Participating Life Insurance Contracts under Solvency II: Inheritance Effects and Allowance for a Going Concern Reserve

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Abstract The final draft of the implementing measures for the Solvency II framework is still under discussion. A crucial aspect in this debate is the appropriate reflection of surplus participation mechanisms that apply to traditional participating life insurance contracts. In particular, the inheritance of profits between existing business and new business resulting from the surplus participation process has to be incorporated in the Solvency II valuation framework which requires a run off valuation of the existing portfolio under going concern assumptions. This paper analyzes the inheritance effects caused by the pre-financing of acquisition cost of new business by cost surplus of existing business which is inherent in traditional German life insurance in the context of Solvency II. We show that an allowance for the inherited funds – denoted as Going Concern Reserve (GCR) – is justified in general and in line with the Solvency II valuation principles. Based on a stochastic balance sheet and cash flow projection model, we present a methodology to quantify the GCR and provide a profound analysis of the GCR and its components. Our results show that the GCR has a significant impact on the overall solvency situation of life insurance companies offering participating contracts.

Keywords Going Concern Reserve · Participating Life Insurance · Inheritance · Run Off · Solvency II

1 Introduction

In November 2013, an agreement on the implementation of Solvency II was reached between the trilogue parties (European Parliament, European Council and European Commis-

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sion) which means that Solvency II will enter into force in 2016. However, discussions on the details of this risk based solvency framework continue.

A crucial aspect in this debate is the appropriate reflection of surplus participation mechanisms that apply to traditional participating life insurance contracts. In this type of business (which plays a major role in old-age provision in Continental Europe), policyholders are entitled to a participation in the surplus of the company, in addition to the contractually guaranteed benefits. In Germany and other member states of the European Union, surplus is derived on an aggregated (portfolio) level and allocated to individual policyholders based on specific actuarial methods. This results in balancing effects between different cohorts of policyholders as well as balancing effects over time. In particular, balancing effects between the existing business and new business arise since cash flows related to new business significantly affect surpluses in subsequent years. As a consequence, the insurer's future obligations towards today's policyholders (as well as its future profits from existing business) are affected by future new business.

Under the Solvency II regime, the valuation of liabilities is restricted to the existing business at the valuation date (run off). However, the valuation method should be based on the assumption that the company will continue to write new business (going concern). This implies that balancing effects and the resulting inheritance of profits between existing business and new business have to be properly taken into account.

The impact of future new business on Future Discretionary Benefits (FDB) was first recognized in the German standard valuation model – the so-called Cash Flow Model – developed by the German Association of Insurance Companies (GDV) for the Quantitative Impact Study 6 (QIS6) in 2012 (GDV, 2012). The methodology was slightly refined for the Long Term Guarantee Assessment (LTGA) in 2013 (GDV, 2013b). Essentially, the present value of these inherited future profits is denoted as Going Concern Reserve (GCR) and treated as a Basic Own Funds (BOF) component under Solvency II. This approach, however, is still very approximate in nature and not based on a reliable modeling of inheritance effects. Furthermore, a sound justification of the allowance for the GCR as BOF is missing. Recently, the Federal Financial Supervisory Authority (BaFin) published some comments on the consideration of inheritance effects under Solvency II (cf. BaFin, 2014); Wagner (2013) also gives a short overview on this topic. Overall, the question of a proper integration of these effects in the Solvency II valuation framework has remained unanswered in scientific literature so far.

Our paper addresses this gap and the weaknesses of the methods proposed by practitioners. More specifically, it analyses the pre-financing of acquisition costs of new business by cost surpluses from existing business which is inherent in traditional German life insurance and provides a profound analysis of the inheritance effects in the context of Solvency II.

In the last decade, the valuation of life insurance companies based on stochastic methods has attracted more and more attention in academic literature as well as among practitioners; this appears to be related to the introduction of risk based solvency frameworks such as Solvency II (cf. Jørgensen, 2004). Therefore, a number of scientific papers discuss the modeling and the market-consistent (fair) valuation of participating life insurance contracts (cf. e. g. Briys and De Varenne, 1997; Grosen and Jørgensen, 2000; Grosen and Jørgensen, 2002; Miltersen and Persson, 2003; Tanskanen and Lukkarinen, 2003; Bacinello, 2003; Gerstner et al., 2008). Some papers explicitly focus on the surplus distribution mechanism in Germany (Bauer et al., 2006; Bohnert and Gatzert, 2012; Maurer et al., 2013; Reuß et al., 2014) and others discuss the problem of a valuation under Solvency II (Kochanski and Karnarski, 2011; Ramos and Simões, 2013). However, none of them takes into account the inheritance effects which are common in traditional German life insurance.

In contrast, we explicitly reflect these effects in a run off valuation under going concern assumptions as required by Solvency II. Our method allows for a quantification of the inherited funds, a detailed analysis of the composition of the GCR, and addresses the impact on the overall solvency situation of life insurance companies offering participating contracts.

The remainder of the paper is organized as follows. Section 2 provides a justification for the allowance of inheritance effects under a Solvency II valuation, based on the surplus participation process in Germany. We also illustrate the inheritance effects in a simplified example. In section 3, we introduce the valuation framework underlying our quantification method. Subsequently, section 4 describes a methodology for the setup of a Solvency II balance sheet for the exisiting business under going concern assumptions that takes into account the inheritance of cost surplus to future new business. The results of our analyses are presented in section 5, with a number of sensitivities discussed in section 6. Finally, section 7 summarizes our findings and concludes.

2 Justification of a Going Concern Reserve

We first discuss how allowance for a GCR can be justified by combining the Solvency II valuation principles (section 2.1) with existing surplus participation mechanisms (section 2.2).

2.1 Principles for valuation of life insurance contracts under Solvency II

In this section we summarize all aspects of the Solvency II valuation framework relevant for the subsequent discussions. The discussion reflects the status of the regulatory requirements as of 30 April 2014. In particular, we rely on the latest version of the *Draft Delegated Acts Solvency II* (*Draft DA*) as of 14 March 2014 and the *Technical Specifications for the Solvency II Preparatory Phase (Part I)* (*TS*) provided by EIOPA on 30 April 2014.

The valuation of insurance contracts under Solvency II is based on the concept of a transfer value. Art. 75 of *Directive 2009/138/EC of the European Parliament and of the Council of 25 November 2009 on the taking-up and pursuit of the business of Insurance and Reinsurance (Solvency II)* (*Directive 2009/138/EC*) requires that assets and liabilities *"shall be valued at the amount for which they could be transferred, or settled, between knowledgeable willing parties in an arm's length transaction."* More specifically, insurance companies have to set up so-called Technical Provisions for its obligations towards policyholders according to art. 76 of *Directive 2009/138/EC*: *"Their value shall correspond to the current amount insurance and reinsurance undertakings would have to pay if they were to transfer their insurance and reinsurance obligations immediately to another insurance or reinsurance undertaking."*

Furthermore, art. 77 of *Directive 2009/138/EC* specifies that Technical Provisions are the sum of a Best Estimate (BE) and a Risk Margin, where the BE corresponds to the expected present value of all future cash in- and outflows required to settle the insurance obligations. Note that cash outflows reflect all expenses (including overhead expenses) that will be incurred in servicing the insurance obligations, including an allowance for future expense inflation (cf. *Directive 2009/138/EC*, art. 78 and *Draft DA*, art. 24 TP11). In case of participating life insurance contracts, the cash flows include the FDB, i.e. payments which are expected to be made in excess of the contractually guaranteed benefits; the value of the FDB is disclosed separately (cf. *Directive 2009/138/EC*, art. 78 and *Draft DA*, art. 20 TP7 and 20bis TP7bis). The calculation of the FDB and BE is typically based on an explicit portfolio projection in an actuarial cash flow projection model.

In general, Solvency II requires a run off approach, i.e. the valuation is based on the existing portfolio of insurance contracts (in-force business) and does not include future new business.[1] This implies that the time horizon for the cash flow projections underlying the BE coincides with the remaining term of the contracts in-force at the valuation date.

However, art. 5 V1 of the *Draft DA* specifies that the valuation shall be based *"on the assumption that the undertaking will pursue its business as a going concern."* Therefore, assumptions underlying the projection of future cash flows for the in-force business have to be derived under the assumption that the undertaking will write new business in the future.

This holds particularly for expense assumptions and includes a proper reflection of overhead expenses (cf. *Draft DA*, art. 24 (4) TP11). As a consequence, overhead expenses need to be split between existing business and future new business; only the overhead expenses attributable to the existing business are included in the calculation of the BE (cf. *TS*, TP.2.55).

The going concern concept also has an impact on the valuation of the FDB. Assumptions underlying the derivation of the FDB should be objective, realistic and verifiable. Existing profit sharing mechanisms need to be properly reflected, e. g. regarding the split of the surplus between policyholders and the insurance company as well as the allocation among policyholders (cf. *TS*, TP.2.114 and TP.2.115). Again, only expected future discretionary benefits attributable to the existing business are included in the calculation of the BE.

The run off approach under going concern assumptions also applies to the calculation of the Risk Margin, which follows a cost-of-capital approach (cf. *Directive 2009/138/EC*, art. 77). Overall, the amount of Technical Provisions needs to be consistent with the transfer value as defined in art. 76 of *Directive 2009/138/EC*. The Technical Provisions are then combined with the Solvency II valuation of investments and other assets/liabilities to derive the BOF.

