Minimum Standards for Investment Performance: A New Perspective on Non-Life Insurer Solvency

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MINIMUM STANDARDS FOR INVESTMENT PERFORMANCE:
A NEW PERSPECTIVE ON NON-LIFE INSURER SOLVENCY

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Abstract

The aim of this paper is to develop an alternative approach for assessing an insurer’s solvency as a proposal for a standard model for Solvency II. Instead of deriving minimum capital requirements—as it is done in solvency regulation—our model provides company-specific minimum standards for risk and return of investment performance, given the distribution structure of liabilities and a predefined safety level. The idea behind this approach is that in a situation of weak solvency, an insurer’s asset allocation can be adjusted much more easily in the short term than can, e.g., claims cost distributions, operating expenses, or equity capital. Hence, instead of using separate models for capital regulation and solvency regulation—as is typically done in most insurance markets—our single model will reduce complexity and costs for insurers as well as for regulators. In this paper, we first develop the model framework and second test its applicability using data from a German non-life insurer.

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\textit{Subject and Insurance Branch Code:} IM01; IM20; IE46; IE53; IB50

\textit{Keywords:} Solvency; Regulation; Investment performance; Non-life insurance; Normal power approximation

1. INTRODUCTION

Insurance supervision in the European Union is undergoing significant change as the European Commission works toward implementation of new risk-based capital standards (Solvency II). The aim of this paper is to contribute to this discussion by developing a risk model for the calculation of minimum solvency requirements.

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We introduce a new perspective on solvency regulation: instead of deriving a minimum amount of equity capital (as usually done in solvency models), minimum standards for investment performance are obtained, given the risk structure of liabilities, equity capital, and a predefined safety level. The functional relationship, which ensures that the minimum safety standard is met, is denoted as the solvency line. The solvency line determines minimum investment requirements in terms of expected return and standard deviation of the rate of return on the insurer’s investment portfolio. In this setting, insurers with higher equity capital will, ceteris paribus, also have a higher degree of freedom in the choice of their asset allocation. We compare results when the safety level is derived under three risk measures typically used in solvency regulation: the ruin probability, tail value at risk, and expected policyholder deficit (see, e.g., Barth, 2000). Having identified risk-return combinations that are compatible with the solvency rules based on the different risk measures, we further investigate to what extent these minimum requirements can be met on the capital market using several benchmark indices. To illustrate the practicability of the model, we include an empirical application based on data from a German non-life insurance company.

The literature focuses primarily on the predictability power of existing solvency models, alternative methodologies for solvency analysis, and the economic effects of solvency regulation. To date, empirical tests of solvency models used by regulators are restricted to the United States. Cummins/Harrington/Klein (1995) and Grace/Harrington/Klein (1998), as well as Cummins/Grace/Phillips (1999), find that the predictive power of the U.S. risk-based capital standards is relatively low compared to other screening mechanisms based on financial ratios used by regulators (e.g., the financial analysis tracking system). Pottier/Sommer (2002) conclude that the solvency models used by the private sector (A.M. Best ratings) are superior in predictive ability compared to the measures produced by regulators (risk-based capital standards, financial analysis tracking system). According to these empirical studies, the predictability power of the solvency models used by U.S. regulators appears to be limited.¹

Consequently, a second stream of literature proposes a number of different methodologies that can be used to study insolvency and to identify factors explaining insolvencies. Brockett et al. (1994) use neural networks, Carson/Hoyt (1995) multiple discriminant analysis, Lamm-Tennant/Starks/Stokes (1996) logistic regressions and a loss cost function, and Baran-

¹ The designers of Solvency II are thus able to take advantage of these research results and the consensus therein that neither the European regulatory rules (Solvency I) nor the current regulatory framework in the United States are entirely successful in meeting regulatory objectives. Furthermore, those involved in the Solvency II process can take advantage of advanced computer systems capable of implementing more complex models. Hence, differences in solvency systems may partially be explained by the different states of knowledge extant when they were first introduced.
off/Sager/Witt (1999) cascaded logistic regressions. An overview of these methods is provided in Carson/Hoyt (2000) as well as in Chen/Wong (2004). An important aspect in the discussion of alternative methodologies is the use of different risk measures, such as the ruin probability or the tail value at risk (see, e.g., Butsic, 1994; Barth, 2000; Tasche, 2002; Cuocco/Hong, 2006).

Several authors discuss the economic effects of regulation on insurance markets, often focusing on different forms of regulation and on different sectors of the insurance industry. Grabowski/Kip Viscusi/Evans (1989), for instance, focus on rate regulation in auto insurance, Barros (1995) analyzes market entry regulation in auto insurance, and Zweifel/Telser/Vaterlaus (2006) study product regulation in health insurance. With a focus on solvency regulation, Munch and Smallwood (1980) find that minimum capital requirements reduce the number of insolencies by reducing the number of small firms; they conclude that capital requirements are a particular burden for small insurers. Recent literature also highlights the particularly strong effect of regulation on small insurers (see, e.g., Van Rossum, 2005, on the interrelation between degree of regulation and corresponding costs). Rees/Gravelle/Wambach (1999) show that insurers always provide enough capital to ensure solvency if consumers are fully informed of insolvency risk; they conclude that regulators should provide information rather than imposing capital requirements. While most studies argue against extensive solvency regulation, this issue is not our main concern; instead we consider solvency regulation as a given component of insurance markets.

