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Abstract: Recent literature discusses the persistence of skewness and tail risk in hedge fund returns. The aim of this paper is to suggest an alternative skewness measure δ which is derived as the normalized shape parameter from the skew-normal distribution. First, we illustrate that the skew-normal distribution is better able to catch the characteristics of hedge fund returns than the normal distribution. And second, we show that using the skewness parameter δ has a number of advantages compared to common measures of skewness, e.g., it has a limpid financial interpretation as a skewness shock on normally distributed returns and tail-risk measures such as Value-at-Risk and Conditional Value-at-Risk are decreasing functions of δ.

JEL Classification: G10, G11, G23, G29

Keywords: Hedge funds, Skew-normal, Skewness coefficients, Azzalini's delta parameter

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1. Introduction

A number of studies have empirically shown that hedge fund returns are far from being Gaussian and show persistent skewness (see, e.g., Fung and Hsieh, 1997; Lo, 2001; Darius et al., 2002; Cvitanic et al., 2003; Agarwal and Naik, 2004). In the literature, many indices are used to measure skewness. The most common is the third standardized moment, which provides comparative information on the tail divergence from the normal distribution (see, e.g., Kim and White, 2004). Additionally, there are several other easy-to-use and easy-to-communicate coefficients, such as the Bowley and Pearson coefficients, which provide standardized divergence between the mean and the mode, or the median, respectively (see, e.g., Groeneveld and Meeden, 1984). One important disadvantage of all these measures is that they can be strongly influenced by outliers (see Kim and White, 2004) and therefore may provide contrary information; for example, not even the same sign is guaranteed among the different measures.

In this paper, we suggest an alternative, clear-cut skewness measure δ, called *Azzalini's skewness parameter*, defined as the normalized shape parameter of the skew-normal distribution (see Azzalini, 1985). This parametric distribution shares the convenient properties of the normal distribution with the flexibility of having an asymmetric shape. Therefore, it is better able to capture the distributional characteristics of hedge fund returns and is thus especially helpful in describing the nature of these funds.

The contribution of this paper is twofold. First, we show that using the skew-normal distribution to estimate hedge fund returns leads to a better goodness of fit than the normal distribution. Second, we illustrate that using δ as a skewness measure has many advantages compared to the classical skewness coefficients, including:

(1) δ has a limpid financial interpretation as a skewness shock on normally distributed returns;

(2) δ is an increasing transformation of the third standardized moment, but due to its boundness between –1 and +1, it better captures the magnitude of skewness;

(3) δ permits easily grasping of the impact of the skewness shock on the tail risk in hedge fund returns; and

(4) tail-risk measures such as Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) are decreasing functions of δ.

The remainder of this paper is organized as follows. Section 2 provides an overview of the skew-normal distribution. Goodness-of-fit tests for the skew-normal distribution are presented in Section 3. In Section 4, the advantages of using δ with respect to the classical skewness coefficients are discussed. In Section 5 we illustrate how δ can be used as an indicator of tail risk.

2. Skew-normal distribution: A review

Although the origins of the skew-normal distribution can be traced back to Del Helguero (1908), only the work of Azzalini (1985, 2005) has raised attention to these non-Gaussian distributions. Given the limitations of the normal distribution to describe financial time series, some applications of the skew-normal distribution have been carried out recently. Liseo and Loperfido (2006) build a skew-in-mean GARCH model using the skew-normal distribution and apply their model to the U.K. FTSE index. Adcock (2005) incorporates the skew-normal distribution into the capital asset pricing model, and Adcock (2007) extends Stein's lemma to multivariate skew-normal returns. In order to make the paper self-contained we recall the original Azzalini definition. We consider a continuous random variable X having a probability density function of the following form:

$$f_\alpha(x) = 2\phi(x)\Phi(\alpha x), \tag{1}$$

where α is a fixed arbitrary number, ϕ denotes the standard normal density function, and Φ is its distribution function. The class of distributions describe by Equation (1) is called skew-normal distributions with *shape parameter* α, or $X \sim SN(\alpha)$. For practical numerical work, we need to add location and scale parameters. The linear transformation $Y = \xi + \omega X$ is said to have a *skew-normal distribution with parameters* (ξ, ω, α), i.e., $Y \sim SN(\xi, \omega^2, \alpha)$. We refer to ξ, ω (with $\omega > 0$), and α as the *location*, the *scale*, and the *shape* parameters, respectively.

