Sufficient Conditions for Expected Utility to Imply Drawdown-Based Performance Rankings

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Preprint Series: 2010-07

Fakultät für Mathematik und Wirtschaftswissenschaften
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When does the choice of performance measure not matter?

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This version: 2009/11/26

Abstract: We analyze the Sharpe ratio and 14 alternative reward-to-risk ratios. Every alternative ratio leads to the same ranking of investment funds as the Sharpe ratio if the funds' return distributions satisfy the location and scale condition (see Meyer, 1987). It then makes no difference whether funds are ranked with the Sharpe ratio or an alternative ratio.

JEL classification: D81; G10; G11; G23; G29

Keywords: Asset management; Performance measurement; Performance; Sharpe ratio; Location and scale condition

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1. Introduction

The most widely known performance measure is the Sharpe ratio, a simple reward-to-risk ratio, which measures the relationship between the mean and the standard deviation (MS) of the excess returns generated by a fund. The Sharpe ratio is an adequate performance measure if the investor wishes to place all his risky assets in a single fund and either the investors' preferences or the investment funds' returns satisfy certain conditions that ensure consistency between the MS and the expected-utility (EU) model (see Meyer and Rasche, 1992, for a summary of these conditions).

During the last two decades, many authors have proposed alternatives to the Sharpe ratio (see Cogneau and Hubner, 2009) because they believe that the conditions that ensure consistency between the MS and the EU model have theoretical defects and/or lack empirical support. A common argument for using another reward-to-risk ratio is that the choice of a performance measure depends on the fund's return distribution (see, e.g., Amin and Kat, 2003). In case of normally distributed returns, ratios that rely on the first two moments of the return distribution (mean, standard deviation), as does the Sharpe ratio, are appropriate. Many studies, however, illustrate the presence of skewness and fat tails in financial market returns (see, e.g., Leland, 1999). For example, hedge funds frequently generate non-normal returns and it is thus commonly believed that these cannot be evaluated using the Sharpe ratio (see, e.g., Sharma, 2004). Consideration of this issue has led to the development of many alternative reward-to-risk ratios.

Eling and Schuhmacher (2007) conducted an empirical study based on hedge fund return data and compared the Sharpe ratio to ten alternative ratios. They found that despite significant deviations of hedge fund returns from a normal distribution, the comparison of the Sharpe ratio to the other reward-to-risk ratios resulted in virtually identical rank ordering across hedge funds. This finding is surprising given the vast literature on alternative performance ratios and has motivated recent attempts to explain this result (see Nguyen-Thi-Thanh, 2008; Schmid and Schmidt, 2009; Ornelas, Almeida Silva and Fernandes, 2009; Zakamouline, 2009).

The aim of this paper is to offer an explanation for the finding of Eling and Schuhmacher (2007). The explanation rests upon the following result which is shown analytically for value-at-risk-based performance measures and numerically with Monte-Carlo simulations for partial-moments- and drawdown-based performance measures. Every alternative reward-to-risk ratio that we analyze leads to the same ranking as the Sharpe ratio if the funds' return distributions satisfy the location and scale (LS) condition (see Meyer, 1987). Among the distributions that satisfy the LS condition are the following distributions commonly used in finance (see Levy and Duchin, 2004): the normal, the beta, the extreme value, the gamma, the student's t, the uniform, and the Weibull distribution.

This work extends the findings of Eling and Schuhmacher (2009) who show this result for a new class of modified drawdown-based performance measures. Similar analytical results exist for normally distributed returns and Omega (see Schmid and Schmidt, 2009), partial moments-based, and VaR-based performance measures (see Zakamouline, 2009). This work also extends the research of Fung and Hsieh (1999) who suggest that the mean-variance analysis of hedge funds approximately preserves the ranking of preferences in standard utility functions. While Fung and Hsieh (1999) show that using the MS model to rank hedge funds will produce rankings which are nearly correct, we show why the ranking of hedge funds under the MS model is very comparable to alternative ratios used in literature.

2. Performance Measures

Following Eling and Schuhmacher (2007) we divide the set of alternative reward-to-risk ratios into three classes. The first class contains ratios that are based on partial moments: Omega, the Sortino ratio, Kappa 3 and the upside potential ratio are defined in Eling and Schuhmacher (2007). The q-Sortino-Satchel ratio and the p-q-Farinelli-Tibiletti ratio are defined in Eling et al. (2009). Ratios based on the value-at-risk (VaR) built the second class. The definitions for the reward-to-VaR ratio and the reward-to-conditional VaR ratio are given in Zakamouline (2009). The α-β-Rachev ratio is defined in Eling et al. (2009). The third class contains ratios that are based on drawdowns. The Calmar ratio, the Sterling ratio, the Burke ratio, the Martin ratio and the Pain ratio are explained in Eling and Schuhmacher (2009). Altogether we analyze 14 alternative reward-to-risk ratios.