An overview of the new Solvency II regulation is provided by Eling/Schmeiser/Schmit (2007), Steffen (2008), and Doff (2008). Currently, a framework directive by the European Commission (2008) is under discussion and quantitative impact studies are being conducted. The final draft of the directive should be submitted for resolution by the EU parliament in 2009 and Solvency II should become the general norm by 2012. In the context of Solvency II, different aspects of harmonization are discussed, such as the convergence of Solvency II and International Financial Reporting Standards (see Esson/Cooke, 2007) or the convergence of solvency regulation around the world (see Karp, 2007; Monkiewicz, 2007). One of Solvency II’s innovations is that it allows an insurer to use internal risk models instead of standardized risk models when determining target capital (i.e., the minimum capital an insurer needs for a given time horizon and a fixed safety level). An internal model is constructed by the insurer for its individual needs; a standard model is designed by the regulator and used uniformly across insurers. Liebwein (2006) discusses some requirements for Solvency II internal risk models. Schmeiser (2004) develops an internal risk model for property-liability insurers based on dynamic financial analysis. Only a few approaches and aspects of a standard model under
Solvency II have been discussed at present (see, e.g., Sandström, 2006, for fundamental modeling ideas, or Schubert and Grießmann, 2007, for the “German” approach). The current framework directive already contains a number of principles and a target capital formula, but the precise implementation is unsettled as yet (see European Commission, 2008).

There are two potential problems with the target capital concept as laid out in Solvency II. First, a common assumption in this calculation is that the insurer’s asset structure remains unchanged in the time period under review, which is rather unrealistic in practice since the risk structure can change substantially in the short term. It seems more reasonable to assume that the liabilities change only marginally in the medium term. Second, in states of weak solvency, raising additional equity capital or implementing other risk management measures will be difficult. In such a situation, an insurer’s asset allocation can be adjusted much more easily in the short term operative business than can the claims costs distribution, operating expenses, or equity capital. We argue that the insurer’s solvency situation should be assessed by estimating the risk structure of liabilities and the available equity capital. Then, based on these data, the asset structure should be regulated, rather than requiring a certain minimum amount of capital.

A further problem related to solvency assessment is that developing theoretically sound internal risk models is expensive and is generally only feasible for large insurance companies with sufficient financial and technical resources. Most insurers will need to use a preset standard model to calculate their target capital. To this end, we believe that the model presented in this paper will prove advantageous compared to existing standard approaches. First, and as shown in the following sections, it is easy to communicate and simple to implement. Second, instead of using separate frameworks, with different scopes and assumptions, to derive capital and solvency requirements (which is currently how it is done in many states of the United States as well as in the European Union; see OECD, 2002, Bijapur et al., 2007), our model framework provides an integrated view of the insurer’s risk situation. Third, the approach proposed in this paper not only provides information relevant to insurance regulators, but can also give insight and useful advice to insurers themselves regarding corporate risk management decisions.

The remainder of the paper is structured as follows. Section 2 describes the model framework for determining the solvency line. Section 3 contains an empirical application of the developed standard model using data of a German property-liability insurer as well as capital market data. Section 4 discusses policy implications and Section 5 concludes.
2. MODEL FRAMEWORK

In this one-period solvency model, the market value of the insurer’s assets at time $t = 0$ consists of equity and debt and is denoted by $A_0$. The debt $L_0$ provided by the policyholders can be divided into two parts: the net premiums (after reinsurance) paid in $t = 0$ for contracts written in $t = 0$ (with claim payments to occur after time 0) and net premiums remaining for contracts signed before $t = 0$ with net claims that are not finally settled in $t = 0$. The debt corresponds to the market value of the liabilities. The residual between the market value of the assets $A_0$ and liabilities $L_0$ is the insurer’s equity (available) capital $U_0$.

Analogously, at time $t = 1$, the equity capital $U_1$ is given by the difference between assets ($A_1$) and liabilities ($L_1$):

$$U_1 = A_1 - L_1 = A_0 \cdot R - L_1 = (U_0 + L_0) \cdot R - (S_1 + B_1).$$

(1)

In Equation (1), $R = 1 + r$ denotes the stochastic rate of return on the insurer’s investment. $S_1$ stands for the stochastic market value of the net claims after reinsurance in $t = 1$ for contracts written in $t = 0$ or earlier, and $B_1$ are the operating expenses of the insurer in $t = 1$ (modeled deterministically).

Given a certain risk measure imposed by a regulatory authority, a required safety level, a fixed time horizon, and input data for the variables $L_0$, $R$, $S_1$, and $B_1$, the typical procedure in standard solvency models is to calculate the minimum (or target) required capital at $t = 0$. Whenever the available capital $U_0$ is not sufficient to satisfy the capital requirements, the insurer may be forced to raise additional equity capital, if this is a feasible option. In a situation of weak solvency, however, this is usually not easy. Changing the asset allocation so as to meet the solvency requirements should be much easier to accomplish.

Therefore, the main purpose of this paper is to not calculate minimum capital requirements for an insurer in $t = 0$, but to derive minimum performance requirements for the insurer’s investment portfolio in a $E(R) - \sigma(R)$ framework, based on a given safety level and a chosen risk measure. Even though there are other, more complex measures of risk and return that could be considered, we use $E(R)$ and $\sigma(R)$ in order to make sure that the model remains simple to implement in practice.

As an example, consider the ruin probability and a fixed safety level $\varepsilon$. In this case, the equation $P(U_1 \leq 0) = P(A_1 \leq L_1) = \varepsilon$ must be solved for different values of $E(R)$ and $\sigma(R)$,
given the distributional assumptions (see Equation (1)). A comparison of alternative risk measures, such as ruin probability ($RP$), expected policyholder deficit ($EPD$), or tail value at risk ($TVaR$), is especially relevant given the debate in the literature over the adequacy of solvency measures (for a discussion of the advantages of the different risk measures see, e.g., Barth, 2000). To what degree an insurer will have freedom of investment choice will depend not only on the risk measure and safety level used, but also on the risk structure of its liabilities and the actual equity capital $U_0$.

For a symmetrically distributed rate of return $R$ and a right-skewed distribution of the claims $S_1$ (see, e.g., Dickson/Tedesco/Zehnwirth, 1998), one obtains a left-skewed distribution of the equity capital $U_1$. In this case, the risk measures listed above generally cannot be calculated explicitly. In the following analysis, we will apply the normal power approximation for $U_1$ (Daykin/Pentikäinen/Pesonen, 1994, p. 129), which—in contrast to a simulation framework—allows for analytical expressions of the risk measures.

