers (2009). A first discussion on the valuation of the rebalancing options of dynamic hybrid products has been done by Menzel (2008), Siebert (2008), Reuß and Ruß (2010) and Ferber (2010). An analysis of the risks of static and dynamic hybrid insurance products and the development of a framework that enables insurers to assess the solvency capital requirement for these products is still due.

This paper is organized as follows. In Section 2 we present relevant product types: static hybrids and so called 3-pot dynamic hybrids with single premiums. The standard formula is discussed in Section 3. The partial internal model is described in Section 4 and is based on the model that has been used in Kochanski (2010a) and Kochanski (2010b) to assess the solvency capital requirement for simple German unit-linked insurance with guaranteed death

[3] Committee of European Insurance and Occupational Pensions Supervisors, now replaced by the European Insurance and Occupational Pensions Authority (EIOPA).

benefits. In Section 5 we present and interpret qualitative and quantitative results from several simulation studies. Section 6 concludes.

2 Hybrid products

Traditional German life insurance products[4] include an annual interest rate guarantee. They also feature a strict regulation of the asset investments of the insurer, guaranteed surrender values and the familiarity to German policyholders. Therefore, a high degree of security and acceptance is associated with these products. Unfortunately, this comes at the cost of high charges and low investment returns during phases of booming markets. Especially the rallies of stock markets during the last two decades enhanced the demand of unit-linked insurance products with little guarantees. The major disadvantage of pure unit-linked insurance products were exposed in the course of financial crises. Many policyholders experienced high losses and therefore lost confidence in these products to be an appropriate corner stone of their retirement provisions.Hybrid products were designed to combine the best of the two product types: strong guarantees and high returns. In the following, we describe the typical features of a hybrid insurance policy.

Accumulation period In order to secure their financial situation after retirement, the policyholder purchases a hybrid insurance policy and pays a single premium P to the insurer at time $t = 0$. The insurer invests the premium according to the investing strategy of the corresponding product type for T years. This period is called the accumulation period. Depending on the product type, the insurer will invest in the premium reserve stock (PRS), an equity fund (EF) or a guarantee fund (GF). The account value of the policy at time t is defined as $\mathrm{AV}_t = \mathrm{PRS}_t + \mathrm{EF}_t + \mathrm{GF}_t$. During the accumulation period the insurer also agrees to pay a death benefit (DB) in case of death and a surrender benefit (SB) in case of surrender. The death benefit at time t has a guaranteed minimum: $\mathrm{DB}_t = \max(\mathrm{P}; \mathrm{AV}_t)$. The surrender benefit equals the account value net of the surrender fee (SF): $\mathrm{SB}_t = \mathrm{AV}_t - \mathrm{SF}_t$ with $\mathrm{SF}_t = sf \cdot \left(\mathrm{P} - t \cdot \frac{\mathrm{P}}{12 \cdot T}\right)$. The insurer also deducts charges in order to cover administrative expenses. Three types of expenses can be identified with insurance business: aquisition expenses, unit expenses and variable expenses. In our model, the insurer deducts aquisition charges ($\mathrm{AC}_0 = \mathrm{P} \cdot ac$) to finance its sales department. All other expenses are financed with fixed unit charges ($\mathrm{UC}_0 = \mathrm{P} \cdot \frac{uc}{2}$ at $t = 0$ and $\mathrm{UC}_t = \frac{\mathrm{P}}{12 \cdot T} \cdot \frac{uc}{2}$ every other month).[5]

Pension period At the end of the accumulation period, the policyholder usually can choose between a lump sum payment, which equals the current account value but is at least the initial single premium, and an annuity. In order to

[4] We refer to participating life insurance products such as the "Kapitallebensversicherung" or the "Kapitalrentenversicherung".

[5] See Table 3 (Appendix A) for parameter values of sf, ac and uc.

simplify our analysis, we assume that all policyholders choose the lump sum payment.

Profit participation The insurer uses prudent assumptions for mortality and expenses and the actuarial interest rate for the calculation of the premiums. Since the best estimates for mortality and expenses are usually less adverse and the earned interest on the premium reserve stock is usually higher than the actuarial interest rate, the insurer is likely to generate profits. In Germany, the policyholder participates in three profit categories. The participation rate on profits generated by the interest on the premium reserve stock is denoted by *ipr*, the participation rate generated from mortality assumptions is denoted by *mpr* and *opr* denotes the participation rate on profits generated from expenses and other sources. Of course, the policyholder does not participate in losses. During the accumulation period, surpluses are reinvested into the account. See Table 3 (Appendix A) for all product parameter values.
In the following, we describe the different investing stategies of the three hybrid products.

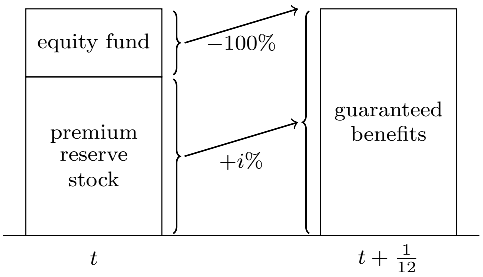

Fig. 1 Static hybrid product - worst case scenario

Static hybrid products Static hybrid products were the first hybrid products introduced in Germany.[6] The simple structure of these products is easily comprehensible for most policyholders and therefore, the success of static hybrids is not startling. The insurer divides the insurance benefits into a guaranteed and a non-guaranteed part. In our model, we have a guaranteed account value of the initial single premium at time T and guaranteed death benefits of the initial single premium during the accumulation period. The insurer invests just enough of the account deposits in the premium reserve stock to meet the guarantees at any point of time. The rest of the account deposits is invested into an equity fund which promises higher returns than the premium reserve stock. The investments are made at the beginning of the accumulation period and there is no rebalancing needed. Therefore, in the worst case scenario (Fig. 1), the policyholder will retrieve at least the initial single premium at time T (or the equivalent whole life annuity). Let G_t denote the account value

[6] Volksfürsorge introduced Best Invest in 1999 (see Schmidt (2010)).

that is needed to meet the guaranteed insurance benefits at time t and i denote the actuarial interest rate, then the investment strategy of a static hybrid insurance product can be expressed as follows:

$$\begin{aligned}\mathrm{PRS}_t &= \frac{\mathrm{G}_{t+\frac{1}{12}}}{(1+i)^{\frac{1}{12}}},\\ \mathrm{EF}_t &= \mathrm{AV}_t - \mathrm{PRS}_t.\end{aligned}$$

Dynamic hybrid products The downside of static hybrid products is that the portion of the account deposit invested in the premium reserve stock is still very high. The worst case scenario suggests that the equity fund could lose all its value at any time and also in any time span. It is clear that the worst case scenario that underlies the investment strategy of static hybrids is way to harsh. Not surprisingly, the extension of the concept of hybrid products[7] is based on a more realistic approach towards the worst case scenario. It is reasonable to define a maximum loss of the equity fund for a short and foreseeable time span. We use a month as the time span and denote the assumed maximum loss of the equity fund during a month as ml. Now, the insurer is able to invest a much higher portion of the account deposits into the equity fund. The insurer invests just enough in the premium reserve stock that all guarantees can be fulfilled after a maximum loss of the equity fund. In this unlikely case (Fig. 2), the insurer would sell all shares of the equity fund immediately and fully invest in the premium reserve stock. Although the maximum loss is usually high[8] there is still some risk, that the equity fund might lose even more value than assumed. In order to eliminate that risk, the insurer invests in a guarantee fund, equivalent to the equity fund with a hedge, that actually can lose ml at most during a month. Of course, this comes with higher management costs but hedging costs are low for this kind of risks. The other implication of this kind of strategy is that the insurer is forced to review the asset allocation whether the guarantees are still met or not and rebalance if needed. In situations where the account deposit is entirely invested in the guarantee fund and where it exceeds the needed account value even after a maximum loss, there is some portion of the account deposit that is hedged unnecessarily. In this case, the investment strategy of a 3-pot dynamic hybrid insurance allows for additional investments in an equity fund:

$$\begin{aligned}\mathrm{PRS}_t &= \begin{cases}\frac{\mathrm{G}_{t+\frac{1}{12}}-(1-\mathrm{ml})\cdot\mathrm{AV}_t}{(1+i)^{\frac{1}{12}}-1+\mathrm{ml}}, & \text{if } \frac{\mathrm{G}_{t+\frac{1}{12}}}{(1-\mathrm{ml})\mathrm{AV}_t} > 1\\ 0, & \text{otherwise}\end{cases}\\ \mathrm{GF}_t &= \begin{cases}\mathrm{AV}_t - \mathrm{PRS}_t, & \text{if } \frac{\mathrm{G}_{t+\frac{1}{12}}}{(1-\mathrm{ml})\mathrm{AV}_t} > 1\\ \frac{\mathrm{G}_{t+\frac{1}{12}}}{1-\mathrm{ml}}, & \text{otherwise}\end{cases}\\ \mathrm{EF}_t &= \mathrm{AV}_t - \mathrm{PRS}_t - \mathrm{GF}_t.\end{aligned}$$

[7] HDI-Gerling introduced Two Trust in 2006 (see Ortmann (2007)).

[8] 20% are usually assumed.