Skew-normal distributions are an extension of the normal distribution. They reduce to the standard normal distribution for α =0 and to the Half-Gaussian when $\alpha \to \infty$. If the sign of α changes, the density is reflected on the opposite side of the vertical axis. This property is highlighted by a characterization of $Y \sim SN(\xi,\ \omega^2,\ \alpha)$, given in Pourahmadi (2007), stating that $Y \sim SN(\xi,\ \omega^2,\ \alpha)$ can be written as a special weighted average of a standard Gaussian and a Half-Gaussian. This suggests an alternative definition of skew normality better suited to financial modeling. A random variable $Y \sim SN(\xi,\ \omega^2,\ \alpha)$ is skew normal if and only if it has the representation:

$$Y = \xi + \omega X = \xi + \omega\left(\delta\left|Z_1\right| + \sqrt{1-\delta^2}Z_2\right), \tag{2}$$

with $\delta = \alpha / \sqrt{1+\alpha^2} \in (-1,1)$. Z_1 and Z_2 are independent N(0;1) random variables; $|\ |$ stands for Half-Gaussian. Formula (2) spotlights the drivers governing the return *Y*:

- a colored noise $\left|Z_1\right|$ shaped as a Half-Gaussian and modulated by $\omega\delta$ and
- a Gaussian noise Z_2 modulated by $\omega\sqrt{1-\delta^2}$.

The parameter δ, called the *Azzalini skewness parameter*, is then a measure of the weight of the Half–Gaussian $\left|Z_1\right|$ on the return *Y*. *Y* collapses into $Y \sim N(\xi,\ \omega)$ if δ =0. The more δ moves toward the extrema ± 1, the higher the relevance of the Half-Gaussian driver, and thus the higher the left or right skewness in *Y*. Moreover, δ is measure of asymmetry around the location parameter ξ and its sign signals the skewness direction. The formula:

$$E(Y) = \xi + \omega\sqrt{2/\pi}\,\delta \tag{3}$$

points out the impact caused by the presence of a half-Gaussian component on the mean. The greater δ, the greater is the mean *E(Y)* because the probability spread over the Half-Gaussian moves on the right. On the contrary, the variance:

$$\operatorname{var}(Y) = \omega^2\left(1 - 2\,\delta^2/\pi\right) \tag{4}$$

shrinks as δ moves toward the extremes and has a maximum for δ=0, which is what one would expect, because as δ moves toward the extremes, distribution tends toward being Half-Gaussian, fading the dispersion on the left or the right side of the location parameter. The third standardized moment, also called the skewness coefficient, for a skew-normal can be expressed in terms of δ:

$$\gamma = \frac{4-\pi}{2} \frac{\left(\delta\sqrt{2/\pi}\right)^3}{\left(1-2\delta^2/\pi\right)^{3/2}} \tag{5}$$

where γ is a strictly increasing and odd function of δ. The two skewness measures have the same sign.

3. Goodness-of-fit tests: Normal vs. skew-normal

As we show in this section, the skew-normal distribution better fits empirical hedge fund data than does the normal distribution. We consider data provided by the Center for International Securities and Derivatives Markets (CISDM), a database widely used in hedge fund research (see, e.g., Ding and Shawky, 2007). It contains 4,048 hedge funds reporting monthly net of fees returns for the period from January 1996 to December 2005.