The basic idea of the normal power approximation is to transform a nonnormally distributed stochastic variable with nonzero skewness into an auxiliary variable $y$—using the transformation $v$—in order to make the distribution approximately symmetric and normal. For the equity capital in $t = 1$, one thus gets $y = v(U_1)$, and for the distribution function of $U_1$, $G(u_1)$, it holds that $G(u_1) \approx \Phi(v(u_1)) = \Phi(y)$, where $\Phi$ stands for the standard normal distribution function. Under the normal power approximation, the auxiliary variable $y$ is given by

$$y = v(u_1) = \frac{1}{\gamma(U_1)} \left\{ \gamma(U_1)^2 + 6 \cdot \gamma(U_1) \left( \frac{u_1 - E(U_1)}{\sigma(U_1)} \right) + 9 \right\}.$$  \hspace{1cm} (2)

Here, $\gamma(U_1) = E(U_1 - E(U_1))^3 / \sigma(U_1)^3$ stands for the skewness of $U_1$. Alternatively, the inverse $v^{-1}(y)$ of the transformation function $v$ results from

$$\frac{u_1 - E(U_1)}{\sigma(U_1)} = y + \frac{\gamma(U_1)}{6} \cdot (y^2 - 1) \cdot v^{-1}(y).$$

The first three central moments of $U_1$ are needed for the normal power approximation. Since, in what follows, we assume that the rate of return is symmetric, we obtain (see Equation (1)):
\[ E(U_1) = A_0 \cdot E(R) - E(S_i) - B_i; \]
\[ \sigma(U_1)^2 = A_0^2 \cdot \sigma(R)^2 + \sigma(S_i)^2 - 2 \cdot A_0 \cdot \text{cov}(R, S); \]
\[ E \left( U_1 - E(U_1) \right)^3 = 3 \cdot A_0 \cdot E \left( (R - E(R)) \cdot (S_i - E(S_i))^2 \right) \]
\[ -3 \cdot A_0^2 \cdot E \left( (R - E(R))^2 \cdot (S_i - E(S_i)) \right) - E \left( S_i - E(S_i) \right)^3. \]

In the following, we apply the normal power concept to three risk measures—the ruin probability, expected policyholder deficit, and tail value at risk.

**Ruin Probability (RP)**

The probability \( \varepsilon \) of an insolvency at \( t = 1 \) is given by

\[ RP = P(U_1 \leq 0) = P(A_i \leq L_i) = \varepsilon, \]

i.e., the probability that the firm’s assets are not sufficient to cover its liabilities. The ruin probability corresponds to the value at risk approach as planned for Solvency II. For a fixed ruin probability \( \varepsilon \), use of the normal power approximation for the distribution of \( U_1 \) leads to

\[ P(U_1 \leq 0) = P(v(U_1) \leq v(0)) = \varepsilon. \]

Since \( v(0) = z_{\varepsilon} \) (\( z_{\varepsilon} \) is the \( \varepsilon \)-quantile of the standard normal distribution), the following equation can be derived using \( 0 = v^{-1} \left( z_{\varepsilon} \right) \):

\[ 0 = E(U_1) + z_{\varepsilon} \cdot \sigma(U_1) + \frac{z_{\varepsilon}^2 - 1}{6} \cdot \gamma(U_1) \cdot \sigma(U_1), \quad (3) \]

where \( z_{\varepsilon} \) denotes the \( \varepsilon \)-quantile of a standard normal distribution. Only when the distribution of \( U_1 \) is symmetric, the last term on the right-hand side of Equation (3) becomes zero and, hence, the normal power approximation merges to a normal approximation.\(^3\)

\(^3\) In general, a left-skewed (right-skewed) distribution for \( U_1 \) will c.p. imply higher (lower) solvency requirements.
**Expected Policyholder Deficit (EPD)**

One shortcoming of the ruin probability approach is that it does not take into account the severity of ruin (Butsic, 1994; Powers, 1995). Therefore, other risk measurers have been proposed in the literature and are widely used in insurance practice. For instance, the expected policyholder deficit (EPD) (Butsic, 1994; Barth, 2000) is, in the present context, defined as

$$EPD = E\left(\max\left(0 - U_1, 0\right)\right),$$

i.e., the expected loss in the case of insolvency, if the equity capital at time 1 becomes negative. The expected policyholder deficit (EPD) can be written as

$$EPD = E\left(\max\left(0 - U_1, 0\right)\right) = -E(U_1) + E\left(\max\left(0, 0\right)\right).$$

For $Y \in N(0,1) = \Phi$, where $\Phi$ is the cumulative distribution function of the standard normal distribution and $\phi$ stands for the respective density function, it holds that (see Winkler/Roodman/Britney, 1972)

$$E_{\sigma(0)}(Y^n) = \int_{\sigma(0)}^{\infty} y^n \phi(y) dy = E(Y^n) - E_{-\infty}^{\sigma(0)}(Y^n),$$

where, in general,

$$E_{-\infty}^{z}(Y^n) = -z^{n-1}\phi(z) + (n-1)E_{-\infty}^{z}(Y^{n-2})$$

$$E_{-\infty}^{z}(Y^0) = \Phi(z)$$

$$E_{-\infty}^{z}(Y^1) = -\phi(z).$$

Using this result and the normal power approximation for the distribution of $U_1$, $E\left(\max\left(U_1, 0\right)\right)$ can be derived from the relationship
Thus, the expected policyholder deficit is given by

$$E \left( \max (U_i, 0) \right) = \int_{0}^{\infty} u_i g(u_i) \, du_i = \int_{\gamma(0)}^{\infty} v^{-1}(y) \varphi(y) \, dy$$

$$= \int_{\gamma(0)}^{\infty} \left[ E(U_i) + \gamma(U_i) \cdot \sigma(U_i) \cdot \left( y^2 - 1 \right) \right] \varphi(y) \, dy$$