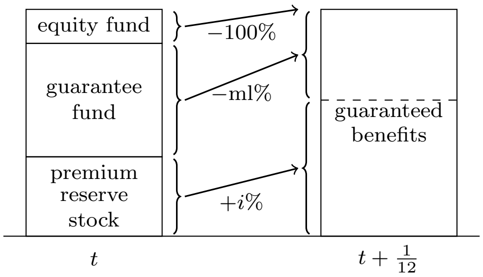

Fig. 2 3-pot dynamic hybrid product - worst case scenario

3 Solvency capital requirement

The main objective of insurance supervision and regulation is to provide adequate policyholder protection. In order to achieve this objective, the Solvency II framework addresses qualitative and quantitative requirements, such as the solvency capital, transparency and accountability, with a three pillar framework. Pillar I is solely devoted to the calculation of the solvency capital requirement (SCR). The SCR is calculated on an "economic risk-based approach" and "corresponds to the economic capital a (re)insurance undertaking needs to hold in order to limit the probability of ruin to 0.5%".[9] In the process of launching Solvency II, CEIOPS already organized five field studies with the intention to motivate insurers establishing the required internal structures early and to refine the directives based on the results. In the course of the field studies (QIS) CEIOPS defines a standard formula which is documented in the technical specifications. The standard formula enables insurers to calculate the SCR if they are not able or not willing to build an internal model on their own. Furthermore, the standard formula can be used as a starting point for the development of (partial) internal models.[10] In our paper, we use the latest version of the standard formula as outlined in the technical specifications of QIS5. National organizations, such as the German Insurance Association (GDV), develop own and more detailed interpretations of the standard formula in order to clarify ambiguities. These versions of the standard formula are usually calibrated and optimized for traditional products. Therefore, they cannot be used for product innovations such as hybrid insurance products. In the following, we highlight the main facts about the structure of the standard formula, as we use it to develop a partial internal model.[11] Since the current standard formula of QIS5 differs from the one of QIS4 only in few details and since the obligatory implementation of Solvency II is imminent we are confident that the final standard formula will be similar. In the standard

[9] See (The European Parliament, 2009, page 5) and (The European Parliament, 2009, page 12), see this document also for further general description of Pillar I.
[10] See Section 4 for more details.
[11] See CEIOPS (2010) for a detailed description.

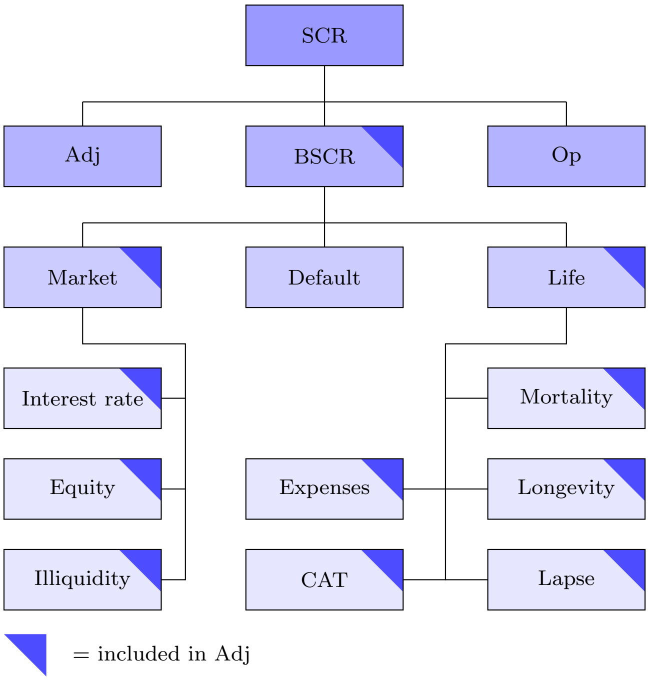

Fig. 3 SCR - modular structure

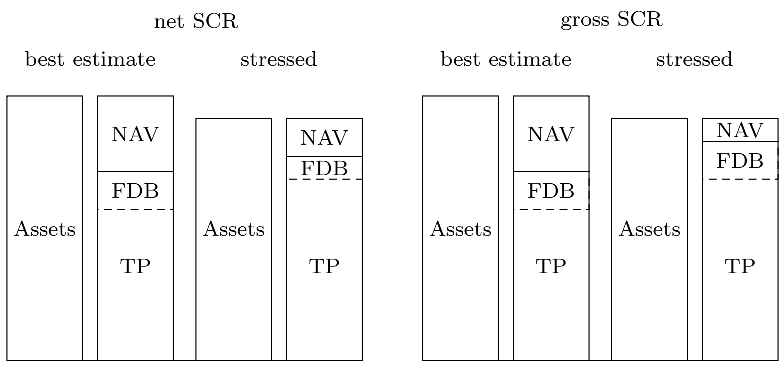

Fig. 4 Δ-NAV method

formula risks are categorized in modules such as the market module or the life underwriting module, and decomposed in submodules such as equity risk or mortality risk.[12] The submodules contain stress scenarios that are calibrated to a significance level of 99.5%. Since the stress scenarios and the correlation

[12] Fig. 3 shows the reduced modular structure (see CEIOPS (2010)) with the relevant (sub-)modules for hybrid products.

matrices[13] of the modules are provided by CEIOPS, the insurer is left with the task of calculating the SCR on the submodule level. For most submodules, the standard formula requires the Δ-NAV (Net Asset Value) approach (Fig. 4). The SCR is then defined as the difference of the NAV of a best estimate economic balance sheet and the NAV of a stressed economic balance sheet. This calculation must be performed twice, with the risk absorbing effect of future discretionary benefits to obtain the net SCR and without to obtain the gross SCR. In some cases, e.g. in case of the operational risk, the SCR is calculated using a factor formula.

4 Partial internal model

The Solvency II framework allows insurers to choose among three approaches in order to determine the solvency capital requirement: the standard formula, an internal model or a partial internal model. Since the GDV standard formula is based on a deterministic calculation and uses approximations for the values of the policyholders' options and guarantees that are calibrated on traditional insurance products, it is not recommended for SCR calculations for hybrid products. On the other hand, a full internal model requires the modeling of the insurance company as a whole: This task requires, amongst others, the determination of all correlations and of stochastic models for all risk factors. This is beyond the scope of our paper. A partial internal model allows us to extend the standard formula by relevant aspects only, namely a stochastic market model.[14]

Setup of the model For the deterministic parts of the partial internal model such as mortality, expenses or lapses, we use a set of best estimate parameters (see Section 5). The financial market model consists of stochastic models for the risky assets and interest rates. We also need management rules that determine managerial actions which are sensitive to different scenarios and the product model as described in Section 2. The product model contains all relevant parameters of the insurance policies and all relevant information about the insurers portfolio. With most of the cash flows being stochastic now, Monte-Carlo simulations are used to obtain an economic balance sheet and to determine the expected discounted value of the insurance portfolio (denoted by PVFP) which is the equivalent of the NAV. In order to obtain the solvency capital requirement, we use the stress scenarios as defined in the standard formula. They affect either the best estimate assumptions or the parameters of the market model. Again, we can use Monte-Carlo simulations in order to obtain the economic balance sheet and to determine the value of the insurance portfolio, now under the assumption that a stress occurs. Applying this procedure to every stress scenario of every relevant risk module and using the

[13] All risks are assumed to be jointly normally distributed.
[14] See CEIOPS (2010) and consulation papers no. 56, 65 and 80 (CEIOPS (2004-2010)) for further information on (partial) internal models.

Δ-NAV approach, the outcomes can be aggregated to the resulting SCR the same way as in the standard formula (see figure 5).

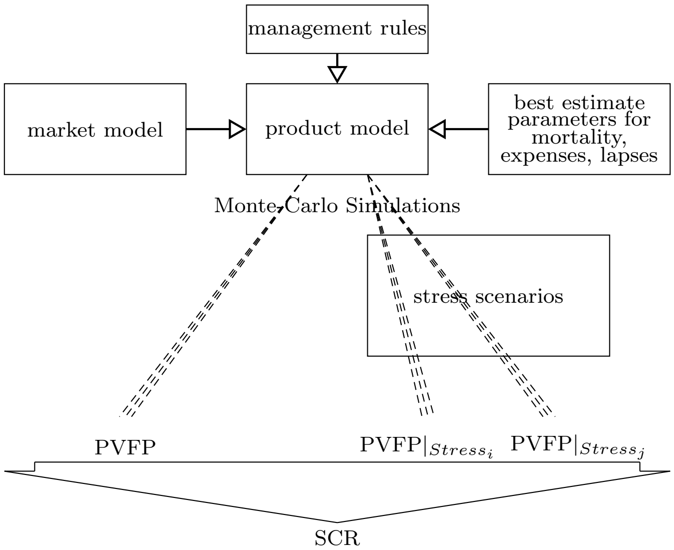

Fig. 5 Partial internal model

Management rules The partial internal model includes management rules for the asset management and profit sharing. The management controls the rebalancing of the account assets as specified in Section 2. The management also controls the asset composition of the premium reserve stock, more precisely, the equity exposure level and the bond investment strategies. The limit of the equity exposure level EE^{limit} is set to 35% by the German regulator but most insurers calculate with significantly lower limits. We define $\text{EE}^{\text{target}}$ as the targeted equity exposure level, the minimum level is naturally at 0%. Now let S_t denote the equity price at time t, then the equity exposure level after applying the management is described by

$$\text{EE}_{t+\frac{1}{12}} = \begin{cases} \min\left(\text{EE}^{\text{limit}}; \text{EE}_t\left(1+0.25\frac{S_{t+\frac{1}{12}}}{S_t}\right)\right) & \text{if } \text{EE}_t \geq \text{EE}^{\text{target}} \text{ and } \frac{S_{t+\frac{1}{12}}}{S_t} > 1 \\ \min\left(\text{EE}^{\text{limit}}; \text{EE}_t\left(1+0.75\frac{S_{t+\frac{1}{12}}}{S_t}\right)\right) & \text{if } \text{EE}_t < \text{EE}^{\text{target}} \text{ and } \frac{S_{t+\frac{1}{12}}}{S_t} > 1 \\ \text{EE}_t\left(\frac{S_{t+\frac{1}{12}}}{S_t}\right) & \text{if } 0.95 < \frac{S_{t+\frac{1}{12}}}{S_t} \leq 1 \\ \max\left(\text{EE}_t\left(2\cdot\frac{S_{t+\frac{1}{12}}}{S_t}-1\right); 0\right) & \text{if } \frac{S_{t+\frac{1}{12}}}{S_t} \leq 0.95. \end{cases}$$