3. The Location and Scale Condition

The LS-condition requires that the random alternatives (i.e., in our case, the returns of $i=1,\ldots,n$ hedge funds) differ only in location and scale parameters (Meyer, 1987). That is, the random hedge funds' returns r_i satisfy the general form $r_i=a_i+b_i\cdot x_i$, whereby a_i and $b_i>0$ are nonrandom and x_i are identically distributed random terms.

To show the result for VaR-based performance measures, we assume for the sake of simplicity that x_i has zero mean and unit variance. Then a_i is the mean and b_i the standard deviation of r_i. Funds' i reward-toVaR ratio is $R2VaR(r_i)=-a_i/(a_i-b_i \cdot VaR(x_i))$. Dividing the numerator and denominator by b_i yields $R2VaR(r_i)=SR_i/(VaR(x_i)-SR_i)$ whereby SR_i is funds' i Sharpe ratio, i.e. $SR_i=a_i/b_i$. It is easy to see that $R2VaR(r_i)$ is strictly increasing in Sharpe's ratio. Using the same argument it can be shown that i's reward-to-CVaR ratio is $R2CVaR(r_i)=SR_i/(CVaR(x_i)-SR_i)$ which is again strictly increasing in Sharpe's ratio. It can also be shown that i's Rachev ratio is $R=(CVaC(x_i)+SR_i)/(CVaR(x_i)-SR_i)$ strictly increasing in Shape's ratio, whereby $CVaC(x_i)$ denotes the expected tail return above VaR_α. Hence, for returns satisfying the LS-condition all VaR-based performance measures lead to the same ranking as the Sharpe ratio.

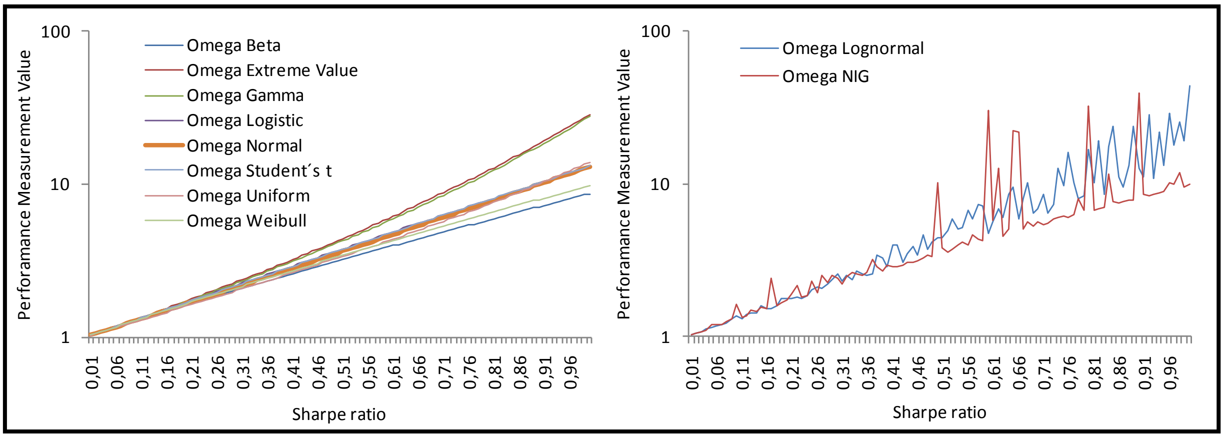
The derivation of an analytical solution for drawdown- and partial-moment-based performance measures is not trivial, but we can easily illustrate the result using Monte-Carlo simulations. We therefore consider the following distributions which satisfy the LS-condition: beta (Johnson et al., 1995, p. 210) with shape parameters a=5 and b=1 (implies that skewness is –1.2 and kurtosis 4.2), extreme value (Johnson et al., 1995, p. 111) with skewness 1.1 and kurtosis 5.4, gamma (Johnson et al., 1994, p. 337) with shape parameter a=4 (skewness 1, kurtosis 4.5), the normal with skewness 0 and kurtosis 3, student's t (Fergusson and Platen, 2006, p. 26) with eight degrees of freedom (skewness 0, kurtosis 4.5), uniform (Johnson et al., 1995, p. 276) with skewness 0 and kurtosis 1.8, and Weibull (Johnson et al., 1994, p. 629) with shape parameter a=30 which (skewness –1, kurtosis 4.6). We also analyze two distributions that do not satisfy the LS-condition: The lognormal and the normal inverse Gaussian with shape parameters a=2 and b=–1 (skewness –1, kurtosis 5.7). For illustration purposes, we have choosen the shape parameters such that skewness is approximately -1 and kurtosis is approximately 4 (the results for other parameters are comparable).