$$= \left( E(U_i) - \frac{\gamma(U_i) \cdot \sigma(U_i)}{6} \right) \int_{\gamma(0)}^{\infty} y \varphi(y) \, dy + \sigma(U_i) \int_{\gamma(0)}^{\infty} y^2 \varphi(y) \, dy$$

$$\varphi(y) \, dy$$

$$= \left( E(U_i) - \frac{\gamma(U_i) \cdot \sigma(U_i)}{6} \right) E_{\gamma(0)}^{\infty} \left( Y^0 \right) + \sigma(U_i) \cdot E_{\gamma(0)}^{\infty} \left( Y^2 \right)$$

$$= \left( E(U_i) - \frac{\gamma(U_i) \cdot \sigma(U_i)}{6} \right) \Phi(-v(0)) + \sigma(U_i) \cdot \varphi(-v(0))$$

$$+ \frac{\gamma(U_i) \cdot \sigma(U_i)}{6} \left( 1 + v(0) \cdot \varphi(v(0)) - \Phi(v(0)) \right)$$

$$= E(U_i) \cdot \Phi(-v(0)) + \left( \sigma(U_i) + \frac{\gamma(U_i) \cdot \sigma(U_i)}{6} \cdot v(0) \right) \varphi(v(0)).$$

Thus, the expected policyholder deficit is given by

$$EPD = -E(U_i) \cdot \Phi(-v(0)) + \left( \sigma(U_i) + \frac{\gamma(U_i) \cdot \sigma(U_i)}{6} \cdot v(0) \right) \varphi(v(0)), \quad (4)$$

where \( v(\cdot) \) is given by Equation (2).

**Tail Value at Risk (TVaR)**

Another common risk measure in the insurance sector is tail value at risk (Artzner et al., 1999) as implemented in the Swiss Solvency Test. Using the value at risk \( \text{VaR} \) for a safety level \( \alpha \),

$$\text{VaR}_{\alpha}(U_i) = q_{\alpha},$$

the tail value at risk (TVaR) for the safety level \( \alpha \) is given by the conditional expectation:
If the normal power approximation to estimate the distribution of $U_1$ is used, the tail value at risk can be derived analytically by

$$TVaR_\alpha(U_1) = E(-U_1 | U_1 \leq q_\alpha).$$

To derive minimum requirements for the insurer's investment performance in an $E(R)$ - $\sigma(R)$ relationship, we fix the company’s safety level using the discussed risk measurers. Equations (3), (4), and (5) are then solved for $E(R)$ for a given $\sigma(R)$. In practical applications, the minimum required safety level is set by the regulatory authority. In case of the EPD or TVaR, the requirements are often defined as a percentage of the business volume so as to ensure comparability between companies of different size.

### 3. Numerical Example Based on Empirical Data

To illustrate the applicability of the model, we obtained data from a representative medium-sized German non-life insurance company that mainly deals in property-casualty and automobile insurance. To protect the anonymity of the company, we transformed all data so as to change the absolute values but not the underlying risk structure.

The market value of the assets in $t = 0$, $A_0$, is €1.582 billion. We derived the stochastic market value of net claims in $t = 1$ with an expected value $E(S_1)$ of €1.171 billion and a standard deviation $\sigma(S_1)$ of €66 million. The skewness of the claims, $\gamma(S_1)$, is 0.3. The operating expenses of the insurer ($B_1$, modeled deterministically) yield €246 million.\(^4\) In the nu-

\(^4\) The market value of the company’s liabilities in $t$ is given as the sum of the market value of the net claims and the operating expenses of the insurer. One way to determine the market value of the lia-
merical example we assume stochastic independence between the rate of return of the investment portfolio and the claim payments. The skewness of the equity capital in $t = 1$, $U_1$, is then given by (Daykin et al., 1994, p. 25)

$$
\gamma(U_1) = \frac{\sigma(A_0 \cdot R)^3 \gamma(R) - \sigma(S_1)^3 \gamma(S_1)}{(\sigma(A_0 \cdot R)^2 + \sigma(S_1)^2)^{3/2}}.
$$

The skewness $\gamma(U_1)$ is needed for deriving the different risk measures presented in Equations (3), (4), and (5).

Given a certain safety level for an insurance company and the data above, requirements for the investment performance in a $E(R) - \sigma(R)$ framework can be derived that should be met at any time between $t = 0$ and $t = 1$ (that is, e.g., after any change in the insurer’s asset allocation).

**Comparison of Solvency Lines for Different Risk Measures**

In accordance with the planned Solvency II rules (see European Commission, 2008, Article 101), the ruin probability $RP$ is set to $\varepsilon = 0.5\%$. For the TVaR, we use a safety level of $\alpha = 1\%$ as is also done, e.g., in the Swiss Solvency Test (see Luder, 2005). In the following, we refer to $r$ instead of $R (= 1 + r)$ for clarity of exposition. To have the same starting value $E(r) = E(R) - 1$ for all three risk measures given $\sigma(r) = \sigma(R) = 0$ and to thus ensure comparability, we set $TVaR_{\alpha_{\%}}(U_1) = \varepsilon 7.827$ million and $EPD = \varepsilon 0.135$ million. Figure 1 presents the resulting solvency lines for $RP$, TVaR, and $EPD$.