The Solvency Capital Requirement (SCR) under Solvency II is based on the BOF and therefore significantly affected by the amount of the Technical Provisions. Art. 101 (3) of *Directive 2009/138/EC* specifies that the time horizon for the SCR is one year and that the SCR covers both, existing business and new business expected to be written over the following 12 months. As a simplification in the standard formula, expected new business is ignored for life insurance companies (cf. *TS*, SCR.1.13). This implies that the SCR is also based on the concept of a run off under going concern assumptions.

The SCR modules in the standard formula are based on instantaneous stresses that are assumed to occur at the valuation date. E. g. the SCR for expense risk (a submodule of the life underwriting risk module) is derived from a pre-defined stress with respect to the expenses taken into account in the calculation of the BE (cf. *Draft DA*, art. 110 LUR5):

- an increase of 10 % in the amount of expenses,
- combined with an increase of the expense inflation rate by one percentage point.

The BE after stress is determined by applying the same valuation model but using stressed assumptions; the BOF are recalculated accordingly (based on the assumption that the Risk Margin does not change). The SCR is then equal to the change of the BOF:

$$\mathrm{SCR}_{exp} = \max\left(BOF - BOF^{(\mathrm{stress})}; 0\right).$$

[1] For more details on recognition and boundary of insurance obligations see *Draft DA*, art. 12 TP1 and 13 TP2.

The somewhat artificial concept of a run off under going concern assumptions described above also needs to be applied to participating life insurance contracts. This type of business is characterized by risk-sharing between different generations of policyholders such that future benefits of existing policyholders are influenced by future new business. As an example, we summarize the main characteristics of the surplus participation mechanism in Germany in the following section.

2.2 Surplus participation for traditional life insurance in Germany

This section covers the main features of the surplus participation mechanism currently in place for traditional German life insurance business. Subsequently, we concentrate on the cost surplus and describe the balancing effects. Similar mechanisms are in place in other European countries (in particular in Austria, but also Italy and France), although details vary.

2.2.1 Premium calculation

A traditional German life insurance contract typically promises the policyholder a guaranteed payment in case of the insured event and a guaranteed annual interest on the policyholder's account, respectively. The calculation of premiums and reserves for these guaranteed benefits is based on the actuarial equivalence principle (cf. Bowers, 1997). Hence, in expectation, the premiums have to be sufficient to pay for the insurance benefits and to cover the costs incurred. Since the premium is binding for the whole contract period, § 11 (1) of the insurance supervision law (VAG) requires the insurer to use prudent (first-order) actuarial assumptions for the pricing, especially regarding discount rates, mortality rates and cost charges, to ensure that obligations can be settled also in case of adverse events.

However, the safety margins included in the pricing result in systematic profits, since the premium income will on average exceed the expenditures for benefit payments and the costs incurred. Since these profits are mainly the consequence of a conservative calculation, policyholders are entitled to appropriate surplus participation according to § 153 (1) of the insurance contract law (VVG). Thus, besides the guaranteed payments, policyholders can expect an additional amount from distributed surpluses.

2.2.2 Surplus distribution

The basis for the surplus participation is the so-called raw surplus under German GAAP (HGB), where the raw surplus Sp_t at time t is broken down as follows:

$$Sp_t = Sp_t^I + Sp_t^R + Sp_t^C + Sp_t^O.$$

This distinction between sources of surplus is prescribed by the corresponding executive guidance order on minimum surplus participation (MindZV):

- Investment surplus Sp_t^I: difference between the actual investment income R_t^* and the amount of guaranteed interest R_t^{gar} credited to the policyholder accounts.
- Risk surplus Sp_t^R: difference between actual mortality experience and mortality assumptions used for premium calculation; accordingly for other biometric assumptions.

- Cost surplus $Sp_t^C = Sp_t^{AcC} + Sp_t^{AdC}$: difference between the charges included in the premium calculation and the actual costs incurred; includes both acquisition and administration charges/costs.
- Other surplus Sp_t^O: profits/losses due to lapses, tax and other effects.[2]

Note that the raw surplus is not determined separately for each generation of policyholders, but for the entire portfolio of insurance contracts.[3] The subsequent surplus participation of policyholders can be split in two stages as illustrated in Fig. 1.

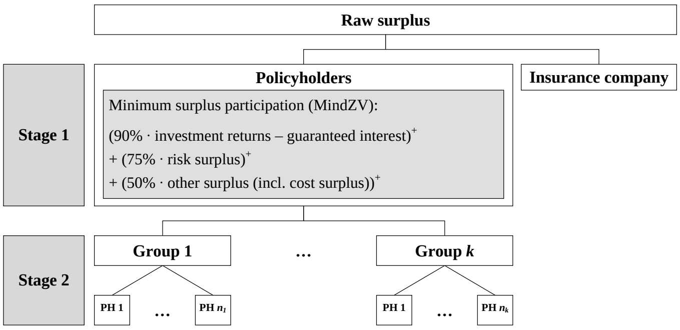

Fig. 1 Surplus participation process

In the first stage, the raw surplus is split between the insurance company and the policyholders. Here, minimum participation rates (by source of surplus) as set out in § 4 MindZV have to be observed. The policyholder's share PS_t has to be at least [4]

$$PS_t = \max\left(90\,\% \cdot R_t^* - R_t^{gar}\,;0\right) + \max\left(75\,\% \cdot Sp_t^R\,;0\right) + \max\left(50\,\% \cdot (Sp_t^C + Sp_t^O)\,;0\right).$$

Note that offsetting between different surplus sources is prohibited (except for cost surplus and other surplus). Hence, losses originating from one source of surplus (e.g. investment) have to be fully covered by the owners of the company and may not be compensated by profits from other sources (e.g. risk). It may even be the case that the raw surplus is negative and at the same time the policyholders are entitled to surplus participation. However, there are some exceptions where the minimum surplus participation rates can be reduced and offsetting between different sources of surplus is possible (cf. § 5 MindZV). In this case, consent of the regulator is required.

In the second stage, the amount designated for surplus participation of the policyholders in the first step is allocated to the individual contracts. Two main restrictions have to be considered:

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[2] We ignore these other sources of surplus for subsequent discussions.

[3] According to § 4 (1) MindZV, the raw surplus needs to be determined separately for "old business in-force" (which includes the contracts issued prior to the deregulation in 1994) and "new business in-force" (all contracts issued after the deregulation in 1994) and all subsequent stages are performed separately for these two groups. For presentational reasons, we ignore this additional split since the balancing effects apply to both sub-portfolios in the same way.

[4] Notice that minimum participation rate of 90 % concerning the investment surplus refers to the investment returns R_t^* and not to the actual investment surplus Sp_t^I.

1. Equal treatment: § 11 (2) VAG requires that under identical preconditions premiums and benefits have to be determined in accordance to the same basic principles. In particular, this applies to the benefits resulting from policyholders' surplus participation and in general results in the same total yield (guaranteed interest plus investment surplus participation) for each contract, independent of the contractually guaranteed interest rate.
2. Cause-orientation of surplus distribution: § 153 (2) VVG requires that the distribution has to reflect the contribution of the individual contract to the surplus. However, it does not require to allocate the profits exactly according to the individual contribution of every single contract.

To satisfy these restrictions, the company's insurance portfolio is classified into groups of similar contracts and surplus participation rates are declared within these group. Usually, several surplus participation rates are specified for each contract, with different reference values (e.g. investment surplus rate as percentage of mathematical reserve, cost surplus rate as percentage of premium).

Policyholders may receive some portion of the profits immediately, but the main part of the surplus is transferred to the reserve for bonuses and rebates (RfB) first and then allocated to individual contracts in subsequent years. The aim of the lagged surplus allocation is to smooth returns for policyholders and buffer fluctuations in raw surplus (especially in the annual investment returns). Since our paper focuses on the cost surplus, which is not subject to high annual variations, for our purposes, the smoothing over time based on the RfB mechanism can be neglected. Hence, for subsequent discussions, we assume that the policyholders' share of surplus is immediately credited to the individual contracts.

2.2.3 Balancing Effects

With respect to the cost surplus, balancing effects occur in two dimensions. On the one hand, the cost surplus "vertically" balances the stand-alone cost surplus of the different generations of policyholders, in particular, of the current year's new business and the existing business. On the other hand, it "horizontally" combines the acquisition and the administration cost surplus. The size of these balancing effects clearly depends on the development of both acquisition and administration cost surpluses over time when considered on a single contract basis.

The administration cost surplus is typically positive both on a single contract and portfolio level since the corresponding first-order charges are calculated sufficiently prudent.[5] On a single contract basis, it may also be rather stable over time (but this depends on the type of charges allowed for in the premium calculation).

The situation regarding acquisition cost surplus is different. The actual costs incurred at inception of a new contract are in general significantly higher than the initial acquisition charges considered in the premium calculation. This is due to the fact that the allowance for initial acquisition charges in the premium calculation is essentially limited to 4 % of the premium sum.[6] Whereas the Zillmerisation procedure ensures that actual acquisition costs up to 4 % of the premium sum can be offset against the corresponding charges (with no effect on cost surplus), the remaing acquisition costs reduce the cost surplus at inception of the contract (cf. § 4 of the policy reserve regulation (DeckRV)). In subsequent years, this additional amount is amortised by corresponding charges included in the recurring premiums which

[5] For detailed statistics on German life insurers see BaFin, 2013.

[6] The average acquisition cost rate of a German life inurance company in 2012 amounts to 5.0 % of the premium sum of the new business (cf. GDV, 2013a).

results in positive acquistion cost surpluses in subsequent years. Given the requirements for prudent premium calculation in § 11 (1) VAG, the overall allowance for acquisition charges over the lifetime of the contract should in general be sufficient to cover the actual acquisition costs.