The bond investment strategy addresses the managerial actions in case the insurer needs to buy bonds or sell them. We assume that the insurer will buy zero coupon bonds with a maturity of 5 years and sell bonds of all maturities according to their proportion of the portfolio. We already outlined the profit participation system in Section 2 but it is worth noticing that the profit participation rule for the investment profits leaves all losses to the insurer and most of the profits to the policyholder. Therefore, insurers aim to smooth those profits. We implemented two mechanisms that have a smoothing effect on profits. Firstly, profits are aggregated throughout the year and paid out at the end of the year. The aggregation allows to offset possible losses in one month by profits in another. Secondly, investment profits and losses are generated through changes of the book value of the premium reserve stock which is less volatile than the market value. Doing so, the calculation of investment profits also takes into account that hidden reserves are amortized while selling assets of the premium reserve stock. Furthermore, a friction of hidden reserves of the equity of the premium reserve stock is amortized monthly. This is done by simply selling and rebuying 2% of the equity every month and does only affect the book value of the equity not the market value. This approach to model the calculation of investment profits on a German premium reserve stock has been carried out by Schneeberger (2010) and Burkhart (2010).

Financial market The financial market model contains stochastic models for the short rate and for the evolution of equity. From these models, we can derive forward rates, bond prices, stock prices and the values of the equity and guarantee fund.
The Cox-Ingersoll-Ross model is used to model the short rate (see Shreve (2000) and Hull (2008)). Let lm denote the constant long run short rate, mrs the constant mean reversion speed, σ_r the volatility of the short rates and W_t^r a standard Brownian motion, then the stochastic differential equation that describes the short rate r_t is given by

$$dr_t = mrs\left(lm - r_t\right)dt + \sigma_r\sqrt{r_t}dW_t^r.$$

The discount rate D_t is then defined as

$$D_t = \exp\left(-\int_0^t r_u du\right).$$

The Cox-Ingersoll-Ross model also provides an analytical formula for the Bond price of a zero coupon Bond at time t and maturity T:

$$P\left(t,T\right) = A\left(T\right)e^{r_t B(T)} \qquad \text{with}$$

$$\begin{aligned} A\left(T\right) &= \left(\frac{2h\left(e^{(mrs+h)\frac{T}{2}}\right)}{2h + \left(mrs+h\right)\left(e^{Th}-1\right)}\right)^{2mrs\ lm/\sigma_r^2} \\ B\left(T\right) &= \frac{2\left(e^{Th}-1\right)}{2h + \left(mrs+h\right)\left(e^{Th}-1\right)} \\ h &= \sqrt{mrs^2 + 2\sigma_r^2}. \end{aligned}$$

Unfortunately, this model has no solution in closed form and we have to use a discretization scheme[15]. Despite this drawback (as opposed e.g. to the Vasiceck

[15] We use the Euler-Maruyama algorithm, see Alfonsi (2006).

model), the interest rates are always positive.
Now, let S_t denote the value of one share of the risky asset with a constant volatility σ, the risk-free short-rate r_t and a Brownian motion W_t (uncorrelated to W_t^r) under the risk-neutral measure at time t, then S_t solves the following stochastic differential equation (see Shreve (2000)):

$$dS_t = r_t S_t dt + \sigma S_t dW_t.$$

The equity fund is holding shares of the risky asset and charging a constant annual rate of management fees *mf*. Let S_t^{EF} denote the value of one share of the equity fund, then

$$dS_t^{\mathrm{EF}} = r_t S_t^{\mathrm{EF}} dt + \sigma S_t^{\mathrm{EF}} dW_t + \log(1 - mf)\, S_t^{\mathrm{EF}} dt$$

describes the equity fund. The equity fund is modeled as a continuous dividend paying share.[16]The analytical solution of this stochastic differential equation can be written as

$$\begin{aligned} S_{t+\frac{1}{12}}^{\mathrm{EF}} &= S_t^{\mathrm{EF}} \exp\left(\int_t^{t+\frac{1}{12}} \left(r_s - \frac{\sigma^2}{2} + \log(1 - mf) \right) ds + \int_t^{t+\frac{1}{12}} \sigma\; dW_s \right) \\ &= S_t^{\mathrm{EF}} \frac{S_{t+\frac{1}{12}}}{S_t} (1 - mf)^{\frac{1}{12}}. \end{aligned}$$

Kickbacks are paid by the equity fund management to the insurer and are financed by the equity fund management fees.[17] The monthly kickback payment per share is denoted by

$$\text{Kickbacks (per share)} = S_t^{\mathrm{EF}} \cdot \left(1 - (1 - kb)^{\frac{1}{12}} \right).$$

The guarantee fund is modeled similarly to the equity fund with the difference that the fund management also hedges the guarantee of a maximum loss of ml% per month by investing in put options.[18] The fund management adds the hedging costs to the constant rate of management fees *mf*. In order to determine the price of the put option, we use the extended Black-Scholes formula for option pricing (see Hull (2008)). The Black-Scholes formula requires the actual short rate and these calculations are rather time consuming. At a short rate of 0%, the put option price $P^{\mathrm{CP}}(K, T)$ with the strike price $K = xS_t$, with $x \in (0, 1]$ and maturity T, has a maximum. Therefore, we can safely simplify calculations by using a short rate of 0% for the pricing. Additionally we assume a volatility σ_{CP} that is higher than the volatility σ of the actual model. These assumptions lead to a price of the put option that is only dependent

[16] Paying negative dividends, see Shreve (2000).
[17] Therefore, the rate of kickbacks should be chosen to be smaller than the rate of investment fund management fees.
[18] A similar approach to model a guarantee fund has also been used by the DAV-Arbeitsgruppe "Bewertung von Garantien" (2010).

on the current price of the equity share and can be interpreted as a prudent estimate:

$$\begin{aligned} P^{\mathrm{CP}}\left(xS_t, \frac{1}{12}\right) &= S_t\left(x\,\Phi\left(-d_2\right) - e^{-\frac{1}{12}\log(1-mf)}\Phi\left(-d_1\right)\right) \qquad \text{with} \\ d_1 &= \frac{\log\frac{1}{x} + \frac{1}{12}\frac{1}{2}\sigma_{\mathrm{CP}}^2 - \frac{1}{12}\log\left(1-mf\right)}{\sigma_{\mathrm{CP}}\sqrt{\frac{1}{12}}}, \\ d_2 &= d_1 - \sigma_{\mathrm{CP}}\sqrt{\frac{1}{12}}. \end{aligned}$$

Now, the price of a share of the guarantee fund can be expressed as follows:

$$S_{t+\frac{1}{12}}^{\mathrm{GF}} = S_t^{\mathrm{GF}} \cdot \max\left(1 - \mathrm{ml}; \frac{S_{t+\frac{1}{12}}}{S_t + P^{\mathrm{CP}}}\left(1-mf\right)^{\frac{1}{12}}\right).$$

Again, the insurer receives kickbacks:

$$\text{Kickbacks (per share)} = S_t^{\mathrm{GF}} \cdot \left(1 - (1-kb)^{\frac{1}{12}}\right).$$

In our model the insurer is using an actuarial interest rate i_0, which is also the guaranteed interest on the premium reserve stock. The parameter settings can be obtained from Table 6 (Appendix B).

Economic balance sheet In order to calculate the SCR, we need to calculate a stochastic economic balance sheet (see Fig. 6) for the best estimate and every stress scenario. The values of the assets are simply expressed by the current market values. The value of the liabilities result from stochastic simulations under the risk neutral measure. The PVFP, the insurance benefits and the management fees equal the average of the corresponding sum of the discounted cash flows. The PVFP includes all positive and negative cash flows to the insurer. The insurance benefits include death benefits, annuity benefits, surrender benefits (net of lapse fees), and lump-sum payments. The balance sheet item "management fees" includes all fees that are deducted by the equity and the guarantee fund management net of kickbacks. The balance sheet item "insurance benefits" can be broken down into benefits that result from profit participation (FDB[19]) and the value of options and guarantees (O&G) and other insurance benefits. For all net SCR calculations, the value of the FDB is again determined by the average of the corresponding sum of the discounted cash flows. For gross SCR calculations, the FDB under a stress scenario is required to equal the best estimate FDB (see Fig. 3). There are several approaches to interpret this equality[20], we opted for the (easily to implement) present value approach and therefore define:

$$\begin{aligned} \mathrm{FDB}_{\text{stressed}}^{\text{gross}} &= \mathrm{FDB}_{\text{stressed}}^{\text{net}} + \max\left(\mathrm{FDB}_{\mathrm{BE}} - \mathrm{FDB}_{\text{stressed}}^{\text{net}}; 0\right) \qquad \text{and} \\ \mathrm{PVFP}_{\text{stressed}}^{\text{gross}} &= \mathrm{PVFP}_{\text{stressed}}^{\text{net}} - \left(\mathrm{FDB}_{\text{stressed}}^{\text{gross}} - \mathrm{FDB}_{\text{stressed}}^{\text{net}}\right). \end{aligned}$$

[19] Future discretionary benefits, see CEIOPS (2010) for a detailed definition.

[20] See consultation paper no. 56 CEIOPS (2004-2010) for a discussion of methods on how to calculate the FDB.