Bilities caused by contracts sold by the insurer before $t = 1$ (without new business) is using risk-neutral valuation, as described by the following expression:

$$
L_t = S_t + B_t = E_t^{Q} \left( \sum_{i=1}^{n} S(t) \cdot e^{-r_f(t-1)} \right) + \sum_{i=1}^{n} B(t) \cdot e^{-r_f(t-1)}.
$$

Here, $r_f$ stands for the risk-free rate of return (here assumed to be constant in time) and $E_t^{Q}(.)$ denotes the conditional expected value with respect to the risk-neutral measure $Q$ given the information available in $t = 1$ (see, e.g., Björk, 2004). Thus, from the viewpoint of $t = 0$, the liabilities in $t = 1$ are stochastic. In our numerical example, the stochastic market value of net claims and the market value of assets have been estimated by the German non-life insurance company.
**Figure 1:** Solvency lines for $RP$, $EPD$, and $TVaR$ based on data from a German non-life insurance company

In Figure 1, three different areas can be identified. The $E(r) - \sigma(r)$ combinations in Area (1) satisfy the requirements of all three risk measures. The $E(r) - \sigma(r)$ combinations in Area (2) satisfy the requirement of $RP$ and $TVaR$, but not those of $EPD$. The $E(r) - \sigma(r)$ combinations in Area (3) do not satisfy the requirements of any of the risk measures. Thus, the $EPD$ appears to be the most restrictive risk measure for the considered company, followed by the $TVaR$, which leads to slightly higher restrictions than the $RP$ concept. For instance, for a given expected rate of return $E(r) = 10\%$, a standard deviation of $\sigma(r) = 6.16\%$ is allowed under the $EPD$, whereas $\sigma(r) = 6.48\%$ is permitted in the case of $TVaR$ and $\sigma(r) = 6.59\%$ in the case of $RP$.

Alternatively, it can be concluded from this example that the required rate of return for a given volatility is higher for the $EPD$ compared to $TVaR$ or $RP$.

Figure 2 displays solvency lines for different ruin probabilities $\varepsilon$. The upper line is calculated for $RP = 0.1\%$, i.e., a ruin occurs on average every 1,000 years. The middle line uses a ruin probability of 0.5% and the lower curve shows the solvency line for $RP = 1\%$.

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5 For a given safety level and the same starting point on the ordinate of Figure 1, the $TVaR$ concept will lead to higher requirements and, hence, to fewer degrees of freedom in choosing the asset allocation compared to the $VaR$ approach.
Figure 2: Solvency line for ruin probability $RP$ under three different safety levels $\varepsilon$ based on data from a German non-life insurance company

The three solvency lines presented in Figure 2 are parallel to each other in the $E(r) - \sigma(r)$ graph. In the case of a risk-free portfolio (i.e., $\sigma(r) = 0$), there is a minimum return that the insurer must achieve to satisfy the intended safety level. A ruin probability of 0.1% implies a minimum risk-free rate of return of 4.00%, which seems rather high, realistically speaking, especially when taking into account the recent average interest rates on high rated government bonds (see Table 1 in the following section). Using a ruin probability of 0.5%, the necessary rate of return is 1.54%. For a ruin probability of 1%, the minimum risk-free rate of return will need to be 0.22% to hold steady at the desired safety level.

**Solvency Lines and the Capital Market**

Having identified $E(r) - \sigma(r)$ combinations that are compatible with solvency requirements for an insurance company, we next link these results with risk and return figures actually available on the capital market. In practical applications, the regulatory authority defines different asset classes and provides an estimation of the risk and return figures the insurer must use in verifying whether the current asset allocation meets the solvency requirements.

For this purpose, we empirically derive the capital market line. Following the approach in Mayers (1972) and Brito (1977), net claims are interpreted as (negative) nonmarketable income.\(^6\) Let $k$ denote the number of different asset classes. The asset type $j = 0$ with the rate of return $r_j$ stands for the risk-free investment; the risky assets yield the rate of return $r_j$ (with $j = 1, \ldots, k$). The vector of returns is given by $\hat{r}$. In addition, $w$ characterizes the vector of the in-

\(^6\) In this framework, nonmarketable income is defined as income that cannot be duplicated by assets traded on the capital market (see also Turner, 1987, pp. 79–99).
vestment proportions in the different asset classes, the superscript $T$ denotes the transposition of a vector, and $C$ stands for the covariance matrix of the returns of the different asset classes. The optimization problem is then described by:

$$\sigma(U_1)^2 = A_0^2 \cdot w^T \cdot C \cdot w + \sigma(S_i)^2 - 2 \cdot A_0 \cdot \text{cov}(w^T \cdot \hat{r}, S_i) \rightarrow \min_w$$

such that

$$E(U_1) = A_0 \cdot (1 + w^T E(\hat{r})) - E(S_i) - B = \text{const.}$$

$$1 = 1^T \cdot w.$$  

For the optimal vector $w$ derived by the optimization, one obtains efficient risk–return combinations for $E(r) = w^T E(\hat{r})$ and $\sigma(r)^2 = w^T \cdot C \cdot w$.\(^7\)

The estimated capital market line is based on benchmark indices that represent investment opportunities available. German insurers typically hold a globally diversified portfolio of stocks, bonds, real estate, and money market instruments (these four asset classes cover approximately 99.50% of all investments made by insurance companies; see El-ling/Schuhmacher, 2007). Within these four asset classes, we consider 11 indices, each having a different regional focus (see Table 1).