Since the actual acquisition costs arise initially, whereas the amortisation takes place over the entire premium-payment period, the acquisition cost surplus of a contract (on a stand-alone basis) is usually negative in the first year and positive in the following years under HGB. The initial funding gap caused by new business can be compensated by both, the acquisition cost amortisation charges of existing business and the administration cost surplus of the entire portfolio. Hence, a pre-financing of some portion of the acquisition costs of the new business takes place.

Consequently, in the first stage of the surplus distribution, the acquisition costs caused by new business reduce the overall cost surplus and therefore lead to lower policyholder participation as well as lower profits for the insurance company. If the cost surplus would be derived separately for each policyholder generation instead, the insurance company would have to fully cover the initial losses from new business.

The second stage is affected by balancing effects regarding cost surplus as well. Since profit sharing from cost surplus is determined based on contract variables like premium or sum insured with no distinction between acquisition and administration costs, there is typically no difference in cost surplus participation rates between existing and new business. This approach is based on the concept that the exisiting business has to bear a share of the acquisition costs since new business is required for maintaining the insurance portfolio over the long-term (cf. Hagelschuer, 1987). However, waiting periods for new policyholders concerning the surplus participation are applied in some cases to reduce this effect.

Thus, focusing on the new business of a certain period, in the first year of those contracts, some portion of the cost surplus is inherited from the existing business to new policyholders. However, starting on from the second contract year, the former new policyholders themselves contribute to the financing of the acquisition costs of future new business reducing the burden of the remaining insurance portfolio and increasing its surplus, respectively. These inheritance effects are illustrated by a simple example in the following section.

2.2.4 Numerical example

We consider a stylized life insurance company which will be described in more detail in section 3. Under normal circumstances, the policyholders receive 70 % of the annual cost surplus (first stage of the surplus participation process). Subsequently, the policyholders' overall share of cost surplus is allocated to individual policyholders based on their gross premium (second stage of the surplus participation process). Table 1 shows the acquisition, administration, and overall cost surplus for the entire insurance portfolio as well as stand-alone for different generations of policyholders (initial in-force business and new business over three years).

Whereas the administration cost surplus is positive for all policyholder generations, the acquisition cost surplus of the new business is negative in the year of inception and positive thereafter. In consequence, the stand-alone cost surplus is also negative in the first year of the contract, since the losses from the acquisition cost surplus exceed the positive administration cost surplus by far. Due to those initial losses regarding the acquisition costs of the new business, the acquisition cost surplus for the whole insurance portfolio is also negative. However, the balancing with the administration cost surplus results in a positive overall cost surplus.

Table 1 Vertical and horizontal decomposition of the cost surplus

(in 1,000 €)	Year 1			Year 2			Year 3		
	Sp_1^{AcC}	Sp_1^{AdC}	Sp_1^{C}	Sp_2^{AcC}	Sp_2^{AdC}	Sp_2^{C}	Sp_3^{AcC}	Sp_3^{AdC}	Sp_3^{C}
Initial portfolio	369	285	654	349	270	619	329	255	584
1st year's new business	-462	15	-447	20	15	35	20	15	35
2nd year's new business	-	-	-	-462	15	-447	20	15	35
3rd year's new business	-	-	-	-	-	-	-462	15	-447
Total portfolio	-93	300	207	-93	300	207	-93	300	207

We now illustrate the balancing effects in both dimensions. Table 2 contains the allocation of cost surplus and shows the difference of the allocation with and without new business being written in the current year. Hence, this difference Δ represents the inheritance of cost surplus to the current new business. As previously described, in the first year of the contract, the new business profits from the cost surplus of the existing insurance portfolio.

Table 2 Vertical balancing effects concerning the cost surplus

(in 1,000 €)	Year 1			Year 2			Year 3		
Surplus allocation incl. the new business?	yes	no	Δ	yes	no	Δ	yes	no	Δ
Cost surplus of:	207	654		207	654		207	654	
Initial portfolio	138	458	-320	131	433	-302	124	408	-284
1st year's new business	7	(-447)[a]	(454)	7	25	-18	7	25	-18
2nd year's new business		-		7	(-447)[a]	(454)	7	25	-18
3rd year's new business		-			-		7	(-447)[a]	(454)
Company	62	196	-134	62	196	-134	62	196	-134

[a] The number represents the initial losses of the current new business and not the share of cost surplus allocated to it.

In year 1, the 1st year's new business causes a negative stand-alone cost surplus of -447, which is combined with a positive cost surplus of 654 for the initial portfolio. In stage 1, the resulting overall cost surplus is split between policyholders ($+145$) and the insurer ($+62$). In stage 2, even the 1st year's new business receives some cost surplus (7). Compared to a situation without new business, the policyholders of the initial portfolio receive much less profit sharing (138 vs. 458); the company's profit related to the initial portfolio is reduced as well (62 vs. 196).

A similar pattern can be observed in year 2. Note that 1st year's new business exhibits a positive acquisition cost surplus in year 2 (due to amortization charges), which is used to finance acquisition costs for 2nd year's new business. Compared to a situation without new business in year 2, the 1st year's new business receives less profits (7 vs. 25).

The inheritance of cost surplus from existing business to new business continues in year 3 (where 3rd year's new business is financed by the initial portfolio as well as 1st and 2nd year's new business). Overall, this example shows that profit participation for the initial portfolio highly depends on the cost surplus of future new business.

Apart from the vertical balancing discussed above, horizontal balancing effects occur as well. Table 3 shows that due to the balancing of acquisition and administration cost surplus, the negative acquisition cost surplus can be fully covered by the remaining cost surplus. If the split between policyholders and the insurer would be performed separately for both components of the cost surplus (such that the insurance company fully covers the losses of a single source of surplus), the insurance company would incur a loss of $93 - 90 = 3$ due to the overall negative acquisition cost surplus caused by the new business.

Table 3 Horizontal balancing effects concerning the cost surplus

(in 1,000 €)	Year 1			Year 2			Year 3		
Surplus allocation with horizontal balancing?	yes	no	Δ	yes	no	Δ	yes	no	Δ
Cost surplus of:	207	(-93+300)		207	(-93+300)		207	(-93+300)	
Initial portfolio	138	(0+199)	-61	131	(0+188)	-57	124	(0+177)	-53
1st year's new business	7	(0+11)	-4	7	(0+11)	-4	7	(0+11)	-4
2nd year's new business		-		7	(0+11)	-4	7	(0+11)	-4
3rd year's new business		-			-		7	(0+11)	-4
Company	62	(-93+90)	+65	62	(-93+90)	+65	62	(-93+90)	+65

Now, we show how the funds normally inherited to new business can be used to absorb losses in case of an adverse event. Therefore, we assume that the investment return (which is arbitrarily assumed for illustrative purposes) persistently drops to 1.25 %, i. e. below the guaranteed interest rate of 1.75 %. Since the insurer still has to credit the guaranteed interest to the policyholder accounts, the investment surplus becomes negative. Table 4 illustrates three scenarios based on different management reactions on the low investment return:

- Scenario *I*: The insurance company continues to write new business and distributes 70 % of the cost surplus to policyholders. Hence, the company can use the remaining 30 % of the cost surplus to cover the negative investment surplus.
- Scenario *II*: Although the insurance company continues to write new business, it makes use of § 5 MindZV which means that the whole cost surplus can be used to cover the losses from the investment surplus.
- Scenario *III*: The insurance company goes into run off, i. e. stops writing new business (thus avoiding the associated acquisition costs). Again applying § 5 MindZV, additional cost surplus is not shared with policyholders but used to close the gap in the investment surplus.

Table 4 shows that making use of § 5 MindZV and simultaneously going into run off represents a substantial relief for the insurance company. Whereas the mere application of § 5 MindZV only reduces the company's losses occurring in the first three years by 16 %, the additional stop of writing new business further reduces the losses by 46 %. Hence, almost half of the losses can be covered by the funds that become available by avoiding the acquistion costs of new business.

2.3 Consequences for valuation of life insurance contracts under Solvency II

We now discuss how the existing surplus participation mechanism in Germany described in section 2.2 should be reflected in a run off valuation of the existing portfolio under going concern assumptions (as required by the Solvency II valuation principles presented in section 2.1).

Table 4 Inheritance of cost surplus to future generations of policyholders

(in 1,000 €)	Year 1			Year 2			Year 3		
Scenario	*I*	*II*	*III*	*I*	*II*	*III*	*I*	*II*	*III*
Sp_t^I	-956	-956	-955	-949	-948	-944	-942	-941	-930
Sp_t^C	207	207	654	207	207	619	207	207	584
Usage of Sp_t^C:									
To policyholders	145	0	0	154	0	0	154	0	0
To cover losses	62	207	654	62	207	619	62	207	584
Remaining losses	894	749	301	887	741	325	880	734	346
Δ (to the preceding scenario)		145	448		146	416		146	355

As illustrated in the previous section, surplus participation is determined on a portfolio level and characterized by balancing effects between different (generations of) policyholders (in particular between existing business and new business) and different sources of cost surplus (in particular between administration and acquisition cost surplus). More specifically, the future surplus participation of existing business at the valuation date depends on the new business written after the valuation date (regarding both volume and characteristics of new business). This holds in particular for the cost surplus and the resulting FDB: a negative acquisition cost surplus in the first year of a contract reduces the overall cost surplus, the policyholders' share of the cost surplus and thus the future surplus participation of the existing business. This means that positive future cost surpluses of the existing business are inherited to future new business under the current surplus participation rules. The impact of new business on other surplus sources is ignored for the moment.[7]

In order to derive the market consistent balance sheet under Solvency II, this effect has to be taken into account in the calculation of the FDB under going concern assumptions. Of course, this is only relevant if new business can reasonably be expected (e.g if the undertaking is not in run off). Even though acquisition costs related to future new business are reflected in the calculation of the FDB, there is no need to set up reserves for these future acquisition costs since they are not related to the existing portfolio.