Table 1 contains descriptive statistics for the return distributions of 30 hedge funds randomly selected out of the database. The table gives mean, standard deviation, and three classical skewness coefficients (third standardized moment, Pearson coefficient, and Bowley coefficient). Furthermore, it shows the moments of the skew-normal distribution along with the resulting theoretical values for the first three moments (calculated using Formulas (3)–(5)). To ensure that the 30 selected funds are not fundamentally different from the full sample of 4,048 hedge funds, we calculate the average over the 30 (second last row of Table 1) and compare these with the average of the full 4,048 hedge funds (last row of Table 1; the full statistics on all 4,048 funds are available upon request). The values in the penultimate and last row are nearly identical, so the selected funds should be a good indicator for the full sample.

Fund No.	Empirical					Skew Normal							Goodness of Fit	
	Mean (%)	St. Dev. (%)	Classical γ	Pearson	Bowley	Location $\xi(\%)$	Scale $\omega(\%)$	Shape α	Delta δ	Mean (%)	St. Dev. (%)	Classical γ	Normal	Skewed Normal
1	1.46	3.62	-0.68	-0.02	-0.04	4.16	4.50	-1.16	-0.76	1.44	3.59	-0.19	217.13	220.07
2	1.91	5.47	-1.04	-0.09	-0.06	7.52	7.71	-2.34	-0.92	1.87	5.24	-0.54	143.72	146.23
3	0.79	0.97	0.03	0.14	0.23	0.44	1.02	0.48	0.43	0.79	0.96	0.02	71.47	73.73
4	1.17	4.24	0.65	0.04	-0.16	-3.40	6.21	2.39	0.92	1.17	4.21	0.55	89.61	92.03
5	0.87	2.53	-0.04	-0.07	-0.09	2.30	2.88	-0.79	-0.62	0.87	2.50	-0.08	131.04	144.26
6	0.88	0.48	0.81	0.32	0.34	0.51	0.56	1.39	0.81	0.87	0.43	0.27	277.26	292.79
7	0.80	2.83	0.23	-0.10	-0.23	-0.99	3.32	0.93	0.68	0.81	2.79	0.12	80.84	107.64
8	1.69	8.33	0.75	0.18	0.28	-7.78	12.51	3.11	0.95	1.73	8.14	0.68	271.57	275.46
9	0.66	3.16	1.19	0.09	-0.09	-2.17	4.07	1.90	0.88	0.70	2.88	0.43	350.33	350.64
10	0.07	4.49	-0.21	0.01	-0.12	3.19	5.41	-1.05	-0.72	0.06	4.42	-0.15	113.96	114.02
11	0.14	2.95	-0.91	-0.11	-0.12	2.74	3.91	-1.61	-0.85	0.09	2.88	-0.34	118.85	118.70
12	0.52	0.83	-0.57	-0.21	-0.31	1.38	1.19	-2.11	-0.90	0.52	0.83	-0.48	251.68	250.50
13	-0.21	20.33	-0.85	-0.17	-0.03	21.16	29.07	-2.49	-0.93	-0.36	19.55	-0.57	291.44	296.95
14	1.71	3.47	0.62	0.08	-0.03	-1.81	4.91	2.08	0.90	1.72	3.41	0.48	188.37	188.35
15	0.89	3.41	0.02	-0.04	-0.20	-0.35	3.61	0.48	0.43	0.89	3.39	0.02	79.89	80.04
16	0.35	7.04	0.05	0.06	0.10	-1.01	7.10	0.25	0.24	0.35	6.97	0.00	271.54	275.37
17	2.12	4.06	0.82	0.27	0.39	-2.02	5.72	2.33	0.92	2.17	3.89	0.54	190.17	191.32
18	-0.37	7.77	1.23	0.03	-0.18	-8.25	10.93	2.38	0.92	-0.21	7.41	0.55	195.62	206.23
19	2.44	15.94	0.34	0.07	0.04	-9.58	19.80	1.18	0.76	2.49	15.70	0.20	252.40	260.77
20	1.42	2.49	0.87	0.14	0.16	-1.09	3.52	2.10	0.90	1.45	2.44	0.48	223.06	222.33
21	0.68	5.59	-0.70	-0.08	0.23	6.74	8.15	-2.63	-0.93	0.66	5.43	-0.60	135.75	138.46
22	5.37	11.62	-0.59	-0.07	0.10	14.84	14.17	-1.50	-0.83	5.45	10.60	-0.30	105.24	110.02
23	2.48	8.26	-0.40	-0.03	-0.08	8.87	10.35	-1.23	-0.78	2.46	8.12	-0.21	239.21	261.98
24	1.78	2.15	0.54	0.36	0.61	-0.50	3.05	2.47	0.93	1.75	2.05	0.57	549.51	555.26
25	1.13	2.80	-2.05	-0.03	0.07	3.74	3.81	-1.90	-0.89	1.05	2.70	-0.43	344.26	346.14
26	1.43	5.67	0.48	0.10	0.17	-3.48	7.48	1.47	0.83	1.46	5.62	0.29	138.74	139.59
27	0.37	0.68	0.23	0.00	-0.01	-0.15	0.85	1.20	0.77	0.37	0.67	0.20	66.99	65.22
28	1.44	3.37	1.15	0.10	0.07	-1.93	4.75	2.16	0.91	1.51	3.28	0.50	251.08	251.73
29	0.91	4.44	-0.66	0.09	0.20	4.76	5.85	-1.50	-0.83	0.88	4.38	-0.30	91.64	93.54
30	0.68	1.76	-0.33	-0.16	-0.26	2.00	2.19	-1.16	-0.76	0.67	1.74	-0.19	151.42	156.20
Mean	0.97	4.67	0.05	0.03	0.03	0.77	6.15	0.26	0.09	0.97	4.54	0.06	196.13	200.85
Mean for the 4,048	0.97	4.37	0.01	0.01	0.03	0.58	5.63	1.52	0.00	0.97	4.17	0.01	192.53	196.58