<table>
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<tr>
<th>Asset class</th>
<th>Index</th>
<th>Illustration</th>
<th>$E(r)$ annualized</th>
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<td>Money</td>
<td>JPM Euro Cash 3 Month</td>
<td>Money market in the EMU = $r_f$</td>
<td>3.95%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Stocks</td>
<td>MSCI World ex EMU</td>
<td>Worldwide stocks without the EMU</td>
<td>6.54%</td>
<td>16.07%</td>
</tr>
<tr>
<td></td>
<td>MSCI EMU ex Germany</td>
<td>Stocks from the EMU without Germany</td>
<td>12.16%</td>
<td>17.86%</td>
</tr>
<tr>
<td></td>
<td>MSCI Germany</td>
<td>Stocks from Germany</td>
<td>10.46%</td>
<td>22.12%</td>
</tr>
<tr>
<td>Bonds</td>
<td>MSCI SDI World ex EMU</td>
<td>Worldwide government bonds without the EMU</td>
<td>4.84%</td>
<td>5.78%</td>
</tr>
<tr>
<td></td>
<td>MSCI SDI EMU ex Germany</td>
<td>Gov. bonds from the EMU without Germany</td>
<td>6.54%</td>
<td>3.82%</td>
</tr>
<tr>
<td></td>
<td>MSCI SDI Germany</td>
<td>Government bonds from Germany</td>
<td>5.59%</td>
<td>3.46%</td>
</tr>
<tr>
<td></td>
<td>MSCI Euro Credit Corporate</td>
<td>Corporate bonds from the EMU</td>
<td>5.84%</td>
<td>3.41%</td>
</tr>
<tr>
<td>Real estate</td>
<td>GPR General PSI Global</td>
<td>Real estate worldwide</td>
<td>8.45%</td>
<td>11.37%</td>
</tr>
<tr>
<td></td>
<td>GPR General PSI Europe</td>
<td>Real estate in Europe</td>
<td>8.19%</td>
<td>7.16%</td>
</tr>
<tr>
<td></td>
<td>DIMAX</td>
<td>Real estate in Germany</td>
<td>7.65%</td>
<td>13.15%</td>
</tr>
</tbody>
</table>

JPM: J.P. Morgan; MSCI: Morgan Stanley Capital International; EMU: European Monetary Union; SDI: Sovereign Debt Index; GPR: Global Property Research; PSI: Property Share Index; DIMAX: Deutscher Immobilien Aktienindex.

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\(^7\) Due to the stochastic independence between the rate of return of the investment portfolio and the claim payments in our example, efficient risk-return combinations can be described by the “classical” capital market line derived from the Tobin separation theorem (Tobin, 1958, pp. 65–86).
The selected market indices are well known and can usually be acquired over index funds with small transaction costs. Furthermore, they are broadly diversified so that they are generally well suited for performance measurement (for criteria appropriate for selecting representative benchmark indices, see, e.g., Sharpe, 1992). For each of these indices, we extract monthly returns between January 1994 and December 2006 from the Datastream database and calculate mean and standard deviation of the annualized returns. To consider returns from changes in prices and dividend payments, we look at performance indices only. All indices are calculated on a Euro basis. Based on the returns and their correlation (see Appendix), we calculate the efficient frontier and the capital market line using the return of the JPM Euro Cash 3 Month as the risk-free rate \((r_f = 3.95\%)\). The capital market line resulting from the optimization problem above is in this case given by

\[
E(r) = 3.9492 + 1.0637 \cdot \sigma(r).
\]

Figure 3 displays the capital market line along with the solvency lines previously illustrated in Figure 1.

**Figure 3**: Capital market line and solvency lines for RP, EPD, and TVaR based on data from a German non-life insurance company

In contrast to Figure 1, which revealed three areas of relevance, Figure 3 shows four different areas. Looking at Figure 3, we see that risk–return combinations in Areas (1) and (4) comply with the requirements of the EPD. However, in contrast to the \(E(r) - \sigma(r)\) combinations in Area (4), Area (1) is not attainable on the capital market. Areas (2) and (3) include risk-return combinations that are only acceptable under RP and TVaR; again, combinations in Area (2) are not in line with capital market opportunities. Overall, including the capital market line in Figure 3 reveals the stronger regulatory restriction imposed by the EPD concept, as the al-
The different asset classes used in calculating the capital market line shown in Figure 3 may not necessarily correspond to the actual investment of the insurance company under consideration. Furthermore, in regard to asset allocation, other tools and decision criteria might be implemented.8 Hence, the capital market line derived in this section should be understood as an interpretation aid for insurance regulators and as an illustration of market restrictions in respect to the asset allocation. From the viewpoint of insurance management, the asset allocation chosen must lead to realizations above the solvency line (i.e., particularly after any reallocation of the investment portfolio between $t = 0$ and $t = 1$). In the objective function of the insurer, the solvency line serves as an (additional) constraint. However, the proposed solvency model should not influence the insurer's objective function itself and, therefore, the problem of regulatory actions having too great an influence on a company's policy should be avoided.

In the following, we support our line of reasoning with an example. Given the risk–return figures of the 11 different asset classes set out in Table 1 (derived by the regulator), we consider four different types of asset allocation in Table 2.

### Table 2: Examples of asset allocation

<table>
<thead>
<tr>
<th>Asset class</th>
<th>Index</th>
<th>Asset allocation Example 1</th>
<th>Asset allocation Example 2</th>
<th>Asset allocation Example 3</th>
<th>Asset allocation Example 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Money</td>
<td>JPM Euro Cash 3 Month</td>
<td>100%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Stocks</td>
<td>MSCI World ex EMU</td>
<td>0%</td>
<td>5%</td>
<td>10%</td>
<td>20%</td>
</tr>
<tr>
<td></td>
<td>MSCI EMU ex Germany</td>
<td>0%</td>
<td>5%</td>
<td>10%</td>
<td>20%</td>
</tr>
<tr>
<td></td>
<td>MSCI Germany</td>
<td>0%</td>
<td>5%</td>
<td>10%</td>
<td>20%</td>
</tr>
<tr>
<td>Bonds</td>
<td>MSCI SDI World ex EMU</td>
<td>0%</td>
<td>15%</td>
<td>10%</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>MSCI SDI EMU ex Germany</td>
<td>0%</td>
<td>15%</td>
<td>10%</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>MSCI SDI Germany</td>
<td>0%</td>
<td>15%</td>
<td>10%</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>MSCI Euro Credit Corporate</td>
<td>0%</td>
<td>10%</td>
<td>10%</td>
<td>0%</td>
</tr>
<tr>
<td>Real estate</td>
<td>GPR General PSI Global</td>
<td>0%</td>
<td>10%</td>
<td>10%</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>GPR General PSI Europe</td>
<td>0%</td>
<td>10%</td>
<td>10%</td>
<td>20%</td>
</tr>
<tr>
<td></td>
<td>DIMAX</td>
<td>0%</td>
<td>10%</td>
<td>10%</td>
<td>20%</td>
</tr>
</tbody>
</table>

| $E(r)$ annualized | 3.95% | 7.02% | 7.63% | 9.00% |
| $\sigma(r)$ annualized | 0% | 4.64% | 6.85% | 12.53% |
| Corresponding RP of the insurer | 0.12% | 0.32% | 1.30% | 7.05% |