Note that reflection of new business typically decreases the FDB, but in some cases can result in an increase of the FDB. To ensure a prudent, reliable, and objective calculation of the Technical Provisions (cf. *Directive 2009/138/EC*, art. 76 (4)), adjustments in both directions need to be considered. In any case, the appropriateness of such an allowance for inheritance effects needs to be assessed from an economic point of view. Overall, the level of Technical Provisions still appears to be sufficient: in case of a transfer of the existing portfolio to a reference undertaking that does not write new business, additional cost surpluses would become available and could be used to cover losses by referring to § 5 MindZV (cf. example in previous section).

If inheritance effects reduce the FDB, it should be checked if the reduction of the FDB is caused by systematic cross-subsidization from existing business to future new business such that future new business receives surplus that cannot be financed by new business itself. This could for example be the case if the actual acquisition costs of new business contracts are not covered by corresponding charges over their lifetime. However, this is typically not the case given regulatory requirements regarding premium calculation (cf. section 2.2.1).

[7] For further discussion about the consideration of the impact of new business on other surplus sources see BaFin (2014).

Inheritance from future new business to existing business is also possible (resulting in an increase of the FDB). For example, the combination of administration cost surplus of existing business and future new business could results in a higher surplus per policy.[8]

The impact of future new business on the FDB was first recognized in the standard valuation model developed by the GDV for the QIS6 exercise in 2012 (GDV, 2012). This so-called Cash Flow Model was further refined for the LTGA exercise in 2013 (GDV, 2013b).

In this model, future cost surpluses are projected without taking into account negative acquisition cost surplus from future new business. The resulting cost surplus is split between policyholders and the insurance company according to the MindZV. As an approximation for the impact of negative future acquisition cost surplus, the policyholders' part is subsequently multiplied by a reduction factor. As a consequence, future surplus participation of policyholders in cost surplus (and thus the overall FDB) is reduced. The present value of these reductions is denoted by GCR. No adjustment is made to the insurer's part of future cost surpluses.

Even though the methodology for deriving the GCR as proposed in the Cash Flow Model needs further refinement, the allowance for a GCR appears in line with the reasoning above and consistent with art. 75 of *Directive 2009/138/EC*. However, as illustrated in the previous section, profits of the insurance company are also affected by a projection under going-concern assumptions. Therefore, it seems appropriate to distinguish two types of GCR: GCR_{PH} as introduced in the Cash Flow Model and GCR_{SH} covering the impact on the insurer's profits.

Essentially, allowance for a GCR reduces the FDB (by GCR_{PH}) and therefore the Technical Provisions under Solvency II. In turn, the excess of assets over liabilities in the Solvency II balance sheet (cf. *Directive 2009/138/EC*, art. 88) is increased. Note that the term GCR is not used in the Solvency II regulatory requirements on a European level and that the GCR is therefore not specified as a separate component of the BOF.[9] This implies that the GCR is part of the reconciliation reserve (cf. definition in art. 58bis COF1bis of the *Draft DA*).[10]

The question is whether the GCR can be classified as a tier 1 BOF item (as defined by art. 94 (1) of *Directive 2009/138/EC*). Art. 58 COF1 (1) (f) of the *Draft DA* specifies that the reconciliation reserve shall be classified as tier 1 if the criteria in art. 59 COF2 of the *Draft DA* are satisfied. For this check, there is no requirement to consider components of reconciliation reserve (such as the GCR) separately (cf. *Draft DA*, art. 58bis COF1bis (3)).

Nevertheless, it seems appropriate to check the GCR against these criteria:

- Art. 59 COF2 (1) (c) and (d) of *Directive 2009/138/EC* (*"the basic own fund item is immediately available to absorb losses"* and *"the basic own fund item absorb losses at least once there is non-compliance with the Solvency Capital Requirement and does not hinder the recapitalization of the insurance or reinsurance undertaking"*): As illustrated in the example in section 2.2.4, future cost surpluses can be used to cover losses from other sources by applying § 5 MindZV. If the company is closed to new business, the entire amount of the GCR is available (as required by *Directive 2009/138/EC*, art. 93 (1) (b)). Note that consent of the regulator is required, but this could reasonably be assumed in case of non-compliance with SCR (cf. § 5 MindZV).
- Art. 59 COF2 (1) (j) of *Directive 2009/138/EC* should usually be satisfied (*"the basic own fund item is free from encumbrances and is not connected with any other*

[8] This effect is also addressed in BaFin (2014) and will be illustrated in section 5.

[9] As opposed to the GCR, Surplus Funds are recognized separately and treated as a tier 1 BOF item.

[10] In particular, the GCR does not fall under the list of deductions from the excess of assets over liabilities.

transaction, which when considered with the basic own fund item, could result in that basic own fund item not satisfying the requirements set out in art. 94 (1) of Directive 2009/138/EC*"*), due to the nature of the GCR.

Therefore, we conclude that the GCR may be classified as tier 1 BOF, which is in line with the classification in the QIS6 and LTGA exercises.

Of course, this requires an appropriate and reliable methodology for determining the GCR. We present our approach in section 4. Furthermore, assumptions regarding new business need to be justified and the uncertainty of the valuation needs to be assessed as illustrated in sections 5 and 6.

3 Asset-Liability Framework

Our analysis of the valuation of insurance contracts under Solvency II is based on a stochastic balance sheet and cash flow projection model for a stylized life insurance company. The model is build on the work of Seyboth (2011); some adjustments were made to enable a detailed analysis of the GCR. Similar models have been introduced by Kling et al. (2007) and Graf et al. (2011). Due to the complexity of the stochastic future cash flows, the valuation is based on Monte Carlo simulations and we do not apply closed-form formulas for our analysis (cf. Gerstner et al., 2008). In this model, assets and liabilities are projected until the complete run off of the initial business in-force, following certain management rules. It is also possible to include future new business. Since the focus of the paper is on the valuation of insurance obligations, we will introduce the model under a risk neutral measure.

3.1 Financial market model

We consider a risk-neutral, frictionless and continuous financial market and assume that the insurer's assets are invested in two types of risky assets: coupon bonds and stocks. The short rate process $(r_t)_{t>0}$ is described by the Vasicek model and the stock price $(S_t)_{t>0}$ follows a geometric Brownian motion:

$$\begin{aligned} dr_t &= \kappa(\theta - r_t)\,dt + \sigma_r dW_t^{(1)}, \\ dS_t &= S_t\left(r_t dt + \rho\sigma_S dW_t^{(1)} + \sqrt{1-\rho^2}\sigma_S dW_t^{(2)}\right), \end{aligned}$$

where $W_t^{(1)}$ and $W_t^{(2)}$ denote two uncorrelated standard Wiener processes adapted to a filtration $\mathscr{F}$ on some probability space $(\Omega, \mathscr{F}, \mathbb{Q})$ with a risk neutral measure $\mathbb{Q}$. The parameters κ, θ, σ_r, σ_S and ρ are deterministic and constant. The initial values r_0 and $S_0 = 1$ are deterministic as well. As described in Zaglauer and Bauer (2008) and Bergmann (2011), the stochastic differential equations from above can be solved and Monte Carlo paths can be generated based on these formulae. Depending on the discretely compounded yield curve at time t, as given in Branger and Schlag (2004), we calculate par yields that determine the coupon rates of the considered coupon bonds. The bank account $B_t = \exp\left(\int_0^t r_u du\right)$ at time t serves as the discount factor in the risk-neutral framework. Therefore, the market value at time $t = 0$ of a stochastic cash flow Y_t occurring at time t is given by $PV(Y_t) = \mathbb{E}^{\mathbb{Q}}\left(\frac{Y_t}{B_t}\right)$ where $\mathbb{E}^{\mathbb{Q}}(\cdot)$ denotes the expected value under the risk neutral measure $\mathbb{Q}$.

In addition to the stochastic valuation, we also consider the deterministic present value of a stochastic cash flow Y_t in a so-called Certainty Equivalent (CE) scenario. This scenario represents the expected development of the financial market under the risk-neutral measure $\mathbb{Q}$. Hence, all assets are assumed to earn the forward rate implied by the initial yield curve $r_0(s)$ (cf. Oechslin et al., 2007). The present value is then given by $PV^{[\mathrm{CE}]}(Y_t) = \frac{Y_t^{[\mathrm{CE}]}}{B_t^{[\mathrm{CE}]}}$, where $(\cdot)^{[\mathrm{CE}]}$ denotes the respective values in the CE scenario.

3.2 Liability model

The company's insurance portfolio consists of different cohorts of identical traditional participating life insurance contracts with a contract duration of n years and policyholder's age x at inception of the contract. The contract provides a guaranteed benefit G (sum insured) at maturity or death against annual premium payments P. The pricing is based on a guaranteed interest rate i and reflects the following charges:[11]

- initial acquisition charge α (as percentage of premium sum),
- amortization charge α^γ (as percentage of sum insured),
- administration charge β (as percentage of premium).

Each cohort (k) initially consists of ${}^{(k)}l_k^*$ contracts incepted at the beginning of year $k+1$. We assume that the sum insured increases over time due to inflation. Therfore, the sum insured ${}^{(k)}G$ of a certain cohort (k) is given by ${}^{(k)}G = {}^{(k-1)}G \cdot (1+ir_k^G)$, where ir_k^G denotes the annual inflation rate and ${}^{(0)}G = G$.