4. Numerical Illustrations With Monte Carlo Simulation

For every return distribution we generate 100,000 returns for 100 hypothetical funds with exogenously fixed theoretical Sharpe ratios which range from 0.01 to 1. We then calculate all performance measures for the 100,000 returns and iterate this procedure 1,000 times. The simulation results are illustrated using the measure Omega (the results for all other ratios are comparable and are available upon request). The left (right) part of Figure 1 shows all distributions from our set that (do not) fulfill the LS-condition. The x-axis represents the fixed values of the Sharpe ratio (the 100 hypothetical funds with Sharpe ratios from 0.01 to 1). The y-axis shows the results for Omega (the mean across 1,000 iterations).

The normal distribution is depicted in the figure by the heavy line. Due to the high number of distributions in the upper part of the figure, it is not possible to identify single distributions, but there are two results worth emphasizing. First, all distributions shown in the left part of the figure are strictly increasing functions of the Sharpe ratio; the distributions in the right part are not strictly increasing. Second, the results for distributions are very comparable to the results for the normal distribution.

Figure 1: Omega (1,000 iterations with 100,000 returns)



Further simulations indicate that every alternative reward-to-risk-ratio leads to the same ranking of investment funds as the Sharpe ratio if the funds' return distributions satisfy a generalization of the location and scale condition (see Meyer and Rasche, 1992). The advantage of this generalization is that funds with different skewness and kurtosis values can satisfy it, whereas the standard LS condition implies identical skewness and kurtosis values for all funds.

5. Conclusions

Random alternatives do not need to be normally distributed to ensure consistency between the MS and the EU model. They might follow any other distribution as long as these fulfil the LS-condition. It then makes no difference whether funds are ranked with the Sharpe ratio or an alternative reward-to-risk ratio. This result might explain the high rank correlations that Eling and Schuhmacher (2007) have found when comparing rankings for Sharpe ratios and alternative measures although hedge fund returns are far from normally distributed.

This finding also has another important implication for the use of alternative reward-to-risk ratios. Many of the alternative measures were developed by practitioners who did not base their work on theoretical considerations such as decision theory. However given our numerical result, the use of alternative ratios is theoretically sound under the same conditions as is the Sharpe ratio, i.e., for random alternatives that satisfy the LS-condition.

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Material available upon request

Figure A1: Sortino Ratio (1,000 iterations with 100,000 returns)

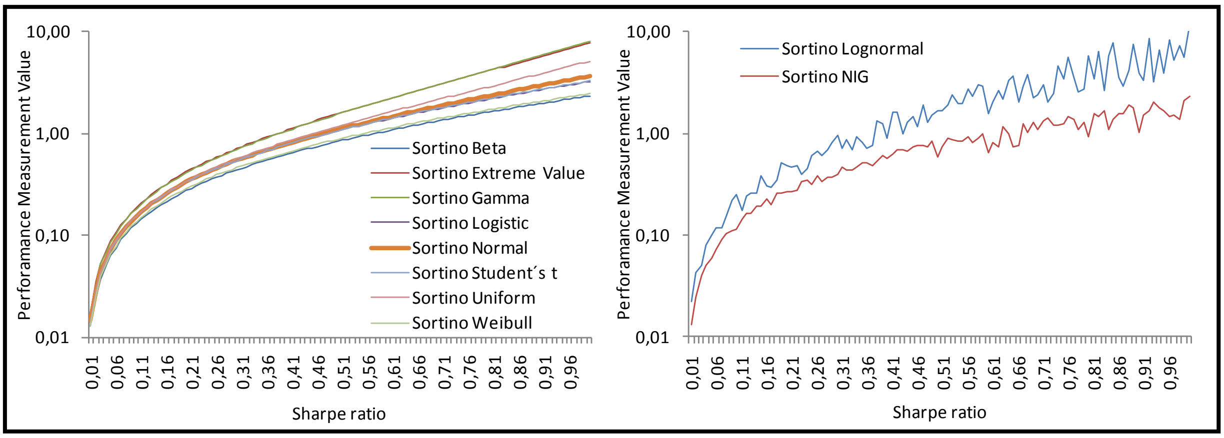

Figure A2: Kappa 3 (1,000 iterations with 100,000 returns)