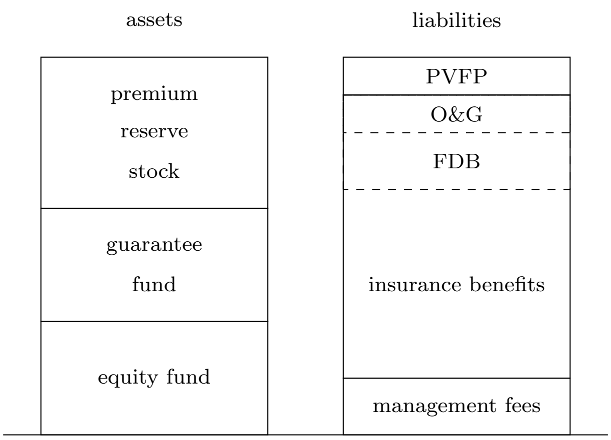

Fig. 6 Economic balance sheet

The value of the O&G is defined as

$$\text{O\&G} = \text{PVFP}^{\text{CE}} - \text{PVFP}^{\text{net}}$$

where PVFP^{CE} denotes the PVFP under the certainty equivalent scenario.[21] Note, that we use a MCEV type approach to determine the value of O&G[22] which differs from the approximation formulas as defined in the GDV standard formula.

Partial internal models often require some interpretation of the standard formula stress scenarios. Since our model is based on a stochastic model for the financial market, we have to define equity stress, interest rate stress, illiquidity premium stress and default stress scenarios according to the stress scenarios of the standard formula. The equity stress is performed immediately and as a whole at the level of the standard formula in the first month. The interest rate stresses and the illiquidity premium stress affect the parameters of the interest rate model and the resulting bond prices.[23] In case of a default of the management of the guarantee fund, the insurer is obligated to close the gap. In this case, we did not adjust the future profit participation for simplicity. In order to assure the validity of our model, we tested our financial market model with the martingale test and our economic balance sheets with the leakage test.[24]

In spite of the elaborate design, the partial internal model has some shortcomings. The complexity of the nature of the German premium reserve stock and the associated management rules increases the model uncertainty. Since the results of the partial internal model strongly depend on the undelying assumptions regarding the premium reserve stock, changes of these assumptions can

[21] See CFO Forum (2009a) for a detailed definition of the certainty equivalent scenario.
[22] Therefore, O&G represents the time value of options and guarantees (see CFO Forum (2009b) and CFO Forum (2009a) for further information).
[23] See Appendix B for details on the calibration of the CIR-model.
[24] See Burkhart (2010) for a description of the martingale test and the leakage test.

have a significant impact. Therefore, in the process of implementing a (partial) internal model it is a major task to identify and recognize managerial actions. Furthermore, our partial internal model does not sufficiently address the risk of illiquid assets. The concept of dynamic hybrid products is based on a rebalancing algorithm, that stipulates major asset sales during or directly after a market distress. The correspondent market risk stress scenarios might underestimate the actual risks, since such asset sales could lead to a market breakdown, given a sufficient large market share of those products.

5 Numerical results

In this section, we present and analyse results from several simulation studies. First, we specify the analysis assumptions in Section 5.1. In Section 5.2, we examine a portfolio of new business in order to aquire unbiased information about the risk structure of hybrid insurance products. We also neglect the pension period assuming all policyholders to decide for the lump sum option at the end of the accumulation period. In Section 5.3 we analyze fictitious sample portfolios that are in force for 2 and 7 years. Finally, in Section 5.4 we perform a sensitivity analysis with respect to the crucial model parameters. Throughout our analyses, we focus on the key indicators PVFP, SCR and SCR ratio[25] as well as on the economic balance sheets items FDB and O&G.

5.1 Historical data and parameter assumptions

For the simulation study, we assume homogeneous portfolios of 5000 policies and all policyholders to be male and 30 years old at the beginning of the policy. We use German mortality tables DAV (2008) to determine prudent and best estimate mortality during the accumulation period. Deaths are assumed to be uniformly distributed over the year. Lapses are assumed to be only dependent on the policy year, the corresponding annual lapse rates can be found in Table 5 (Appendix A). The fixed unit expenses are assumed to increase at an annual cost inflation rate *infl* ($\mathrm{UE}_0 = \mathrm{P} \cdot \frac{\mathrm{ue}}{2}$ at $t = 0$ and $\mathrm{UE}_t = \frac{\mathrm{P}}{12 \cdot T} \cdot \frac{\mathrm{ue}}{2} \cdot (1 + \mathit{infl})^{\frac{t}{12}}$ every other month). Parameter assumptions can be found in Table 4 in Appendix A.

We use historical financial market data to generate existing portfolios. For this purpose, we obtained a series of short term interest rates from the Bundesbank-Database[26] and Bloomberg data for 5-year German interest rates[27] as well as the DAX index. Since the DAX index represents the historical development of the risky asset, we obtain historical data of the equity fund and the guarantee fund by applying the financial market model as outlined in Section 4. Fig. 5.1 shows the historical financial market data.

[25] Defined as SCR ratio $= \frac{\mathrm{PVFP}^{\mathrm{BE}}}{\mathrm{SCR}}$.

[26] See SU0101 (2010).

[27] For the interest rates, we used the GDBR5 index (synthetic German zero-coupon bonds).

The results in 5.2 and 5.3 are based on 100,000 Monte-Carlo simulation paths while the results in 5.4 are based on 1,000 Monte-Carlo simulation paths.

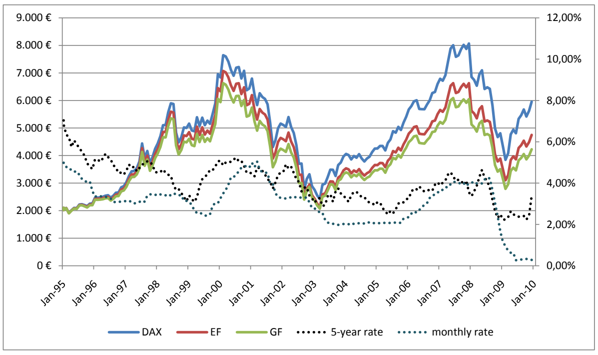

Fig. 7 Historical financial market data

5.2 Numerical results for new business

The results of the simulation study of a portfolio of new business are presented in columns "SP 0" of table 1. A modular representation of the composition of the respective SCR is shown[28] in Fig. 8. Some relevant economic balance sheets can be found in Appendix C (Fig. 21 and Fig. 22). More data is available upon request. The initial single premium is invested according to the product specificatons. The accumulation period of 35 years implicates a rich compound interest rate effect, therefore both products can invest strongly in risky assets. Almost half of the account of the static hybrid is invested in the equity fund, the other half is invested in the premium reserve stock. The 3-pot dynamic hybrid account is split into the guarantee fund to two thirds and the equity fund to one third.

Table 1 Numerical results - new business (SP 0) and sample (SP 2/7) portfolios

	static hybrid			3-pot dynamic hybrid hybrid		
	SP 0	SP 2	SP 7	SP 0	SP 2	SP 7
PRS-Bonds	225.89 mln €	185.16 mln €	150.05 mln €	0.00 mln €	100.39 mln €	0.00 mln €
PRS-Equity	24.64 mln €	44.75 mln €	36.01 mln €	0.00 mln €	24.97 mln €	0.00 mln €
GF	0.00 mln €	0.00 mln €	0.00 mln €	308.60 mln €	116.84 mln €	214.01 mln €
EF	221.05 mln €	130.13 mln €	278.70 mln €	158.87 mln €	0.00 mln €	304.12 mln €
PVFP (BE)	19.17 mln €	14.58 mln €	19.69 mln €	24.66 mln €	10.21 mln €	24.82 mln €
O&G (BE)	6.14 mln €	4.15 mln €	3.83 mln €	0.75 mln €	2.09 mln €	0.25 mln €
FDB (BE)	68.41 mln €	68.90 mln €	67.98 mln €	40.81 mln €	41.53 mln €	33.54 mln €
TP (BE)	358.00 mln €	257.40 mln €	344.08 mln €	337.68 mln €	162.93 mln €	377.87 mln €
MF (BE)	20.16 mln €	15.18 mln €	28.07 mln €	64.34 mln €	25.61 mln €	80.17 mln €
SCR	15.98 mln €	15.31 mln €	15.35 mln €	11.92 mln €	14.62 mln €	11.05 mln €
SCR ratio	1.20	0.95	1.28	2.07	0.70	2.25

[28] The boxes show (top - middle - bottom): (sub)module - gross SCR - net SCR.