Based on the actuarial principle of equivalence ant the assumptions above the annual premium ${}^{(k)}P$ is given by

$$^{(k)}P = \frac{^{(k)}G \cdot (A_{x:\overline{n}|} + \alpha^\gamma \cdot \ddot{a}_{x:\overline{n}|})}{(1-\beta)\cdot \ddot{a}_{x:\overline{n}|} - \alpha \cdot n}\,.$$

Following the Zillmerisation procedure, the actuarial reserve ${}^{(k)}AR_t$ under HGB at time t before payment of the premium can be calculated recursively by[12]

$$^{(k)}AR_t = \begin{cases} -\alpha \cdot n \cdot {}^{(k)}P, & t = k \\ \Big(\Big({}^{(k)}AR_{t-1} + {}^{(k)}P \cdot (1-\beta) - {}^{(k)}G \cdot \alpha^\gamma \Big) \cdot (1+i) & \\ \qquad - {}^{(k)}G \cdot q_{x+(t-k)-1} \Big) \frac{1}{1-q_{x+(t-k)-1}}\,, & t = k+1, \ldots, k+n. \end{cases}$$

Besides the guaranteed annual interest i on their actuarial reserve, the policyholders participate in the company's annual raw surplus. The part of surplus ${}^{(k)}Sp_t$ assigned to a policyholder of cohort (k) at time t is credited to his bonus reserve ${}^{(k)}BR_t$. In subsequent years,

[11] As a simplification, we do not allow for other types of charges (e. g. administration charges on the sum insured or the actuarial reserve and unit charges).

[12] For simplification, we do not explicitly consider the balance sheet presentation under HGB (in particular regarding Zillmer receivables). Since we do not consider lapses in our projection model, we also ignore further restrictions imposed by insurance contract law (in particular, the requirement to distribute initial acquisition charges over five years for the purpose of determining surrender values including the consequences regarding the balance sheet presentation). Overall, the treatment of initial acquisition charges in Germany allows to cover initial acquisition costs of $\alpha \cdot n \cdot P$ at inception of the contract and this is also the case in our simplified model.

the guaranteed interest rate i also applies to the bonus reserve such that the development of the bonus reserve is given by ${}^{(k)}BR_t = {}^{(k)}BR_{t-1} \cdot (1+i) + {}^{(k)}Sp_t$ with ${}^{(k)}BR_k = 0$.

In the event of a claim (i. e. in case of the policyholder's death or at maturity of the contract), the bonus reserve is paid out in addition to the contractually guaranteed benefit ${}^{(k)}G$.[13] Consequently, the benefits (liabilities) ${}^{(k)}L_t$ the insurer has to pay for a policyholder of this cohort at time t in the event of a claim at the end of the year amount to ${}^{(k)}L_t = {}^{(k)}G + {}^{(k)}BR_t$.

All relevant figures determined for each cohort are summed up to determine the respective figures for the entire insurance portfolio.

3.2.1 Liability portfolio development

At the beginning of each year $k+1$ a new cohort of ${}^{(k)}l_k^*$ policyholders joins the insurance portfolio. The number of policyholders ${}^{(k)}l_t^*$ still in the cohort at the end of year t depends on the number of policyholders at the beginning of the year and on the actual mortality rate $q^*_{x+(t-k)-1}$, i. e. for $t = k+1, ..., k+n-1$

$$ {}^{(k)}l_t^* = {}^{(k)}l_{t-1}^* \cdot (1 - q^*_{x+(t-k)-1}) , $$

with ${}^{(k)}l_{k+n}^* = 0$ (maturity of contracts of cohort (k)). Since we want to single out the inheritance effects, we do not take into account lapses and we further assume that the actual (best estimate) second-order mortality rates equal the first-order mortality rates used for the premium calculations which are based on the German standard mortality table (DAV 2008 T). The overall size of the company's insurance portfolio at time t is equal to

$$ l_t^* = \sum_{k=0}^{n-1} {}^{(t-k)}l_t^* . $$

3.2.2 Cost model

We assume that costs considered in the model arise at the beginning of the year and we differentiate between acquisition costs AcC_t^* and administration costs AdC_t^*. The former consist of internal acquisition costs ${}^{int}AcC_t^*$ for each new contract, e. g. expenses for the initial examination of the policyholder and the entry of the new contract into the system, and a commission payment α_t^* to the insurance broker given in percentage of the premium sum of the new business ${}^{(t)}l_t^*$ written in this year. The administration costs are divided into a fixed part ${}^{fix}AdC_t^*$ and a variable part ${}^{var}AdC_t^*$, taking into account that a part of the costs arises from the general maintenance of the insurance operation independent of the total number of contracts l_t^* in the insurance portfolio.

Furthermore, future internal acquisition costs as well as future administration costs are affected by annual cost inflation rates ir_t^C. Therefore, based on the cost parameters at time $t = 0$, the expenses arising at time t are determined as follows:

$$ \begin{aligned} AcC_t^* &= \left({}^{int}AcC_{t-1}^* \cdot (1 + ir_t^C) + \alpha_t^* \cdot n \cdot {}^{(t)}P \right) \cdot {}^{(t)}l_t^* , \\ AdC_t^* &= \left({}^{fix}AdC_{t-1}^* + {}^{var}AdC_{t-1}^* \cdot l_t^* \right) \cdot (1 + ir_t^C) . \end{aligned} $$

[13] For technical reasons we assume that the death of a policyholder always occurs at the end of the year.

As introduced in section 2.1, the following stressed cost parameters are used to derive SCR_{exp}:[14]

$$AcC_t^{(\text{stress})} = \Big(1.1 \cdot {}^{int}AcC_{t-1}^{*} \cdot (1 + ir_t^C + 1\,\%) + \alpha_t^{*} \cdot n \cdot {}^{(t)}P\Big) \cdot {}^{(t)}l_t^{*}\,,$$
$$AdC_t^{(\text{stress})} = 1.1 \cdot ({}^{fix}AdC_{t-1}^{*} + {}^{var}AdC_{t-1}^{*} \cdot l_t) \cdot (1 + ir_t^C + 1\,\%)\,.$$

3.3 Asset model

The asset portfolio of our life insurance company consists of coupon bonds and stocks. The company invests in coupon bonds yielding at par with fixed initial maturity T_B and these bonds are considered as held to maturity for accounting purposes. Therefore, the maximum remaining time to maturity of all coupon bonds in the portfolio is T_B years. Following HGB accounting rules, both, stocks and coupon bonds, are considered as fixed assets and therefore recognized at acquisition costs which may result in unrealized gains or losses (UGL) due to the difference between market values MV_t^A and book values BV_t^A of the assets at time t.

The projection of the asset portfolio needs to reflect cash flows from liabilities and assets. The cash flow CF_t^+ occurring at the beginning of the year (premium payments P_t as well as costs AcC_t^* and AdC_t^*) is invested in a riskless bank account earning the interest rate $r_t(1)$. At the end of the year, the asset portfolio is adjusted, also taking into account cash flows at the end of the year (coupon payments CP_{t+1}, nominal repayment $N_{t+1}^{(t+1-T_B)}$ of bonds at maturity and benefit payments L_{t+1} to the policyholders).

Following a constant strategic asset allocation, the asset portfolio is rebalanced based on the targeted stock ratio q (in terms of the market values). If bonds need to be sold, this is done proportionally to the market values of the bonds in the company's portfolio. UGL may be realized due to this rebalancing. In addition, the company realizes a certain ratio d of the UGL of stocks in order to stabilize the investment income.

Overall, the investment return rate is given by

$$i_{t+1}^{*} = \frac{CF_t^+ \cdot r_t(1) + CP_{t+1} + N_{t+1}^{(t+1-T_B)} + UGL_{t+1}^{real}}{BV_t^A + CF_t^+}\,,$$

where UGL_{t+1}^{real} denotes the realized portion of the UGL.

3.4 Surplus distribution

After the asset allocation has been adjusted, the annual raw surplus can be derived on an HGB book value basis, including the breakdown in its sources as introduced in section 2.2.2. Note that the risk surplus is zero since we assume that first- and second-order mortality rates coincide. Other surplus is also zero since lapses, tax payments etc. are not considered.

[14] In line with the technical specifications in *TS*, SCR.7.61. the acquisition commission is not included in the stress.

3.4.1 Cost surplus

As discussed in section 2.2.2, the cost surplus is based on the difference between the charges reflected in the pricing of the contract and the actual costs incurred. In our model, the cost surplus Sp^{C}_{t+1} at time $t+1$ is the sum of the acquisition cost surplus Sp^{AcC}_{t+1} and the administration cost surplus Sp^{AdC}_{t+1} and given by

$$Sp^{AcC}_{t+1} = \left(1+i^*_{t+1}\right) \cdot \left(\alpha \cdot n \cdot {}^{(t)}P \cdot {}^{(t)}l^*_t + \sum_{k=0}^{n-1} \left(\alpha^\gamma \cdot {}^{(t-k)}G \cdot {}^{(t-k)}l^*_t \right) - AcC^*_t \right),$$

$$Sp^{AdC}_{t+1} = \left(1+i^*_{t+1}\right) \cdot \left(\sum_{k=0}^{n-1} \left(\beta \cdot {}^{(t-k)}P \cdot {}^{(t-k)}l^*_t \right) - AdC^*_t \right).$$

Note that both charges and costs occur at the beginning of the year. Therefore, the cost surplus also includes the investment return i^*_{t+1} on the difference between charges and costs.