Table 1: Descriptive statistics and goodness of fit for 4,048 hedge funds

Comparing the three classical skewness coefficients illustrates how classical skewness coefficients can provide ambiguous results. For example, for Fund 4, the third standardized moment is positive, indicating positive skewness; however the Pearson coefficient for this same fund is close to zero (no skewness), and the Bowley coefficient is negative, indicating negative skewness.

To investigate parameter fit, we compare the empirical mean, standard deviation, and skewness coefficient with the corresponding skew-normal values. The correlation between the empirical mean, standard deviation, and skewness coefficient with the corresponding skew-normal values for the 30 selected hedge funds are 1.00, 1.00, and 0.90. It thus seems that the skew-normal fits the empirical parameters quite well.

Table 1 also presents the log likelihood value of the normal and the skew-normal to compare the goodness of fit of different distributions. For 25 of the 30 hedge funds, the goodness of fit is higher for the skew normal. The log likelihood value of the skew normal is on average 200.85, which is higher than the goodness of fit for the normal distribution (196.13 on average). The skew normal thus better fits the distributional characteristics of hedge fund returns than does the normal distribution (additional tests, available upon request, show that the goodness of fit of the skew normal is also good compared to other distributions used to model hedge fund returns, e.g., the logistic or the normal inverse Gaussian; see Kassberger and Kiesel, 2006).

4. Ability of skewness parameter δ to spotlight asymmetry

As known from the literature and empirically shown in Section 2, the Bowley and Pearson coefficients can provide contradictory information on the direction of skewness with respect to the classical standardized third moment γ. Azzalini's δ has the advantage of being a strictly increasing function in γ and of having the same sign as γ. In addition δ is bound between –1 and +1, whereas γ may be arbitrarily large. Figure 1 shows the distribution of the skewness parameter δ for the 4,048 hedge funds. The plots show a clear-cut bang-bang behavior: δ is bimodal, with a mean and median of zero. The values of δ are mostly gathered at the extremes, e.g., 96% (81%) of all hedge funds have an absolute value of δ larger than 0.5 (0.75), and it is almost always nonzero, thus confirming the strong influence of the Half-Gaussian factor and the nonnormality of hedge funds.