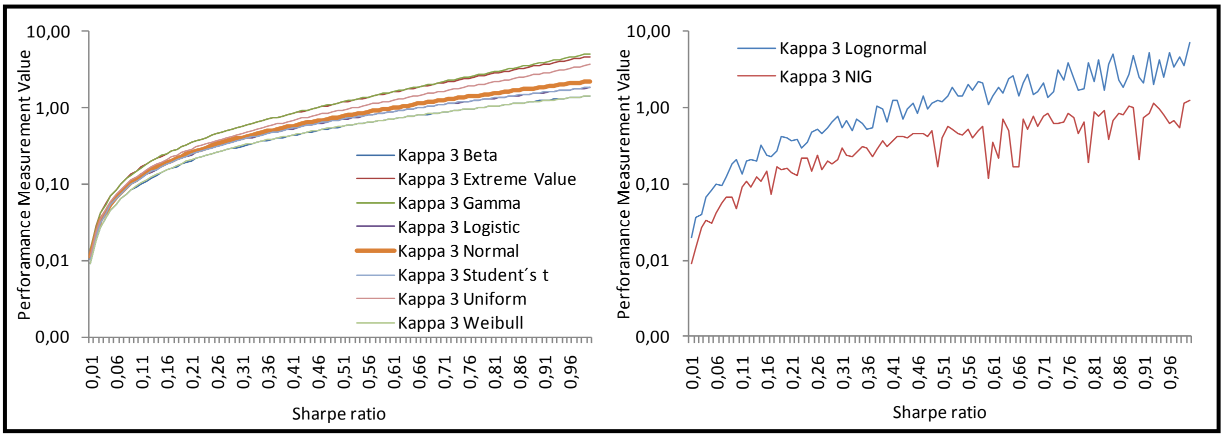

Figure A3: Upside Potential Ratio (1,000 iterations with 100,000 returns)

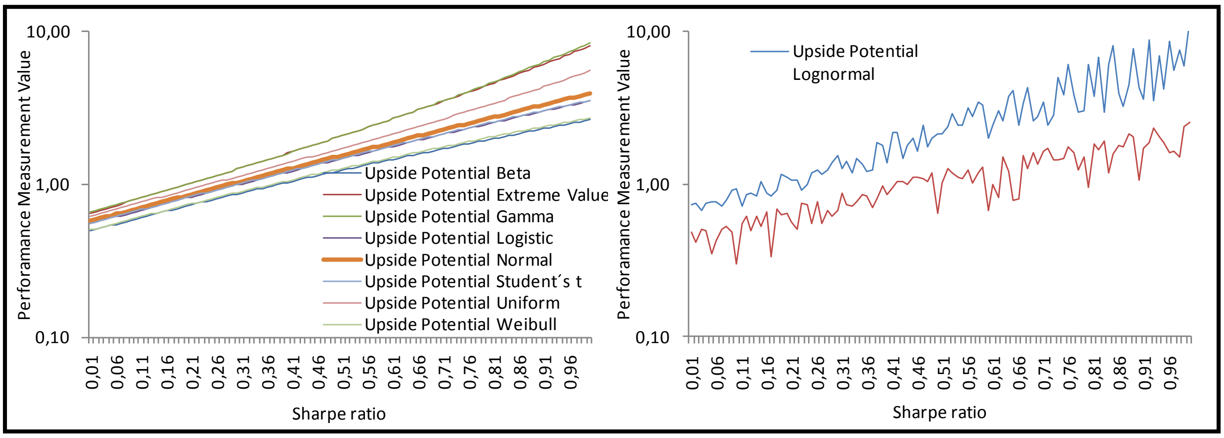

Figure A4: Sortino Satchell Ratio (q = 0.2; 1,000 iterations with 100,000 returns)