3.4.2 Investment surplus

To obtain the investment surplus Sp^{I}_{t+1}, the guaranteed interest R^{gar}_{t+1} credited to the policyholders' accounts is subtracted from the actual realized investment return R^*_{t+1} of the current year. Following the description in section 2.2.2, we get

$$\begin{aligned} Sp^{I}_{t+1} &= R^*_{t+1} - R^{gar}_{t+1} \\ &= \sum_{k=0}^{n-1} \left(\left({}^{(t-k)}AR_t + {}^{(t-k)}BR_t + {}^{(t-k)}P \cdot (1-\beta) - {}^{(t-k)}G \cdot \alpha^\gamma \right) \cdot {}^{(t-k)}l^*_t \right) \cdot (i^*_{t+1} - i). \end{aligned}$$

3.4.3 Splitting of surplus

Minimum surplus participation rules (based on MindZV) are reflected by separate participation rates (p^C, p^I) for each of the relevant surplus sources; these rates are fixed over the projection period. Thereby, losses in a single source are fully covered by the insurer. Hence, the policyholders' share of cost and investment surplus at time $t+1$ equals

$$PS^{C}_{t+1} = \max\left(p^C \cdot Sp^{C}_{t+1}\,;0\right),$$

$$PS^{I}_{t+1} = \max\left(p^I \cdot R^*_{t+1} - R^{gar}_{t+1}\,;0\right).$$

The remaining part of surplus represents the insurer's profit X_{t+1} which results in a respective cash out-/inflow if the profit is positive/negative.[15]. For technical reasons, the cash flow occurs at the beginning of the next year and is given by

$$X_{t+1} = X^{I}_{t+1} + X^{C}_{t+1} = (Sp^{I}_{t+1} - PS^{I}_{t+1}) + (Sp^{C}_{t+1} - PS^{C}_{t+1}).$$

Note that the model can easily be adjusted to surplus participation mechanisms in other countries. E. g. in Austria, a minimum surplus participation rate of 85 % is applied, with no distinction between sources of surplus.

[15] In case of negative profits, we assume that the insurance company does not exercise its limited liability option (cf. Doherty and Garven, 1986; Gatzert and Schmeiser, 2008). This assumption is in line with the Solvency II valuation principles (cf. Bauer et al., 2012).

3.4.4 Allocation of surplus to policyholders

The policyholders' share of surplus has to be further allocated to individual policyholders. Whereas all contracts of a certain cohort (k) receive the same amount of surplus participation ${}^{(k)}PS_{t+1}$, consisting of cost surplus and investment surplus participation, it may differ between contracts from different cohorts. Applying a so-called natural distribution system (cf. Wolfsdorf, 1997) policyholders' profits are allocated as follows:

- Actual costs incurred are typically considered as unit costs whereas all charges reflected in the pricing of the contract directly or indirectly depend on the premium. In order to reflect the contribution of the individual contract to the cost surplus, the allocation of the cost surplus PS^C_{t+1} is also based on premiums. Therefore, the cost surplus participation ${}^{(k)}PS^C_{t+1}$ allocated to a single policyholder of cohort (k) amounts to
$$ {}^{(k)}PS^C_{t+1} = \frac{PS^C_{t+1}}{\sum_{j=0}^{n-1} {}^{(t-j)}P \cdot {}^{(t-j)}l^*_t} \cdot {}^{(k)}P \,. $$
- The investment return R^*_{t+1} is distributed such that all policyholders receive the same total yield (sum of the guaranteed interest rate and the surplus return rate) on their accounts. In any case, all policyholders receive at least the guaranteed interest rate i. Hence, the investment surplus credited to a single policyholder of cohort (k) is determined by
$$ {}^{(k)}PS^I_{t+1} = \left({}^{(t-k)}AR_t + {}^{(t-k)}BR_t + {}^{(t-k)}P \cdot (1-\beta) - {}^{(t-k)}G \cdot \alpha^\gamma\right) \cdot \max\left(p^I \cdot i^*_{t+1} - i\,; 0\right) . $$

3.5 Economic balance sheet

The stochastic projection model is typically applied to an existing portfolio of insurance contracts and a corresponding asset portfolio. Based on the stochastic projection of the company until complete run off of this initial business in-force, we derive market consistent values of all assets and liabilities and set up the economic balance sheet at time $t = 0$.

As illustrated in Table 5, the left-hand side of the balance sheet contains the market value MV^A_0 of the asset portfolio which is determined by the initial data of the financial market described in section 3.1. The right-hand side consists of two items. The BE represents the present value of the company's future obligations towards policyholders (as defined in section 2.1). The company's portion of the expected future surpluses is given by the Present Value of Future Profits (PVFP). It represents the contribution of the considered portfolio to the company's BOF.[16]

Given the stochasticity of the projected cash flows, the risk neutral valuation approach described in section 3.1 is applied. Therefore, the two items are given by

$$ \widetilde{\text{PVFP}} = \mathbb{E}^{\mathbb{Q}}\left(\sum_{t=1}^{T} \frac{X_t}{B_t}\right) , $$
$$ \widetilde{\text{BE}} = \mathbb{E}^{\mathbb{Q}}\left(\sum_{t=1}^{T} \frac{(AcC^*_{t-1} + AdC^*_{t-1}) - P_{t-1}}{B_{t-1}} + \frac{L_t}{B_t}\right) . $$

[16]For the concept of the PVFP see CFO Forum (2009).

However, since the exact distribution of the two above-mentioned items cannot be determined analytically, we use Monte-Carlo-Simulation to estimate the expected values. Based on J realizations of the stochastic processes, PVFP and BE are estimated by (cf. Schmidt, 2014)

$$\text{PVFP} = \frac{1}{J}\sum_{j=1}^{J}\sum_{t=1}^{T}\frac{X_t^{[j]}}{B_t^{[j]}}\,, \tag{1}$$

$$\text{BE} = \frac{1}{J}\sum_{j=1}^{J}\sum_{t=1}^{T}\left(\frac{(AcC_{t-1}^{*}+AdC_{t-1}^{*})-P_{t-1}}{B_{t-1}^{[j]}}+\frac{L_t^{[j]}}{B_t^{[j]}}\right), \tag{2}$$

where $(\cdot)^{[j]}$ denotes the respective values/cash flows in scenario j.[17]

For a closer analysis, we identify the individual components of these two items. In line with the differentiation of the company's profits into a cost and an investment part, the PVFP can be broken down into $\text{PVFP} = \text{PVFP}^I + \text{PVFP}^C$.

Table 5 Economic balance sheet at time $t = 0$

Assets	Liabilities
MV_0^A	PVFP
	- PVFP^C
	- PVFP^I
	BE
	- BE^{gar}
	- FDB^I
	- FDB^C

Concerning the BE, the individual benefits ${}^{(k)}L_t$ paid to a policyholder of cohort (k) leaving the company at time t are decomposed into the future benefits ${}^{(k)}L_t^{gar}$ already locked-in (guaranteed) at time $t = 0$ and the future discretionary benefits ${}^{(k)}L_t^{\text{FDB}}$ resulting from participation in future surplus:

$$ {}^{(k)}L_t^{gar} = {}^{(k)}G + {}^{(k)}BR_0 \cdot (1+i)^t\,,$$

$$ {}^{(k)}L_t^{\text{FDB}} = {}^{(k)}BR_t - {}^{(k)}BR_0 \cdot (1+i)^t = \sum_{s=1}^{t}\left({}^{(k)}PS_s^C \cdot (1+i)^{t-s} + {}^{(k)}PS_s^I \cdot (1+i)^{t-s}\right).$$

Following this decomposition, the BE can be divided into the best estimate for the guaranteed obligations towards policyholders BE^{gar} (including the cost, premiums, and guaranteed benefits) and the future discretionary benefits from the cost and investment surplus (FDB^I, FDB^C), i. e.

$$\text{BE} = \text{BE}^{gar} + \text{FDB}^I + \text{FDB}^C\,.$$

The economic balance sheet can also be set up for the CE scenario as introduced in section 3.1. The resulting $\text{PVFP}^{[\text{CE}]}$ can be used to quantify the asymmetry of the insurer's profits caused by financial options and guarantees included in the insurance contracts. The

[17] Premiums and costs do not depend on the scenario.

Market Consistent Embedded Value (MCEV) framework (CFO Forum, 2009) defines the Time Value of Financial Options and Guarantees (TVFOG) by

$$\text{TVFOG} = \text{PVFP}^{[\text{CE}]} - \text{PVFP}.$$

Of course, similar calculations are possible for the BE and the GCR. Since the TVFOG has a significant impact on the BOF, it will be analyzed in detail and compared with the GCR.

4 Methodology for quantification of the GCR within the Solvency II framework

Following the reasoning in section 2.3, we now introduce a methodology for the setup of a Solvency II balance sheet for the existing business under going concern assumptions that takes into account that future cost surpluses of existing business are inherited to future new business. The methodology for the quantification of the GCR is based on the general valuation framework introduced in section 3 where the projection model is run under three different sets of assumptions. Based on the results of theses three projections, we can identify the components of the GCR and discuss their characteristics in terms of loss absorbency.

4.1 Description of the three projection runs

The basic idea behind our method to quantify the GCR is to compare the projected future cash flows related to the existing business between a situation with new business (going concern) and without new business (run off).

Therefore, run 1 includes both, the initial in-force business as well as the expected future new business. The impact of future new business on the future cash flows of the existing portfolio is explicitly taken into account. As a result, going concern assumptions regarding future costs can be extracted from run 1. Conversely, in run 2 only the existing business is reflected and the projection is based on run off assumptions (in particular regarding costs). Finally, run 3 is an artificial projection run that reflects the run-off (as in run 2) under going concern assumptions (from run 1). As detailed below, the GCR is derived from the differences between run 2 and run 3.