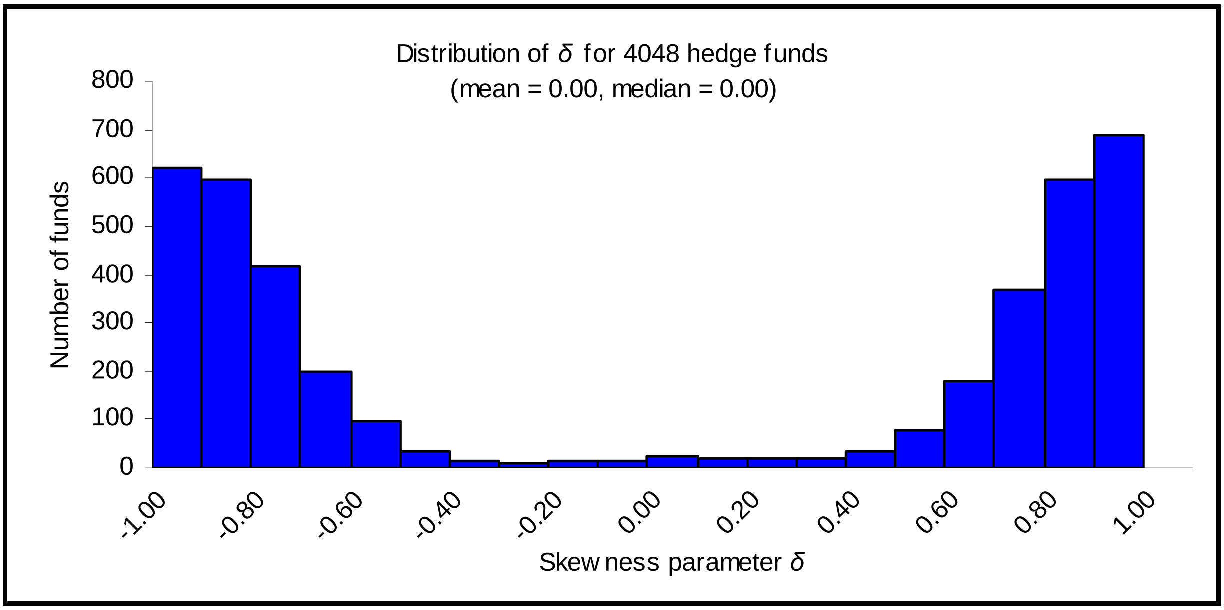

Figure 1: Skewness parameter δ for 4,048 hedge funds

The fact that the median coincides with the mean shows that 50% of the time, Y is negatively skewed, so the hedge fund return is more inclined to undergo Azzalini's location parameter ξ. In this context, the location parameter ξ can be interpreted as a benchmark for the hedge fund under consideration. For the other 50% of the time, the hedge fund return Y is more inclined to outperform its benchmark ξ because Y is positively skewed. However, the location ξ and the scale ω parameters may vary greatly from one fund to another (see Table 1) and thus it would be wrong to evaluate performance only on the basis of δ. Such a performance ranking would be appropriate only in the special case that the location ξ and the scale ω parameters are equal for all candidates under comparison; therefore, only skewness would be the tradeoff.

In conclusion, although δ and γ provide exactly the same information, in the case of skew normality, the use of δ is more advisable because it guarantees short-cut information on the fatter tail and how close to Half-Gaussian it is.

5. Skewness parameter δ illustrates hedge fund tail risk

The literature points out that hedge funds exhibit significant tail risk and that their returns can be explained by option strategies (see Fung and Hsieh, 1997; Darius et al., 2002). For example, a large number of hedge funds exhibit payoffs resembling a short position in an out-of-the-money put option on the market index (see Mitchell and Pulvino, 2001; Agarwal and Naik, 2004). The returns of these hedge funds are positive most of the time until a tail event makes the option pay out and the fund experiences a large loss. These funds thus bear significant tail risk.