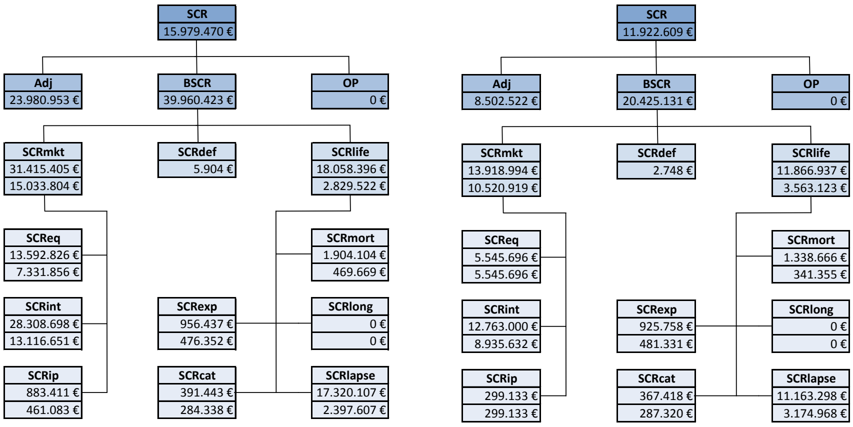

Fig. 8 SCR - static and 3-pot dynamic hybrid (new business portfolio)

The annual guaranteed interest rate and the profit participation system (investment profits) of the premium reserve stock have huge impact on the O&G and the FDB of the static hybrid, while the dynamic hybrid includes enormous management fee payments. Overall, the PVFP of the dynamic hybrid product significantly exceeds the PVFP of the static hybrid. Both products are primarily exposed to market risks. Again, the annual guaranteed interest rate of the premium reserve stock has a heavy impact as it increases the market risk of static hybrid products. The relevant interest rate stress scenario is the down-scenario throughout all simulation studies in this paper. The only relevant underwriting risk is the massive lapse scenario. The portfolio of static hybrid products requires about 30% more SCR than the dynamic hybrid product portfolio, while having about 20% less PVFP and therefore a much lower SCR ratio of 1.14 compared to 2.07. In some market stress scenarios, the stressed value of the FDB for dynamic hybrid products exceededs the best estimate FDB. This effect is due to a high investment in the premium reserve stock during a market stress and the implicated profit participation on investment returns. Unfortunately, this leads to a gross SCR lower than the net SCR. We prohibited this by setting the net SCR as a lower bound of the gross SCR. In the mortality and lapse submodules the gross SCR significantly differs from the net SCR. Both scenarios implicate a massive reduction of business in force and therefore also a reduction of the FDB. This is mainly not an effect of the loss absorbing capacity of technical provisions. In our model, the SCR calculations of the operational risk module use expenses that have been experienced in the past as a reference. Therefore, the operational risk is zero for new business.

5.3 Numerical results for sample portfolios

At first, we begin with a fictitious portfolio that is in force for seven years with underlying historical data from 2003 to 2010 (Fig. 5.1). That period is characterized by relatively low interest rates, a stock market rally followed by a financial crisis with a beginning recovering. Overall, stock markets never fall below the starting value. Fig. 9 shows the impact of the last financial crisis as the account value drops by a half for dynamic hybrid products. The account value of the static hybrid is affected less severely. Fig. 9 also shows that no rebalancing is needed during this period.

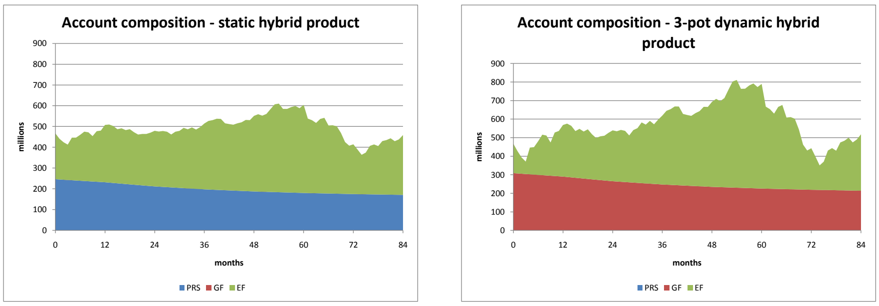

Fig. 9 Account composition - historical data (2003 to 2010)

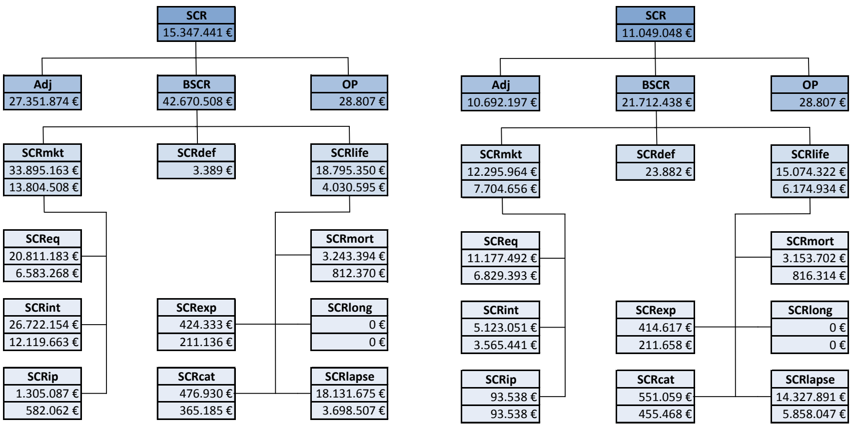

Fig. 10 SCR - static and 3-pot dynamic hybrid (7 years in force)

The results of the simulation study are presented in columns "SP 7" of table 1 while the modular representation of the composition of the respective

SCR is shown in Fig. 10. The structure of the composition of the account assets remains similar to the new business portfolios, the portion of risky assets increased through all products. The dynamic hybrid outperformed the static hybrid comparing the account value. The PVFP, most of the liabilities and the SCR are similar compared to the new business portfolios.

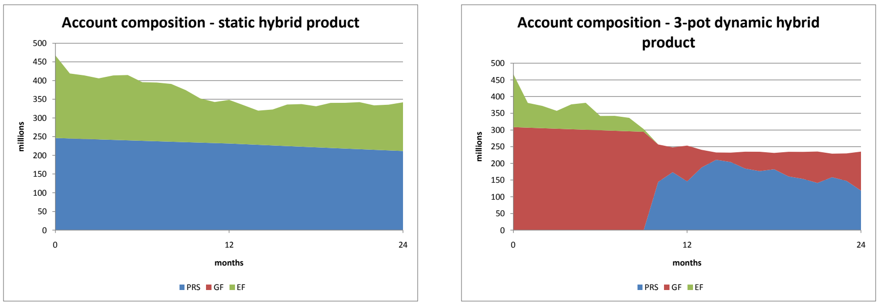

Fig. 11 Account composition - historical data (2008 to 2010)

The value of the O&G decreased significantly for both products, the value of the management fees increased. While the structure of the SCR remained almost unchanged for the static hybrid, it changed for the dynamic hybrid. Market risks are less dominant since the probability of an investment in the premium reserve stock is lower while the SCR for lapse risk increased. The massive lapse of policyholders implies a loss of future profits that result from kickbacks. Overall, the SCR ratio of the static hybrid increased to 1.28 while the SCR ratio of the dynamic hybrid product increased to 2.25. Note that we used the assumption that all profits earned by the insurer are distributed to the shareholders immediately.

The second sample portfolio analysis is performed on market data from the last financial crisis (2008-2010). This period is characterized by a massive fall of stock markets followed by a beginning recovering. Interest rates are very low, close to the guaranteed annual interest rate of 2.25%. This period was critical for most insurers since both, stock markets and interest rates, were falling. Fig. 11 shows that both products deal with extreme decline of the account values. Here, the dynamic hybrid product takes the biggest toll since the account is composed by only risky assets at the beginning of the crisis. The dynamic hybrid has to be rebalanced shifting most of the account to the premium reserve stock. Again, results can be obtained from table 1, columns "SP 2". The PVFP of the static hybrid drops by 24% compared to the new business portfolio and the O&G, TP and MF decrease significantly. The decline of the account value implies a decline of policyholders benefits.

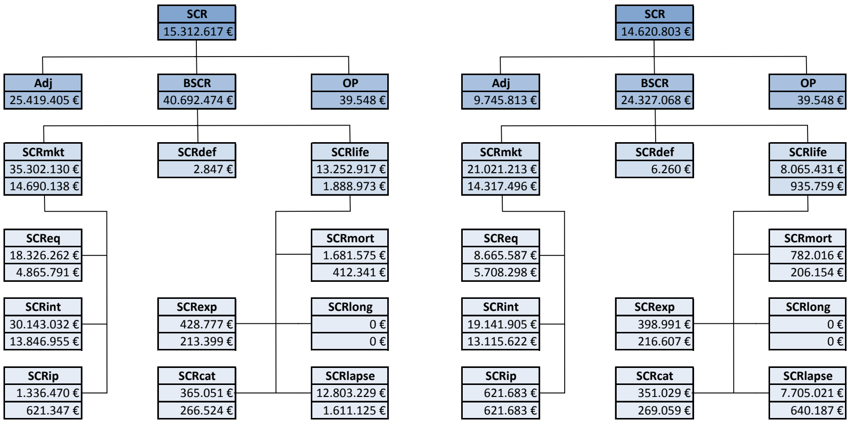

Fig. 12 SCR - static and 3-pot dynamic hybrid (2 years in force)

The changes of the results of the static hybrid product are mostly due to the losses in the equity fund value. The results of the 3-pot dynamic hybrid product show an extreme drop of the PVFP by 59%. Because of the shift to the premium reserve stock the O&G increased by 277%. Insurance benefits decline similar to the static hybrid. The SCR of both products is almost equal now as a result of a strong increase of the dynamic hybrid SCR (23%). Both products have very low SCR ratios of about 0.95 and 0.70. Fig. 12 shows that the composition of the SCR is similar in both products, with only the market risks (especially the interest rate risk) being relevant.

5.4 Sensitivity analysis

The results from above analyses indicate that the solvency capital requirement of both, static and dynamic hybrid products, depends strongly on the developments of the account values. In case of dynamic hybrid products, the solvency capital requirement is also very sensitive to past rebalancing actions.

Table 2 Sensitivity analysis - parameters

sensitivity analysis	7 year sample portfolio		2 year sample portfolio		volatility analysis		actuarial interest rate analysis	accumulation period analysis	
parameter	5-year rate (p.a.)	stock index rate (p.a.)	5-year rate (p.a.)	stock index rate (p.a.)	σ_r	σ	i_0	T	policyholder age
range	$[0\%; 5\%]$	$[-10\%; 10\%]$	$[0\%; 5\%]$	$[-30\%; 30\%]$	$[0.1; 0.3]$	$[0.0125; 0.0375]$	$[1\%; 3.5\%]$	$[15; 45]$	$[50; 20]$
increments	0.5%	1%	0.5%	1%	0.02	0.0025	0.25%	3 y	3 y

These dependencies can be revealed by means of a sensitivity analysis. Therefore, we determine the solvency capital requirement of 7 year and 2 year sample portfolios subject to different fictitious historical market data. The parameters used can be found in Table 2. All historic interest rates and the historic stock index rate of return are assumed to be constant. The historical monthly interest rate is set to half of the 5-year interest rate. The interest rates for all other maturities are obtained performing a linear interpolation between the 5-year inerest rate and the monthly interest rate.