4.1.1 Run 1: going concern projection under going concern assumptions

Run 1 presumes that the life insurance company continues its business as a going concern. In addition to the existing portfolio of insurance contracts, expected new business of future periods is included in the projection (including the related acquisition costs).

As described in section 2.2.3, the initial funding gap regarding acquisition costs of future new business is compensated by the cost surpluses generated by the entire insurance portfolio. Essentially, the portion of the acquisition costs that is not covered by initial acquisition charges (and the Zillmerisation procedure) is split between all policyholders. The inherited acquisition costs at time t per unit of premium can be expressed by

$$^{inh}AcC_t^{pP} = \frac{^{inh}AcC_t^*}{\sum_{k=0}^{n-1} {}^{(t-k)}P \cdot {}^{(t-k)}l_t^*} = \frac{AcC_t^* - \alpha \cdot n \cdot {}^{(t)}P \cdot {}^{(t)}l_t^*}{\sum_{k=0}^{n-1} {}^{(t-k)}P \cdot {}^{(t-k)}l_t^*}.$$

The allocation based on premiums is chosen in order to ensure consistency with the allocation of cost surplus described in section 3.4. Of course, other allocation approaches are possible.

Future new business may also influence the administration cost surplus. Two aspects need to be considered here. First, some portion of the administration costs is fixed and does not depend on the size of the insurance portfolio (so-called overhead costs). Therefore, a higher number of contracts reduces the average costs per policy and increases the overall cost surplus. Second, the administration charges included in the premium are fixed over the lifetime of the contract. Hence, the insurer cannot react to an increase of the administration costs (e. g. caused by cost inflation) by raising the corresponding charges. However, new contracts and the included charges are typically priced in such a way that they are sufficient for the current cost situation (in particular reflecting cost inflation in the past). Of course, higher charges (in absolute terms) of the new business increase overall cost surplus and resulting profits for each contract.

In line with the allocation of inherited acquisition costs and cost surplus, these effects are also allocated to individual contracts based on premiums where the administration costs at time t per unit of premium is equal to

$$AdC_t^{pP} = \frac{AdC_t^*}{\sum_{k=0}^{n-1} {}^{(t-k)}P \cdot {}^{(t-k)}l_t^*}.$$

Note that the entire administration costs are considered here, whereas ${}^{inh}AcC_t^{pP}$ only reflects the portion of acquisition costs not covered by initial acquisition charges. Overall, new business may reduce the administration cost burden of the existing policyholders which results in an inheritance in the opposite direction compared to acquisition costs.

4.1.2 Run 2: run off projection under run off assumptions

Run 2 represents the life insurance company's development in case of a run off, i.e. we assume that the company does not write new business in the future. Consequently, there are no acquisition costs, i. e. $AcC_t^* = 0\ \forall t$; this results in a higher acquisition cost surplus.

As already discussed in the previous section, the lack of future new business may increase the administration costs (expressed as a percentage of premium income) due to the company's fixed administration costs and due to cost inflation. As a consequence, a higher portion of the individual premium of each policyholder may be needed to cover the administration costs.

4.1.3 Run 3: run off projection under going concern assumptions

Run 3 is a combination of run 1 and run 2. As in run 2, the projection in run 3 is limited to the initial in-force business. Although future new business is not modeled explicitly, it is implicitly taken into account by using going-concern assumptions regarding cost.

This is done by adjusting the cost model for the projection in such a way that it represents the situation with new business. More specifically, acquisition and administration costs in run 3 are derived from run 1 parameters by multiplication with the projected premium income from existing business:

$$AcC_t^* = {}^{inh}AcC_t^{pP} \cdot \sum_{k=0}^{n-1} {}^{(t-k)}P \cdot {}^{(t-k)}l_t^* ,$$
$$AdC_t^* = AdC_t^{pP} \cdot \sum_{k=0}^{n-1} {}^{(t-k)}P \cdot {}^{(t-k)}l_t^* .$$

Note that the different amount of future costs compared to run 2 influences the projection of the asset portfolio. In particular, the (re)investment volume is affected which results in different investment income in subsequent years.

Overall, run 3 is an artificial projection setup that cannot occur in reality, but is useful for deriving the Solvency II balance sheet. Since the acquisition costs AcC_t^* assumed in run 3 are not caused by the existing business at the valuation date, they are not taken into account for the calculation of the BE using formula (2).

4.2 Calculation of the GCR and its components

The projections in run 2 and 3 are both limited to the existing business at the valuation date. Due to different assumption sets used, the resulting PVFP and BE differ. Since these differences reflect how future new business affects future cash flows required to settle the existing insurance obligations, the GCR as defined in 2.3 can be determined by

$$\mathrm{GCR} = \left(\mathrm{PVFP}_{(\mathrm{run}\,2)} - \mathrm{PVFP}_{(\mathrm{run}\,3)}\right) + \left(\mathrm{BE}_{(\mathrm{run}\,2)} - \mathrm{BE}_{(\mathrm{run}\,3)}\right) .$$

The impact on future cash flows of the insurance company is captured by the insurer's (shareholder) part GCR_{SH}. It can be decomposed into the cost surplus component GCR_{SH}^C and the investment surplus component GCR_{SH}^I:

$$\mathrm{GCR}_{SH} = \mathrm{GCR}_{SH}^C + \mathrm{GCR}_{SH}^I = \left(\mathrm{PVFP}_{(\mathrm{run}\,2)}^C - \mathrm{PVFP}_{(\mathrm{run}\,3)}^C\right) + \left(\mathrm{PVFP}_{(\mathrm{run}\,2)}^I - \mathrm{PVFP}_{(\mathrm{run}\,3)}^I\right) .$$

The same type of split is also possible for the policyholder part. Since the policyholders' share of surplus is not paid out immediately but stored in the bonus reserve and then included in benefit payments at maturity or death, the quantification of the GCR_{PH} is based on the future discretionary benefit payments as follows:

$$\mathrm{GCR}_{PH} = \mathrm{GCR}_{PH}^C + \mathrm{GCR}_{PH}^I = \left(\mathrm{FDB}_{(\mathrm{run}\,2)}^C - \mathrm{FDB}_{(\mathrm{run}\,3)}^C\right) + \left(\mathrm{FDB}_{(\mathrm{run}\,2)}^I - \mathrm{FDB}_{(\mathrm{run}\,3)}^I\right) .$$

Overall, the cost component of the GCR clearly represents the pre-financing of acquisition costs in the proper sense, whereas the investment component valuates the side effect arising from the influence of the different cost assumptions on the (re)investment volume, the asset performance and the split of investment income between policyholders and the insurer in subsequent years.[18]

If administration costs (as a percentage of premiums) are the same in run 2 and 3, the best estimate of guaranteed benefits is identical, i.e. $BE_{(\mathrm{run}\,2)}^{gar} = BE_{(\mathrm{run}\,3)}^{gar}$. In this case, the GCR only consists of the components described above. As discussed in the previous section,

[18] The side effects will be further analyzed in section 5.

administration costs may differ due to overhead costs and cost inflation. If this is reflected in the setup of the runs, the GCR includes a third component which we denote by GCR_{AdC}:

$$\mathrm{GCR}_{AdC} = \mathrm{BE}^{gar}_{(\mathrm{run}\,2)} - \mathrm{BE}^{gar}_{(\mathrm{run}\,3)}\,.$$

GCR_{AdC} represents the increased administration cost burden of each contract in case of a run off compared to the going concern situation.

Obviously, the total value of the overall GCR equals the present value of the acquisitions costs included in run 3.

4.3 Accounting for the GCR within the Solvency II balance sheet

The Solvency II balance sheet consists of the BE from run 3, which is combined with the PVFP from run 2 and the GCR as defined in the previous section. The company's BOF are then derived as follows:

$$\mathrm{BOF} = \mathrm{PVFP}_{(\mathrm{run}\,3)} + \mathrm{GCR}_{SH} + \mathrm{GCR}_{PH} = \mathrm{PVFP}_{(\mathrm{run}\,2)} + \mathrm{GCR}_{PH}\,.$$

Note that GCR_{AdC} is not included in the BOF since it is not available to compensate unexpected losses. In a run off situation, these funds are needed to cover incurring expenses. Instead, it seems appropriate to include GCR_{AdC} into the BE such that

$$\mathrm{BE} = \mathrm{BE}_{(\mathrm{run}\,3)} + \mathrm{GCR}_{AdC} = \mathrm{BE}^{gar}_{(\mathrm{run}\,2)} + \mathrm{FDB}_{(\mathrm{run}\,3)}\,.$$

In order to determine the SCR for expense risk (cf. definition in section 2.1), all three runs are repeated with stressed cost assumptions. Then both GCR and PVFP are recalculated to determine

$$\mathrm{SCR}_{exp} = \max\left(\mathrm{BOF} - \mathrm{BOF}^{(\mathrm{stress})}; 0\right).$$

This means that additional own funds due to allowance for a GCR also have an impact on the SCR for expense risk. Therefore, in order to analyze the overall impact of the introduction of a GCR, the Excess Capital (ExC) should be considered. Ignoring the impact of the GCR on other SCR modules, we define

$$\mathrm{ExC} = \mathrm{BOF} - \mathrm{SCR}_{exp}\,.$$

Finally, we compare our approach with the methodology used in the Cash Flow Model developed by the GDV. As indicated in section 2.3, it is a simplified model that is based on a single projection run (consistent with run 2 in our setup). The GCR is defined as a certain percentage p of the present value of policyholders' share of future cost surpluses. In our notation, this corresponds to

$$p \cdot \sum_{t=1}^{T} \frac{PS^{C\,[\mathrm{CE}]}_t}{B^{[\mathrm{CE}]}_t}\,.$$

The adjusted policyholder part of cost surplus is then used to determine the FDB.