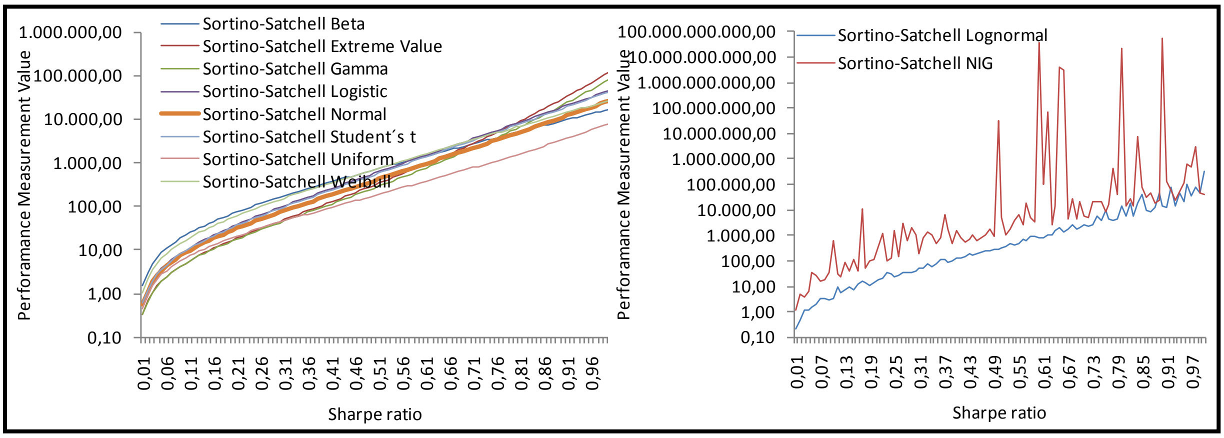

Figure A5: Sortino Satchell Ratio (q = 1; 1,000 iterations with 100,000 returns)

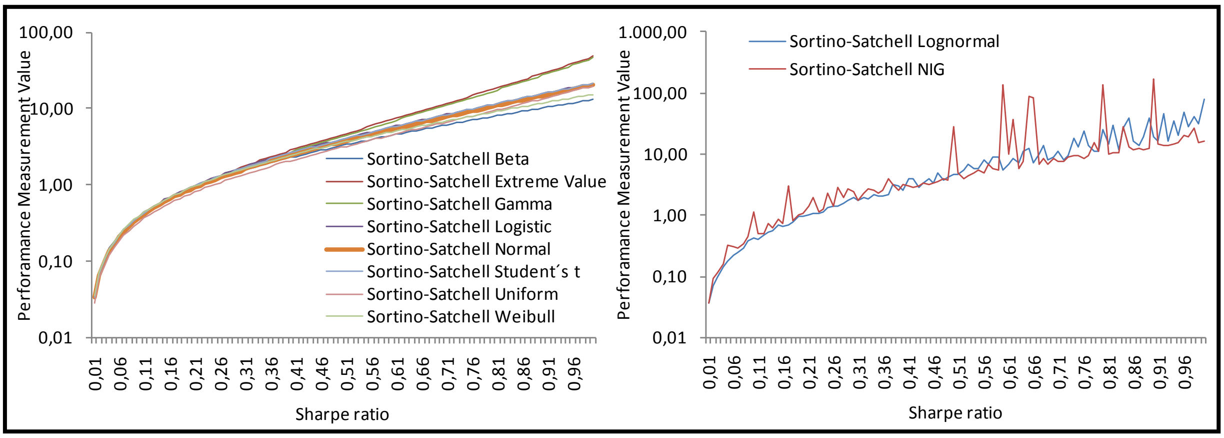

Figure A6: Sortino Satchell Ratio (q = 4; 1,000 iterations with 100,000 returns)

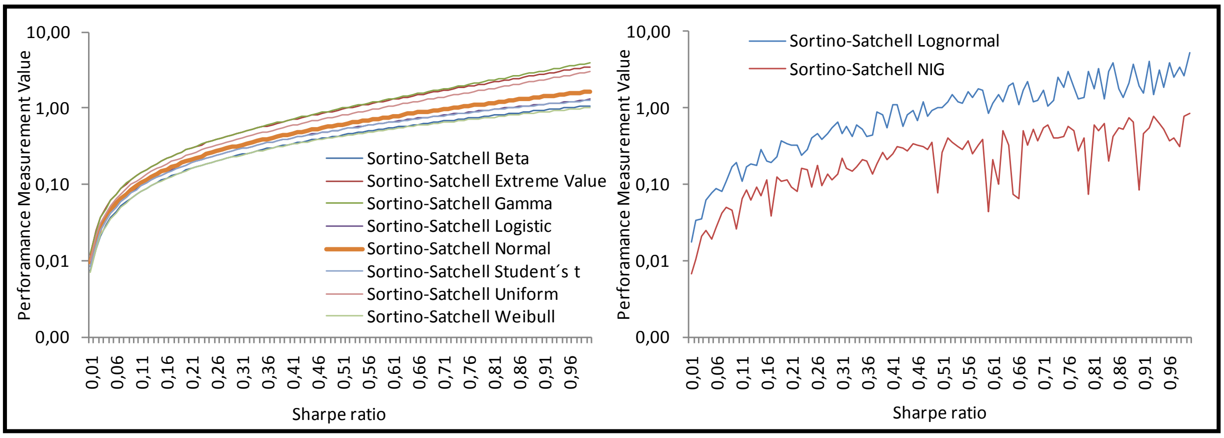

Figure A7: Farinelli Tibiletti Ratio (p = 0.2, q = 2; 1,000 iterations with 100,000 returns)