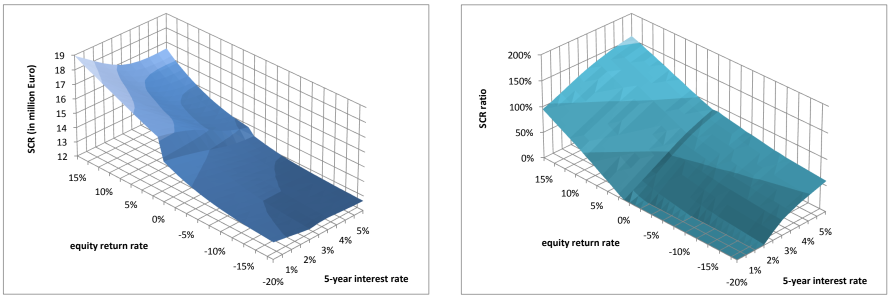

Fig. 13 SCR and SCR ratio - static hybrid - sample portfolios (7 years in force)

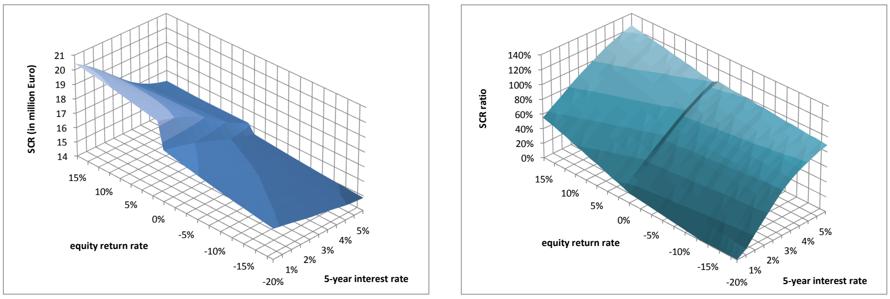

Fig. 14 SCR and SCR ratio - static hybrid - sample portfolios (2 years in force)

The results of the partial internal model depend on many parameters. Out of these, the market volatility, the actuarial interest rate and the length of the accumulation period are supposed to have a significant influence on the solvency capital requirement of a new business portfolio. In case of the volatility analysis, the volatility of both, the interest rates and the stock market, evolve simultaneously and proportionally. In case of the accumulation period analysis, we also adjust the age of the policyholders in order to let the accumulation period end at the age of 65.

The results (SCR and SCR ratio) of the sensitivity analysis with respect to different market developments for 7 and 2 years in force static hybrid portfolios are illustrated in Fig. 13 and Fig. 14. These figures display that both, the SCR and the SCR ratio, are influenced by the equitiy return rates as well as by interest rates. Higher return rates lead to a higher SCR but also a higher SCR ratio since they have a stronger impact on the PVFP. High interest rates reduce the SCR and lead to higher SCR ratios. The buckle at the equity return rate of 0% and −1% is a consequence of the asset management rules (asset composition) of the premium reserve stock.

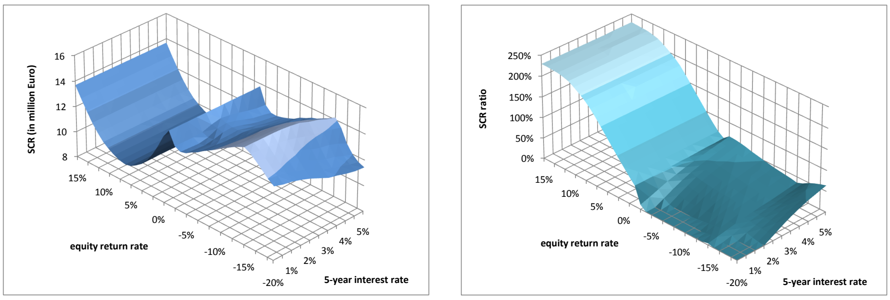

Fig. 15 SCR and SCR ratio - 3-pot dynamic hybrid - sample portfolios (7 years in force)

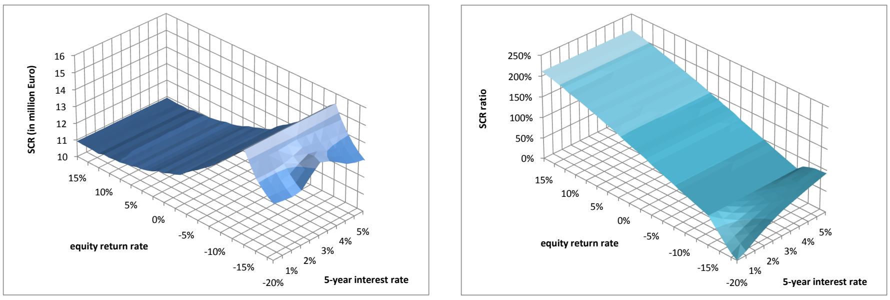

Fig. 16 SCR and SCR ratio - 3-pot dynamic hybrid - sample portfolios (2 years in force)

The results (SCR and SCR ratio) of the outlined sensitivity analysis for dynamic hybrid portfolios are illustrated in Fig. 15 and Fig. 16. For most scenarios, decreasing equity return rates induce increasing SCRs and strongly decreasing SCR ratios while the interest rates have only negligible impact. Decreasing equity return rates come with an increasing probability of rebalancing into the premium reserve stock. This result demonstrates the pro-cyclical nature of dynamic hybrid products. For equity return rates less than −3% p.a. (for 7 years in force) and less than −15% p.a. (for 2 years in force), the portfolio starts to be rebalanced into the premium reserve stock. Since the rebalancing has a severe impact on the policy account's assets, the nature of the SCR and SCR ratio of dynamic hybrids quickly resembles the one of static hybrids.

Overall, the SCR of static hybrids exceeds the SCR of dynamic hybrids significantly in most of the analyzed scenarios while the SCR ratios are lower for most scenarios. The SCR ratio of static hybrid products exceeds the SCR ratios of dynamic hybrids only in extreme scenarios.[29] The results show that the SCR ratios of dynamic hybrid products are more volatile than those of static hybrids.

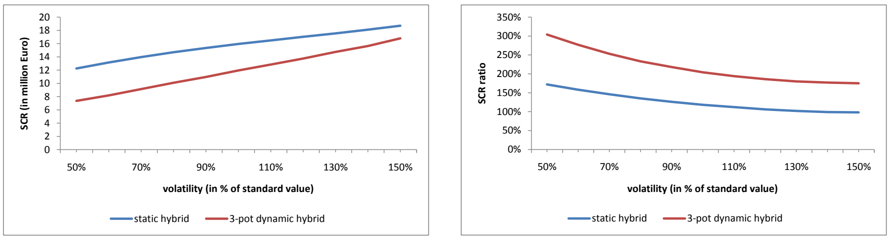

Fig. 17 SCR and SCR ratio - volatility analysis - new business portfolios

Figures 17, 18 and 19 show the SCR and the SCR ratio for new business portfolios of static and dynamic hybrid products dependent on the volatilities of interest rates and equity, the guaranteed interest rate and the length of the accumulation period. Not surprisingly, high volatilities lead to a high SCR and a low SCR ratio. The difference of both products decreases with increasing volatilities. The new business portfolio of static hybrid products induces a lower SCR and a higher SCR ratio than the dynamic hybrid product portfolio in all cases.
The SCR of both products is also increasing with an increasing guaranteed interest rate, while the SCR ratios decrease. The difference between the static and the dynamic hybrid product decreases with a decreasing guaranteed interest rate. Again, the new business portfolio of static hybrid products induces a lower SCR and a higher SCR ratio than the dynamic hybrid product portfolio in most cases. For a guaranteed interest rate of 1% and below, the SCR and SCR ratio of a static hybrid portfolio is lower and higher respectively than the SCR and SCR ratio of a dynamic hybrid portfolio as a result of a low interest rate risk in these scenarios.
Finally, the accumulation period length has a major influence on the SCR and the SCR ratio of both products. The SCR of a static hybrid product portfolio is increasing with an increasing accumulation period length while the SCR of a dynamic hybrid product portfolio is decreasing. This results in a higher SCR for dynamic hybrids for an accumulation period of 21 years and less. The SCR ratios decrease both for an decreasing accumulation period length, while the difference between both ratios is decreasing, too. The SCR ratio of the

[29] 7 years in force: scenarios with constantly negative equity return rates. 2 years in force: scenarios with interest rates lower or equal to 2.25% p.a. and equity return rates lower than −20% p.a.

dynamic hybrid product portfolio exceeds the SCR ratio of the static hybrid product portfolio for all analyzed scenarios.