JPM: J.P. Morgan; MSCI: Morgan Stanley Capital International; EMU: European Monetary Union; SDI: Sovereign Debt Index; GPR: Global Property Research; PSI: Property Share Index; DIMAX: Deutscher Immobilien Aktienindex

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8 A variety of different risk and return measures might be used in this context. See, e.g., Rachev/Menn/Fabozzi, 2005, for an overview. It might also be that the asset allocation is optimized having as a criterion the risk measure itself; see Consiglio et al., 2001, 2003, 2008 for related analyses. In such a situation the optimal weights are determined that minimize the risk measure for a given level for a given $E(U1)$. 

lowed standard deviation of the insurer’s investment return is more limited compared to the other risk measures. In addition, the TVaR concept is more restrictive than the RP for the firm under consideration.
If the regulatory authority requires a ruin probability of (at most) 0.5%, the insurance company can meet this requirement by, e.g., investing in the money market only (Example 1 in Table 2). In this case, the insurer’s ruin probability will be approximately 0.12%. Thus in order to obtain a ruin probability of exactly 0.5%, the equity capital $U_0$ could, ceteris paribus, be reduced by around € 36.86 million in order to realize a ruin probability of 0.5%. Given the asset allocation of Example 2, the corresponding expected return and standard deviation of the return are $E(r) = 7.02\%$ and $\sigma(r) = 4.64\%$. This asset allocation leads to $RP = 0.32\%$; in addition, it can be seen from Figure 3 that this combination lies above the solvency line.

In case of Example 3 ($E(r) = 7.63\%$, $\sigma(r) = 6.85\%$) and Example 4 ($E(r) = 9.00\%$, $\sigma(r) = 12.53\%$), the ruin probabilities are 1.30% and 7.05%, respectively. Instead of substantially increasing equity capital to achieve the required target level of $RP = 0.5\%$, the insurer could shift the asset allocation to satisfy solvency requirements set by regulatory authorities (as in case of asset allocation examples 1 and 2), which in an operative business would appear to be more reasonable and easier to obtain than raising additional capital. However, if raising additional capital is feasible and is also management’s preferred course of action, doing so would be considered an admissible form of risk management under the proposed framework. Hence, one main advantage of the model is that when an insurer is experiencing financial distress (i.e., the current asset allocation and the actual underwriting risk situation do not meet safety requirements), there is a wide variety of asset allocations to choose from to remedy the situation.

4. Policy Implications

The proposed solvency model merges two regulatory frameworks concerning the asset and liability side into a single framework. Compared to having to separate capital regulation and solvency regulation, using different scopes and model implications, as is done in many insurance markets, our single framework model reduces complexity and costs for insurance companies and regulatory authorities. From this point of view, we propose the concept to be used as a standard model for Solvency II. The low cost argument may be especially relevant for smaller firms.

Minimum standards with respect to the investment performance require regulators to estimate parameters of different asset classes. In this respect, robust estimation is vital to avoid maximization of estimation errors, as may occur in a Markowitz setting (see Broadie, 1993). In addition, insurers need to be aware that the asset parameters are estimated based on historical data and expected returns. Hence, unexpected changes in parameters will not be reflected in
the solvency line, and yet such could have a considerable impact on the insurer’s financial situation.

Regulators could use our proposed solvency model as an assessment tool, keeping in mind that it is not intended as a guideline regarding the insurer’s objective function. However, for a profit maximizing firm, our approach may act as a constraint to its objective function. Binding constraints may affect the insurer’s capital structure and can imply higher premiums for policyholders, which the policyholders may find to be an acceptable tradeoff if it means an increased safety level.

Another implication of our solvency model has to do with the possibility of incentives for insurers to invest in low-risk asset portfolios in order to achieve the regulator’s requirements. However, this will reduce certain positive opportunities associated with risky asset allocations, which may be considered a disadvantage from the viewpoint of shareholders and policyholders. In addition, market prices for assets with low systematic risk may rise given a higher demand by the insurance industry. Only empirical analyses will generate credible information on the extent to which a specific solvency model—including ours—is appropriate with respect to reducing external (and internal) costs (see Rees/Gravelle/Wambach, 1999; Posner, 1974; Meier, 1991).

Solvency II focuses on the amount of capital necessary to ensure insurer solvency, while our model takes a different viewpoint by focusing on insurers’ asset allocation opportunities. This is a useful perspective because it reveals that numerous investment strategies—all in compliance with the regulator’s requirements—can be devised, leaving the insurer with more degrees of freedom and lessening the chance of systemic risk in the capital market. Furthermore, modifying an insurer’s investment portfolio, as opposed to its underwriting, operating expenses, or capital position, is appealing since changes to the investment portfolio can be accomplished both more easily and more quickly (compared to the insurer’s liabilities). In

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9 In the context of capital regulation, this is done in, e.g., Munch/Smallwood (1980) and Freixas/Loranth/Morrison (2007).

10 A standard model designed by the regulator and used uniformly across insurers might increase systemic risk. An unusual event in the capital or insurance market may force insurers to respond identically, causing a market run. An example is the 2001 capital market plunge, where many insurers shifted their asset allocation from stocks to bonds, which led to a further expansion of the stock supply and thus to a further plunge (see Cummins and Doherty, 2002, pp. 6–8). In this context, using an individualized solvency line for each insurer might reduce the danger of similar behavior and, in turn, systemic risk.
addition, the costs of modifying the investment portfolio may be less than the cost of raising additional capital.11

5. CONCLUSION

In this paper, we propose a new model for solvency assessment. The key element of the model is the solvency line, which provides risk and return combinations for the insurer’s capital investment that will ensure the maintenance of a maximum permitted risk of insolvency. As risk measures, we compare the ruin probability, the expected policyholder deficit, and the tail value at risk. The model framework of the insurance company takes into account individual factors such as equity capital and the probability distribution of claims.