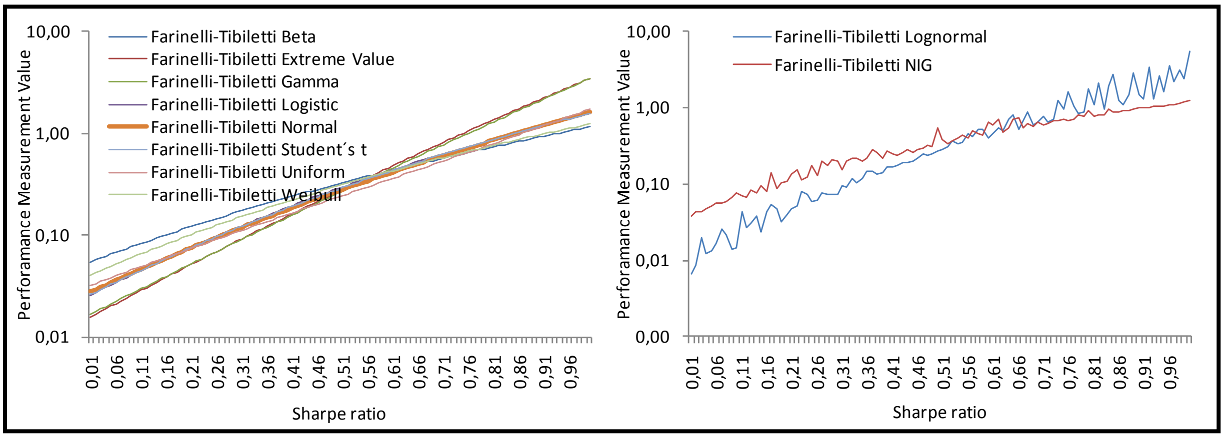

Figure A8: Farinelli Tibiletti Ratio (p = 1, q = 1; 1,000 iterations with 100,000 returns)

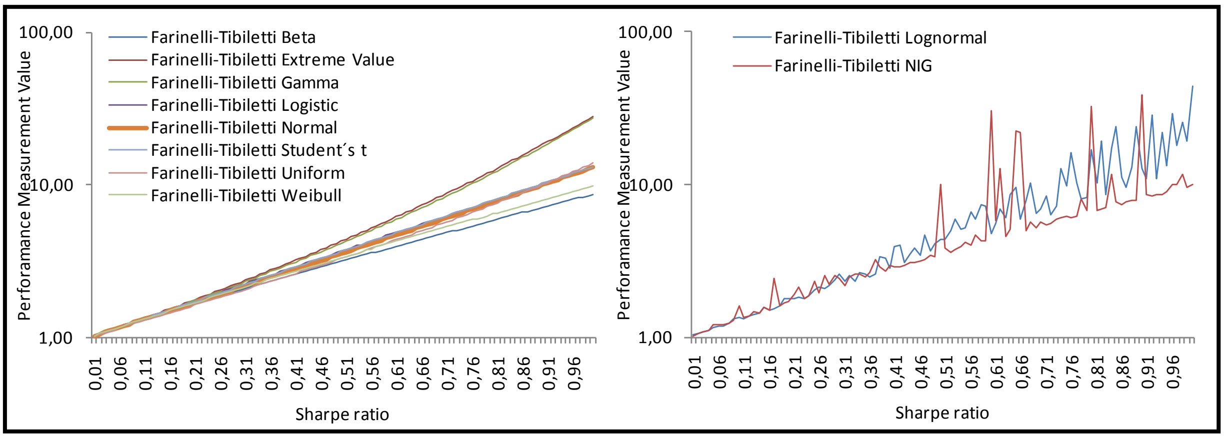

Figure A9: Farinelli Tibiletti Ratio (p = 4, q = 4; 1,000 iterations with 100,000 returns)