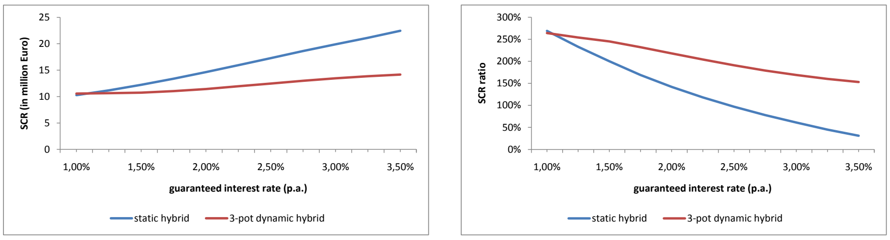

Fig. 18 SCR and SCR ratio - actuarial interest rate analysis - new business portfolios

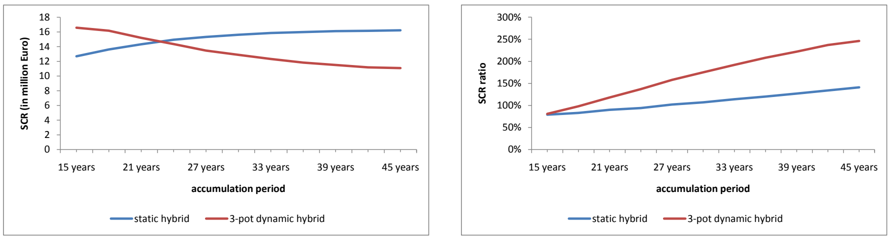

Fig. 19 SCR and SCR ratio - accumulation period analysis - new business portfolios

6 Conclusions

In this paper, we present a partial internal model based on the Solvency II modular formula to assess the solvency capital requirement for static and dynamic hybrid insurance products. The SCR is calculated using risk-neutral valuation methods similar to the MCEV approach. In the partial internal model, the economic balance sheets are derived using a stochastic financial market model. Therefore, we think that our model is superior to the GDV approach which is deterministic and not calibrated to innovative life insurance products.

We use this model to perform various simulation studies. The corresponding results reveal that market and lapse risks dominate. Among market risks, the interest rate risk, more precisely, the downward shift of the interest rate term structure, is the most important risk. The only relevant underwriting risk is the lapse risk (a massive lapse scenario). This result is also in line with the general results from QIS4 and QIS5 (see CEIOPS (2008) and CEIOPS (2011)). Furthermore, the results reveal that the SCR ratio of dynamic hybrid products is strongly volatile. The rebalancing of dynamic hybrid products has a pro-cyclical effect. After a stage of rising stock markets, the account of dynamic hybrid products does not include an investment in the premium reserve stock. The SCR, and in particular the interest rate risk, is low and the product has a more linear structure since the value of options and guarantees is also low. In this case, the SCR and its strucure of dynamic hybrid products are comparable to pure unit-linked products (see Kochanski (2010a)). After a crisis on the stock markets, the accounts of dynamic hybrid products have a high probability of beeing rebalanced or already consist for a large part of the premium reserve stock. The SCR and the interest rate risk and also the value of options and guarantees are very high. In contrast to dynamic hybrid products, static hybrid products are not rebalanced, the investment in the premium reserve stock does not change. Therefore the SCR of static hybrid products is almost always at a high level.

As a conclusion, we think, that our model is of interest for small and mid-sized insurers that have not enough capacities to design a full internal model. The results are important for product designers as well as risk managers. Our analysis can be extended in various ways, for example to other premium paying types and a dynamic pension period. The model is also useful to test more sophisticated and risk reducing management rules as well as smoothing mechanisms for the asset management of the premium reserve stock. As the issue of dynamic policyholder behavior is of concern, our model is also useful to analyze the impact of various dynamic lapse models and models for the lump sum option. Furthermore, we modeled the premium reserve stock without the presence of a portfolio of traditional insurance products. Often, only one premium reserve stock is used for all products. Finally, our model and the results of the simulation studies can be used to derive approximation formulas in order to avoid stochastic simulation techniques.

A Parameter assumptions

The following tables list the values of the parameters used in the model.

Table 3 Product parameters

Parameter	Description	Value
P	single premium payment	100,000 €
T	accumulation period length (years)	35
sf	surrender fee parameter	4%
ac	aquisition charges parameter	4%
uc	accumulation period UC parameter	5%
ipr	investment participation rate	90%
mpr	mortality participation rate	75%
opr	other participation rate	50%

Table 4 Portfolio parameters

Parameter	Description	Value
ue	accumulation period expense parameter	2%
$infl$	cost inflation rate (p.a.)	0.7%

Table 5 Lapse rate table

policy year	lapse rate (p.a.)	policy year	lapse rate (p.a.)
1	7.80%	21	2.34%
2	10.40%	22	2.21%
3	8.45%	23	2.21%
4	7.02%	24	2.08%
5	5.72%	25	1.95%
6	4.81%	26	1.82%
7	4.16%	27	1.69%
8	3.64%	28	1.69%
9	3.25%	29	1.56%
10	2.99%	30	1.43%
11	2.73%	31	1.30%
12	2.60%	32	1.17%
13	2.47%	33	0.94%
14	2.47%	34	0.62%
15	2.47%	35	0.00%
16	2.47%		
17	2.47%		
18	2.47%		
19	2.34%		
20	2.34%		

B Financial market model parameter assumptions

The following tables list the values of the parameters used in the financial market model. The interest rate term structure obtained from the CIR model should approximate the QIS5 Government interest rate term structures. Therefore, CIR-parameters have been set using the least squares optimization method.

Table 6 Financial market model parameters

Parameter	Description	Value
EE^{limit}	limit of the equity exposure level	15%
EE^{target}	targeted equity exposure level	10%
mrs	mean reversion speed (of r_t)	see Table 7
lm	longterm mean (of r_t)	see Table 7
σ_r	volatility of r_t (p.a.)	2.5%
σ	volatility of S_t (p.a.)	20%
mf	management fee (of EF and GF p.a.)	1%
kb	kickback rate p.a.	0.5%
σ_{CP}	volatility used for put option pricing (p.a.)	40%
i_0	initial actuarial interest rate (p.a.)	2.25%

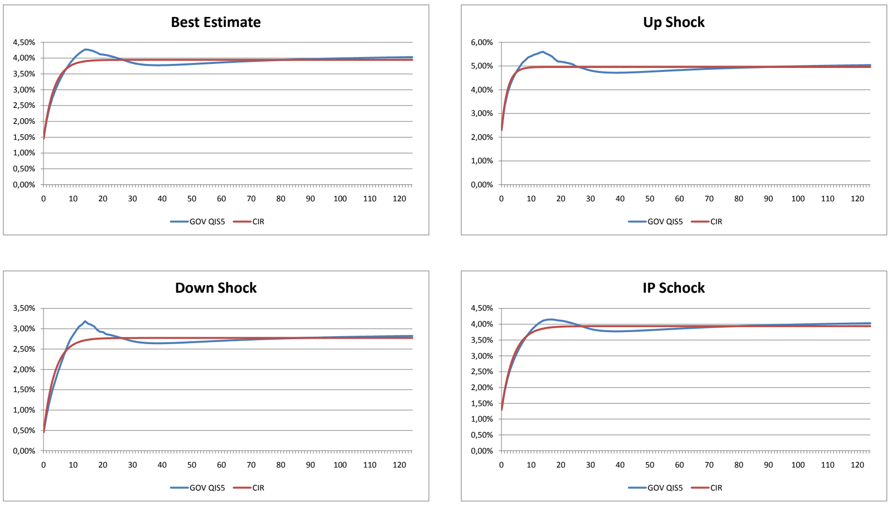

Fig. 20 QIS5 Government Curve vs. CIR (0 to 120 years)

Table 7 Cox-Ingersoll-Ross parameters

Parameter	mrs	lm
Best estimate	0.283256	0.038703
Up-shock	0.487659	0.048417
Down-shock	0.263057	0.027354
IP-shock	0.249183	0.038638

C Economic balance sheets for new business

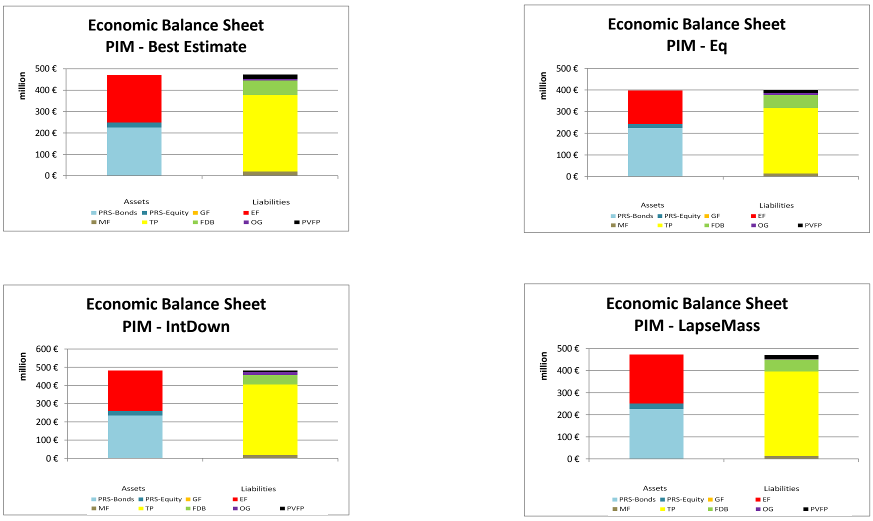

Fig. 21 Economic balance sheets - static hybrid (BE and Eq, Int and Lapse)