The skewness parameter δ is an easy-to-use indicator of tail risk. In fact, for general distributions, an increase in skewness does not guarantee a decrease in tail risk, due to the possible presence of extreme events with low probability of occurrence. We now show that for skew-normal distributions, the two most commonly used risk measures, namely, Value-at-Risk (VaR) and expected shortfall (ES; also called Conditional Value-at-Risk, CVaR), are decreasing functions of δ. The higher δ, the lower the influence of Half-Gaussianity on the left tail and, consequently, the lower the tail risk measured by VaR and ES.

Let L be the random loss of Y, i.e., $L = -Y$. Then the VaR at level $0 < c < 1$ (typically, 0.95 or 0.99) denoted by $VaR_c\left(L\right)$ is the loss level that will be exceeded only with probability c, i.e.,

$P\left(L > VaR_c\left(L\right)\right) = c$, then in 100(1 – *c*)% of the cases the loss is smaller or equal to $VaR_c\left(L\right)$. The skew normal is an absolutely continuous distribution; thus, $VaR_c\left(L\right) = -F_Y^{-1}\left(c\right) = -y_c$, where F_Y is the cumulative distribution function of *Y*, and y_c its *c-th quantile.* Note that $y_c = \xi + \omega x_c$, where x_c is the *c-th quantile* of $X \sim SN\left(\alpha\right)$.

The cumulative function F_X is defined through a parametric integral, then after differentiating:

$$\frac{\partial F_X\left(x_c\right)}{\partial \delta} = \int_{-\infty}^{x_c} 2\left[\phi'\left(t\right)\Phi\left(\alpha t\right) + \phi\left(t\right)\phi\left(\alpha t\right)\alpha'\right]dt + 2\alpha'\phi\left(x_c\right)\Phi\left(\alpha x_c\right),$$

where $\alpha = \frac{\delta}{\sqrt{1-\delta^2}}$ and $\alpha' > 0$. Since $c < 0.5$ and, consequently, $x_c < 0$ and $\phi' > 0$, it follows that $\frac{\partial F_X\left(x_c\right)}{\partial \delta} > 0$. So we get $\frac{\partial F_X^{-1}\left(c\right)}{\partial \delta} > 0$ as well. In conclusion, $y_c = \xi + \omega x_c$ is an increasing function of δ, and VaR is a decreasing function of δ. Thus, the greater δ, the smaller the tail risk measured by the VaR.

We now consider the expected shortfall (ES) (see Acerbi and Tasche, 2002). ES at level c is defined as:

$$\mathrm{ES}_c(Y) = -\frac{1}{c}\int_0^c F_Y^{-1}\left(p\right)dp .$$

Again after differentiating, we get:

$$\frac{\partial \mathrm{ES}_c(Y)}{\partial \delta} = -\frac{1}{c}\int_0^c \frac{\partial F_Y^{-1}(p)}{\partial \delta}dp .$$

Since $\frac{\partial F_X^{-1}\left(c\right)}{\partial \delta} > 0$, $\frac{\partial F_Y^{-1}\left(c\right)}{\partial \delta} > 0$ as well, then $\frac{\partial \mathrm{ES}_c(Y)}{\partial \delta} < 0$. So, ES_c is a decreasing function of δ. In conclusion, the greater δ, the smaller the tail risk of a skew normal, if the tail-risk is measured by the VaR or the expected shortfall. The values of VaR_c and ES_c for δ=0, collapsing to the case of $Y \sim N\left(\xi,\omega\right)$, may be used as reference points. In such a Gaussian case, $VaR_c(L) = -\xi - \omega x_c$ and $ES_c\left(Y\right) = -\xi + \frac{\omega e^{-x_c/2}}{c\sqrt{2\pi}}$ (see Panjer, 2002), where x_c is the *c*-th quantile of $N\left(0,1\right)$.

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