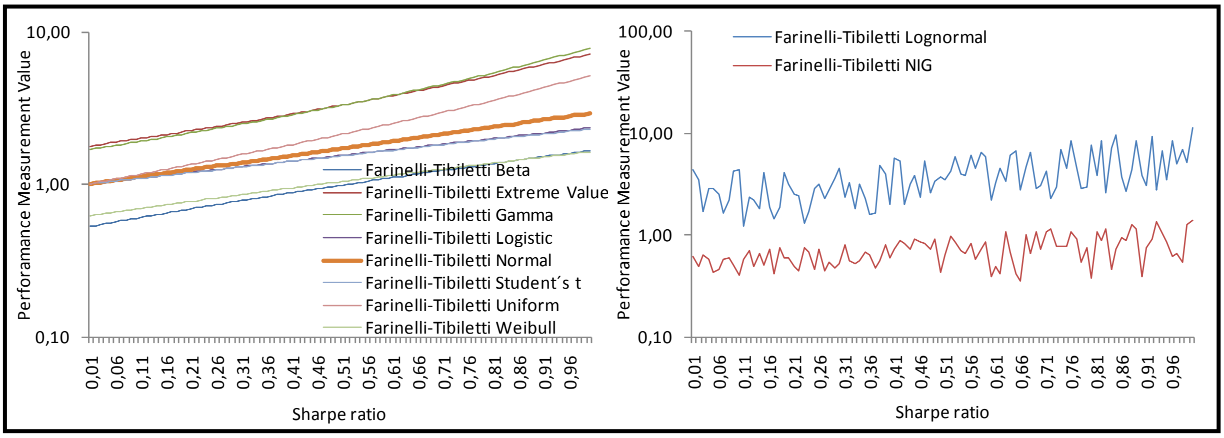

Figure A10: Calmar Ratio (1,000 iterations with 100,000 returns)

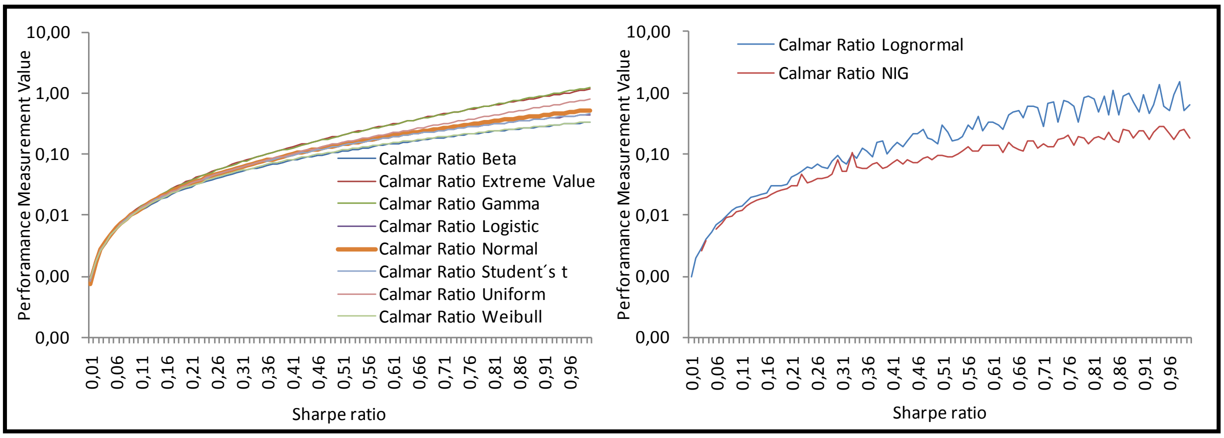

Figure A11: Sterling Ratio (1,000 iterations with 100,000 returns)