A numerical example using empirical data from a German non-life insurer allows deeper insights into how the proposed model performs. Implementation is achieved using the normal power approximation to obtain explicit expressions for the considered risk measures. Permissible risk-return combinations are then related to allocation opportunities actually offered on the capital market. The risk-return figures are provided by the regulatory authority to check whether the current asset allocation meets the imposed capital requirements. In this setting, asset investment regulation and solvency rules are unified in one model framework.

There are two promising fields of application for our model framework. The first is its use for solvency assessment, especially as a standard model within the new Solvency II framework. Regulators in the European Union are still searching for a feasible, flexible, and low-cost standard model to be used in determining the target capital, especially for small insurers. The proposed solvency model meets the practical demands of a standard model and, in addition, is based upon a sound theoretical foundation. In this context, the results show that for the German insurance company examined here, the expected policyholder deficit is more restrictive compared to the tail value at risk, which in turn is more restrictive than the ruin probability approach. Taking into account the available investment opportunities shows that the regulatory restriction is even more severe than appeared to be the case at first glance, as it further limits the maximum volatility of the rate of return.

11 Empirical evidence for the United States suggests that raising capital via a security offering has a significant negative impact on the firm’s market value in terms of adverse stock price reactions (for an overview, see Masulis and Korwar, 1986; Eckbo, Masulis, and Norli, 2007). Our line of reasoning is also supported by modern capital structure theory. According to the pecking order theory, raising additional equity is seen as a financing means of last resort (see Myers and Majluf, 1994). However, firm conclusions on the costs of different risk management measures would require additional analyses.
The second important application for our model is risk control in an operative business. Typically, claims costs and operating expenses cannot be as easily adjusted as can the amount of investment in capital markets. Especially in case of financial distress, it is more reasonable (and easier) to change asset allocation than liabilities or equity capital. Therefore, our model focuses primarily on asset allocation opportunities instead of liabilities or minimum equity capital, resulting in an integrated solvency tool for insurance companies.
Table A1: Correlation between returns of the benchmark indices

<table>
<thead>
<tr>
<th></th>
<th>MSCI World ex EMU</th>
<th>MSCI EMU ex Germany</th>
<th>MSCI Germany</th>
<th>MSCI SDI World ex EMU</th>
<th>MSCI SDI EMU ex Germany</th>
<th>MSCI SDI Germany</th>
<th>MSCI Euro Credit Corporate</th>
<th>GPR General PSI Global</th>
<th>GPR General PSI Europe</th>
<th>DIMAX</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSCI World ex EMU</td>
<td>1.00</td>
<td>0.83</td>
<td>0.75</td>
<td>-0.16</td>
<td>-0.03</td>
<td>-0.21</td>
<td>-0.04</td>
<td>0.51</td>
<td>0.53</td>
<td>0.32</td>
</tr>
<tr>
<td>MSCI EMU ex Germany</td>
<td>0.83</td>
<td>1.00</td>
<td>0.89</td>
<td>-0.19</td>
<td>-0.01</td>
<td>-0.20</td>
<td>-0.02</td>
<td>0.43</td>
<td>0.45</td>
<td>0.33</td>
</tr>
<tr>
<td>MSCI Germany</td>
<td>0.75</td>
<td>0.89</td>
<td>1.00</td>
<td>-0.25</td>
<td>-0.12</td>
<td>-0.26</td>
<td>-0.11</td>
<td>0.37</td>
<td>0.37</td>
<td>0.38</td>
</tr>
<tr>
<td>MSCI SDI World ex EMU</td>
<td>-0.16</td>
<td>-0.19</td>
<td>-0.25</td>
<td>1.00</td>
<td>0.32</td>
<td>0.48</td>
<td>0.41</td>
<td>0.10</td>
<td>0.05</td>
<td>-0.02</td>
</tr>
<tr>
<td>MSCI SDI EMU ex Germany</td>
<td>-0.03</td>
<td>-0.01</td>
<td>-0.12</td>
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<td>0.83</td>
<td>0.13</td>
<td>0.09</td>
<td>-0.14</td>
</tr>
<tr>
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<td>-0.20</td>
<td>-0.26</td>
<td>0.48</td>
<td>0.74</td>
<td>1.00</td>
<td>0.89</td>
<td>-0.01</td>
<td>0.01</td>
<td>-0.18</td>
</tr>
<tr>
<td>MSCI Euro Credit Corporate</td>
<td>-0.04</td>
<td>-0.02</td>
<td>-0.11</td>
<td>0.41</td>
<td>0.83</td>
<td>0.89</td>
<td>1.00</td>
<td>0.13</td>
<td>0.16</td>
<td>-0.11</td>
</tr>
<tr>
<td>GPR General PSI Global</td>
<td>0.51</td>
<td>0.43</td>
<td>0.37</td>
<td>0.10</td>
<td>0.13</td>
<td>-0.01</td>
<td>0.13</td>
<td>1.00</td>
<td>0.59</td>
<td>0.26</td>
</tr>
<tr>
<td>GPR General PSI Europe</td>
<td>0.55</td>
<td>0.45</td>
<td>0.37</td>
<td>0.05</td>
<td>0.09</td>
<td>0.01</td>
<td>0.16</td>
<td>0.59</td>
<td>1.00</td>
<td>0.33</td>
</tr>
<tr>
<td>DIMAX</td>
<td>0.32</td>
<td>0.33</td>
<td>0.38</td>
<td>-0.02</td>
<td>-0.14</td>
<td>-0.18</td>
<td>-0.11</td>
<td>0.26</td>
<td>0.33</td>
<td>1.00</td>
</tr>
</tbody>
</table>

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REFERENCES