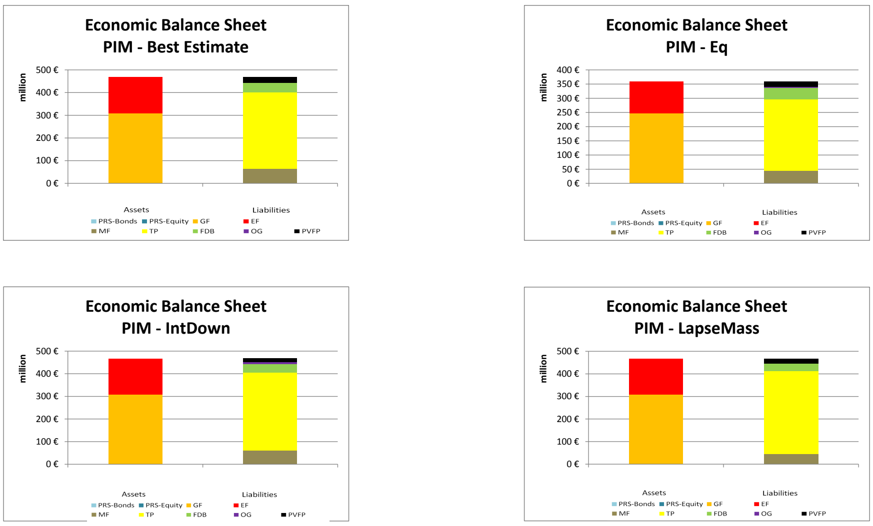

Fig. 22 Economic balance sheets - 3-pot dynamic hybrid (BE and Eq, Int and Lapse)

References

Alfonsi A (2006) On the discretization schemes for the CIR (and bessel squared) processes. Preprint URL http://cermics.enpc.fr/~alfonsi/SC_preprint.pdf

Bauer D, Kiesel R, Kling A, Ruß J (2006) Risk-neutral valuation of participating life insurance contracts. Insurance: Mathematics and Economics 39(2):171–183

Bauer D, Kling A, Ruß J (2008) A universal pricing framework for guaranteed minimum benefits in variable annuities. ASTIN Bulletin 38(2):621–651

Bauer D, Bergmann D, Reuß A (2009) Solvency II and nested simulations: a least-squares monte carlo approach. Preprint URL http://www.cb.wsu.edu/aria2009/ARIA2009Papers/Full%20Papers/session6D_Bauer_Bergmann_Reuss.pdf

Bauer D, Bergmann D, Reuß A (2010) On the calculation of the solvency capital requirement based on nested simulations. Preprint

Brennan MJ, Schwartz ES (1976) The pricing of equity-linked life insurance policies with an asset value guarantee. Journal of Financial Economics 3(3):195–213

Börger M (2010) Deterministic shock vs. stochastic value-at-risk - an analysis of the Solvency II standard model approach to longevity risk. Preprint URL http://www.uni-ulm.de/fileadmin/website_uni_ulm/mawi/forschung/PreprintServer/2009/Solvency_Longevity_01.pdf

Burkhart T (2010) Implementierung und Untersuchung eines Modells für den MCAV, Bachelor thesis, Ulm University

CEIOPS (2004-2010) Consultation papers. URL https://eiopa.europa.eu/consultations/consultation-papers/index.html

CEIOPS (2008) QIS4 Report. URL https://eiopa.europa.eu/consultations/qis/quantitative-impact-study-4/index.html

CEIOPS (2010) QIS5 technical specification. URL https://eiopa.europa.eu/consultations/qis/quantitative-impact-study-5/index.html

CEIOPS (2011) QIS5 Report. URL https://eiopa.europa.eu/consultations/qis/quantitative-impact-study-5/index.html

CFO Forum (2009a) Market Consistent Embedded Value basis for conclusions. URL http://www.cfoforum.eu/downloads/CFO_Forum_MCEV_Basis_for_Conclusions.pdf

CFO Forum (2009b) Market Consistent Embedded Value principles. URL http://www.cfoforum.eu/downloads/MCEV_Principles_and_Guidance_October_2009.pdf

Christiansen M, Denuit M, Lazar D (2010) The Solvency II square-root formula for systematic biometric risk. Preprint URL http://www.uni-ulm.de/fileadmin/website_uni_ulm/mawi/forschung/PreprintServer/2010/MC_SqRootF.pdf

Daalmann S, Bause C (2011) FLV-Update 2010. Towers Watson, URL http://www.towerswatson.com/germany/press/4495

Daalmann S, Märten G (2009) Fondsgebundene Versicherungsprodukte mit Garantien trotzen der Finanzmarktkrise. Towers Perrin, URL http://www.towersperrin.com/tp/showdctmdoc.jsp?country=deu&url=Master_Brand_2/DEU/Press_Releases/2009/20090511/2009_05_11.htm

Daalmann S, Märten G (2010) FLV-Update 2009. Towers Watson, URL http://www.towerswatson.com/germany/press/1812

DAV (2008) Herleitung der Sterbetafel DAV 2008 T für Lebensversicherungen mit Todesfallcharakter. URL http://www.aktuar.de/download/dav/veroeffentlichungen/2008-12-04-DAV-2008-T.pdf

DAV-Arbeitsgruppe "Bewertung von Garantien" (2010) Bewertung von Fondsgebundenen Rentenversicherungen mit Garantiefonds und Variable Annuities

Deichl W (2008) Planbarkeit, Sicherheit und Rendite. VW 15:1265–1269

Devineau L, Loisel S (2010) Risk aggregation in Solvency II: How to converge the approaches of the internal models and those of the standard formula? Preprint URL http://hal.archives-ouvertes.fr/docs/00/40/85/35/PDF/Devineau-Loisel-RiskAggr-30072009.pdf

Doff R (2008) A critical analysis of the Solvency II proposals. The Geneva Papers 33:193–206

Duverne D, Le Douit J (2009) IFRS Phase II and Solvency II: Key issues, current debates. The Geneva Papers 34:47–55

Elderfield M (2009) Solvency II: Setting the pace for regulatory change. The Geneva Papers 34:35–41
Eling M, Schmeiser H, Schmit J (2007) The Solvency II process: overview and critical analysis. Risk Management and Insurance Review 10:69–85
Ferber K (2010) Optionen in dynamischen Hybridprodukten, Bachelor thesis, Ulm University
Fix W, Käfer I (2008) Dynamische Hybridprodukte - eine neue Produktfamilie mit viel Potential. VW 3:172–176
Grosen A, Jorgensen P (2000) Fair valuation of life insurance liabilities: The impact of interest rate guarantees, surrender options, and bonus policies. Insurance: Mathematics and Economics 26:37–57
Hammers B (2009) Full Fair Value-Bilanzierung von Lebensversicherungsprodukten und mögliche Implikationen für die Produktgestaltung, vol 54. Versicherungswirtschaft
Herr H, Kreer M (1999) Zur Bewertung von Optionen und Garantien bei Lebensversicherungen. Blätter der DGVFM 24(2):179–193
Holzmüller I (2009) The United States RBC Standards, Solvency II and the Swiss Solvency Test: A comparative assessment. The Geneva Papers 34:56–77
Hull JC (2008) Options, Futures, and Other Derivatives with Derivagem CD (7th Edition), 7th edn. Prentice Hall
Kling A, Richter A, Ruß J (2007) The interaction of guarantees, surplus distribution, and asset allocation in with-profit life insurance policies. Insurance: Mathematics and Economics 40(1):164–178
Kochanski M (2010a) Solvency capital requirement for German unit-linked insurance products. German risk and insurance review 6:33–70
Kochanski M (2010b) Solvenzkapital für FLV unter Berücksichtigung von dynamischem Storno. Zeitschrift für die gesamte Versicherungswissenschaft 99(5):689–710
Menzel P (2008) Optionen in dynamischen Hybridprodukten. Der Aktuar 1:9–11
Ortmann M (2007) HDI-Gerling Two Trust. Perfomance 6:28–34
Pfeiffer D, Strassburger D (2008) Solvency II: stability problems with the SCR aggregation formula. Scandinavian Actuarial Journal 1:61–77
Reuß A, Ruß J (2010) Vorteile innovativer Lebensversicherungsprodukte und ihre Risiken unter Solvency II. DAV-LEBENS-Gruppe, Herbsttagung, Köln, URL http://www.ifa-ulm.de/downloads/Innovative-Produkte-Solvency-II.pdf
Ronkainen V, Koskinen L, Berglund R (2007) Topical modelling issues in Solvency II. Scandinavian Actuarial Journal 2:135–146
Sandström A (2007) Solvency II: Calibration for skewness. Scandinavian Actuarial Journal 2:126–134
Schmidt K (2010) Ruin am Rollator. Das Investment 11:46–56
Schneeberger K (2010) Berechnung und Analyse des Market Consistent Appraisal Value, Master thesis, Ulm University
Shreve SE (2000) Stochastic Calculus for Finance 2. Springer
Siebert A (2008) Aktuarielle Fragen zu dynamischen Hybridprodukten. Der Aktuar 2:79–81
Steffen T (2008) Solvency II and the work of CEIOPS. The Geneva Papers 33:60–65
SU0101 (2010) Zeitreihe SU0101: Geldmarktsätze am Frankfurter Bankplatz / Tagesgeld / Monatsdurchschnitt. URL http://www.bundesbank.de/statistik/statistik_zeitreihen.php?func=row&tr=SU0101&year=
Tanskanen AJ, Lukkarinen J (2003) Fair valuation of path-dependent participating life insurance contracts. Insurance: Mathematics and Economics 33(3):595–609
The European Parliament (2009) Directive of the European Parliament and of the council on the pursuit of the business of insurance and reinsurance (Solvency II). URL http://register.consilium.europa.eu/pdf/en/09/st03/st03643-re06.en09.pdf
Van Hulle K (2011) Solvency II: state of play and perspectives. Zeitschrift für die gesamte Versicherungswissenschaft pp 1–16
Zaglauer K, Bauer D (2008) Risk-neutral valuation of participating life insurance contracts in a stochastic interest rate environment. Insurance: Mathematics and Economics 43(1):29–40, URL http://ideas.repec.org/a/eee/insuma/v43y2008i1p29-40.html
Zwiesler HJ (2007) Garantieprodukte. DAV-LEBENS-Gruppe, Herbsttagung, München, URL http://www.aktuar.de/download/dav/termine/HT07_LEBEN_Zwiesler.pdf