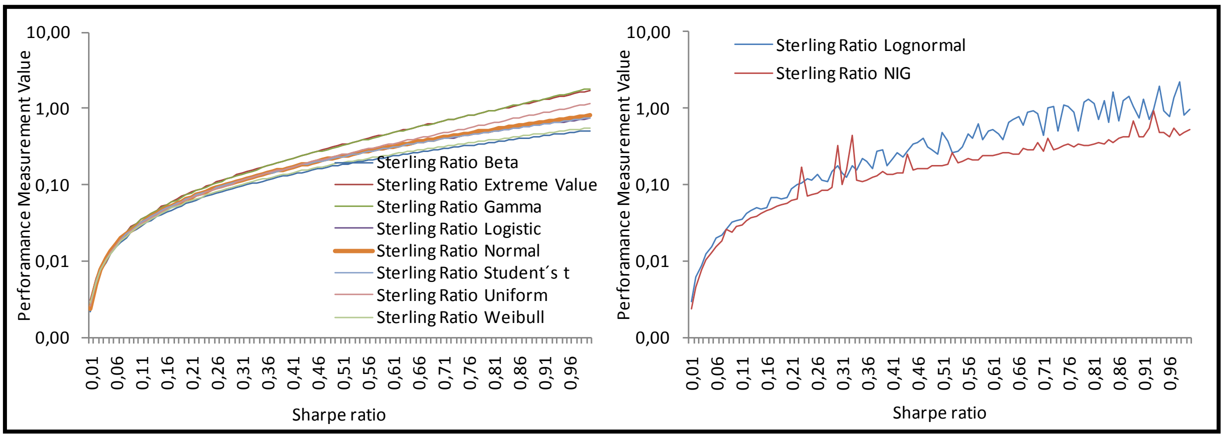

Figure A12: Burke Ratio (1,000 iterations with 100,000 returns)

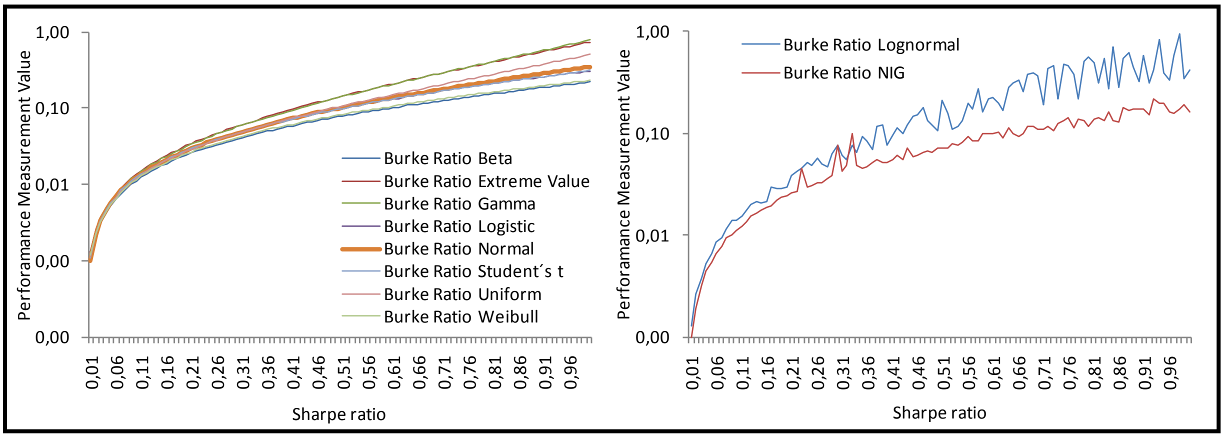

Figure A13: Martin Ratio (1,000 iterations with 100,000 returns)

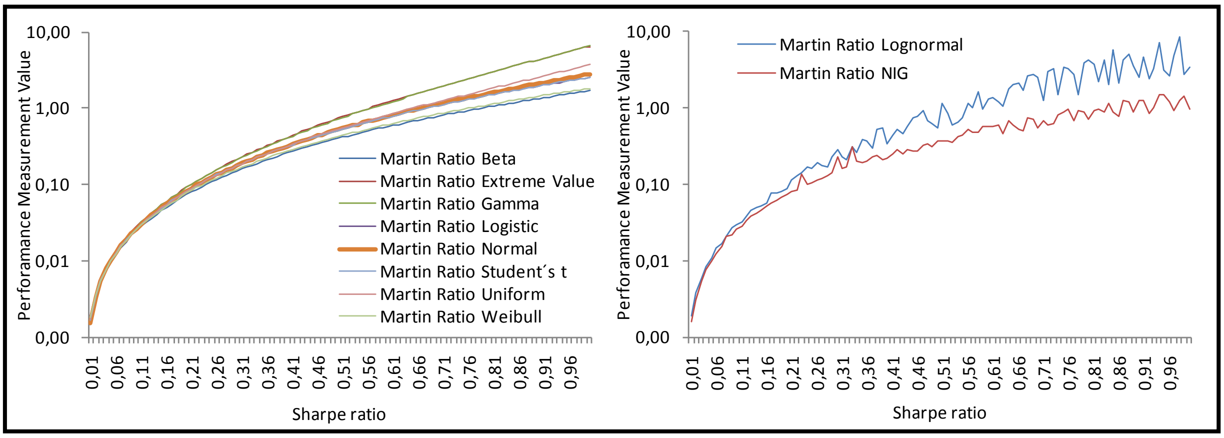

Figure A14: Pain Ratio (1,000 iterations with 100,000 returns)