Compared to our methodology, this approach is clearly less precise as all inheritance effects need to be captured by a single parameter p. Instead of future cash flows, the formula refers to future HGB cost surplus and therefore ignores side effects. Furthermore, the insurer's part of the GCR is not disclosed separately. Note that the latter has no impact on the BOF, but reduces transparency.

5 Base case

5.1 Base case assumptions

The stochastic projection model from section 3 is now applied to a stylized life insurance company. The insurance portfolio consists of identical life insurance contracts with contract parameters as given in Table 6.

Table 6 Insurance contract parameters

x	n	G	i	α	α^{γ}	β	P	ir_k^G
40	20 years	20,000 €	1.75 %	4.0 %	0.1 %	4.0 %	964.19 €	0.0 %

We assume a steady state. This means that in the past as well as in the future ${}^{(k)}l_k^* = 1{,}000$ contracts are sold at the beginning of each year $k+1$. For the base case, we do not take inflation into account.

The insurance portfolio at the valuation date $t = 0$ is built up over 20 years and therefore consists of 19 cohorts of policyholders with time to maturity from 1 to 19 years. Therefore, the time horizon for the projection is $T = 19$ years. The actuarial reserve AR_0 and the bonus reserve BR_0 are derived from a projection in a deterministic scenario. In this deterministic scenario, we use a flat yield curve of 3.0 % (consistent with the mean reversion parameter θ of the stochastic model after time $t = 0$).

The best estimate cost parameters for the projection are shown in Table 7. The commission rate α_t^* is constant over time. The other parameters are chosen such that under a going concern perspective the acquisition and the administration cost rate coincide with the current average of the German life insurance market of 5.0 % of the new business premium sum and 2.4 % of the gross written premium income, respectively (cf. GDV, 2013a). All administration costs are considered as variable.

Table 7 Best estimate cost parameters

${}^{int}AcC_t^*$	Commisson (α_t^*)	${}^{fix}AdC_t^*$	${}^{var}AdC_t^*$	ir_t^C
192.84 € (1.0 %)	771.35 € (4.0 %)	0 €	23.14 € (2.4 %)	0.0 %

At time $t = 0$, the book value of the asset portfolio coincides with the book value of liabilities. We assume a stock ratio of $q = 5\,\%$ with unrealized gains on stocks of 10 % of the book value of stocks. The coupon bond portfolio at $t = 0$ consists of bonds with a uniform coupon of 3.0 % where the time to maturity is equally split between 1 and $T_B = 10$ years.

The parameters for the management rules are shown in Table 8. For illustrative reasons, we use a cost surplus participation rate p^C that is higher than the prescribed minimum rate. The other parameters are consistent with current regulation and practice in the German insurance market.[19]

[19] We acknowledge that accounting restrictions regarding unrealized losses on stocks could be reflected more precisely. However, this has no impact on our conclusions.

Table 8 Parameters for management rules

q	T_B	d	p^I	p^C
5 %	10 years	20 %	90 %	70 %

The financial market parameters for the projection are shown in Table 9. The parameters are directly adopted from Reuß et al. (2014).

Table 9 Financial market parameters

r_0	θ	κ	σ_r	σ_S	ρ
2.5 %	3.0 %	30.0 %	2.0 %	20.0 %	15.0 %

The stochastic projection is performed for 5,000 scenarios of the financial market. Further analyses showed that this allows for a precise estimation of the relevant figures (cf. Glasserman, 2010).

5.2 Base case simulation results

Based on the methodology described in section 4, we derive the Solvency II balance sheet for the in-force business at time $t = 0$ under going concern assumptions. Fig. 2 shows the Solvency II balance sheet and how it is derived from run 2 and run 3 for the best estimate and the expense stress scenario.[20]

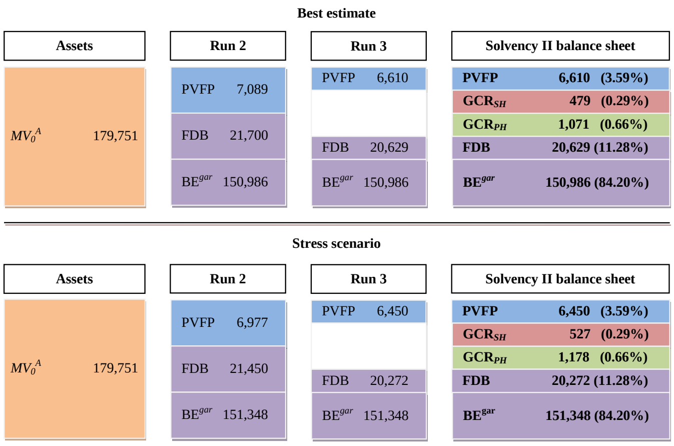


Fig. 2 Solvency II balance sheet at time $t = 0$ for the best estimate and the expense stress scenario (in 1,000 €)

Based on the assumptions described in the previous section, the actual acquisition costs exceed the corresponding charge α by 1.0 % of the premium sum of new business. Hence,

[20] The results are also shown as a percentage of the balance sheet total (in brackets).

a significant amount of the acquisition costs has to be covered by the in-force business and the company. This results in a total GCR of 1.550 m €. GCR_{AdC} is equal to zero since we do not allow for overhead costs or cost inflation. Overall, the BOF are equal to 8.160 m € and 19 % of the BOF originate from cost surpluses inherited from the existing business to cover acquisition costs of future new business.

Compared to a valuation under a pure run off perspective (cf. run 2), only the policyholder part GCR_{PH} increases the BOF. In contrast, the insurer's part GCR_{SH} coincides with the decrease of the PVFP between run 2 and run 3 such that the net impact on the BOF is zero. This may explain why the Cash Flow Model developed by the GDV only discloses the policyholder part separately and determines the PVFP under a pure run off perspective (cf. discussion in section 4.3).

Further insights can be obtained from the stressed balance sheet under the expense stress. As expected, the increase of expenses reduces the cost surplus and eventually the PVFP and the FDB in runs 2 and 3 (with a corresponding increase of BE^{gar}). However, the reduced PVFP is almost completely compensated by the increase of the GCR, because a higher share of the cost surplus is inherited to the future new business to cover the increased (internal) acquisition costs. From a policyholder perspective, this means that profits which are paid out to the in-force business under best estimate assumptions and thus recognized in the best estimate FDB are used to cover acquisition costs in a stress situation and therefore recognized as BOF after stress.

Overall, SCR_{exp} is almost negligible (approximately 5,000 €). If we ignore the inheritance of cost surplus, the resulting SCR is approximately $\text{SCR}^{RO}_{exp} = 112{,}000$ €, as calculated in run 2. Thus in the base case, the allowance for a GCR not only increases the BOF, but also reduces SCR_{exp} such that the ExC increases.

The composition of the GCR presented in Table 10 shows that the main part of the GCR results from inherited cost surplus. The side-effects on the investment surplus (cf. discussion in section 4.2) account for less than 5 % of the GCR. The policyholder part corresponds to approximately 69 % of the total GCR which roughly coincides with the cost surplus participation rate of $p^C = 70\,\%$. This indicates that cost surplus is positive under going concern assumptions (in run 1) and therefore initial losses from new business are split between policyholders and the insurance company based on the cost surplus participation rate.

Table 10 Composition of the GCR

(in 1,000 €)	Total	Cost surplus	Investment surplus
Insurance company	$\text{GCR}_{SH} = 479$	$\text{GCR}^C_{SH} = 465$	$\text{GCR}^I_{SH} = 14$
Policyholders	$\text{GCR}_{PH} = 1{,}071$	$\text{GCR}^C_{PH} = 1{,}021$	$\text{GCR}^I_{PH} = 50$

Another aspect of the results is the degree of asymmetry of the company's profits, represented by the TVFOG. Table 11 shows the TVFOG for the different components of the BOF. The PVFP exhibits some asymmetry which is mainly caused by the investment surplus component PVFP^I. Note that the TVFOG is significantly lower than GCR_{PH}, which emphasizes the importance of the GCR.

As expected, the GCR hardly changes the overall TVFOG, since the main components (GCR^C_{SH}, GCR^C_{PH}) are driven by the cost surplus which is largely unaffected by the stochasticity of the financial market. Hence, we will partly limit sensitivity analyses in the next section to deterministic projections under the CE scenario (as far as it does not affect the reliability of our findings).

Table 11 Time Value of Financial Options and Guarantees

(in 1,000€)	BOF	PVFP	GCR
$\mathbb{E}^{\mathbb{Q}}(\cdot)$	8,160	6,610	1,550
$PV^{[CE]}(\cdot)$	8,800	7,250	1,550
TVFOG	640	640	< 1,000€

6 Sensitivity analyses

The base case showed that allowance for a GCR has a significant impact on the BOF and the SCR. In order to asses the robustness of our results, we vary model parameters that have an impact on the size of the GCR. The main focus is on the cost model and the corresponding parameters. The methodology described in section 4 is repeated for each of the analyzed parameter sets.

6.1 Level of best estimate cost parameters

We separately vary ceteris paribus, the best estimate acquisition and administration cost assumptions, respectively.[21] We first analyze the impact on the BOF and its components. Then, we explore the impact on the SCR and the ExC.

As a simplification, the results of this section are based on a single projection in the CE scenario. We verified that the level of best estimate cost parameters does not have a material impact on the TVFOG. As a consequence, the results presented below differ from the base case presented in the previous section (e.g. for the acquisition cost rate of 5.0%).

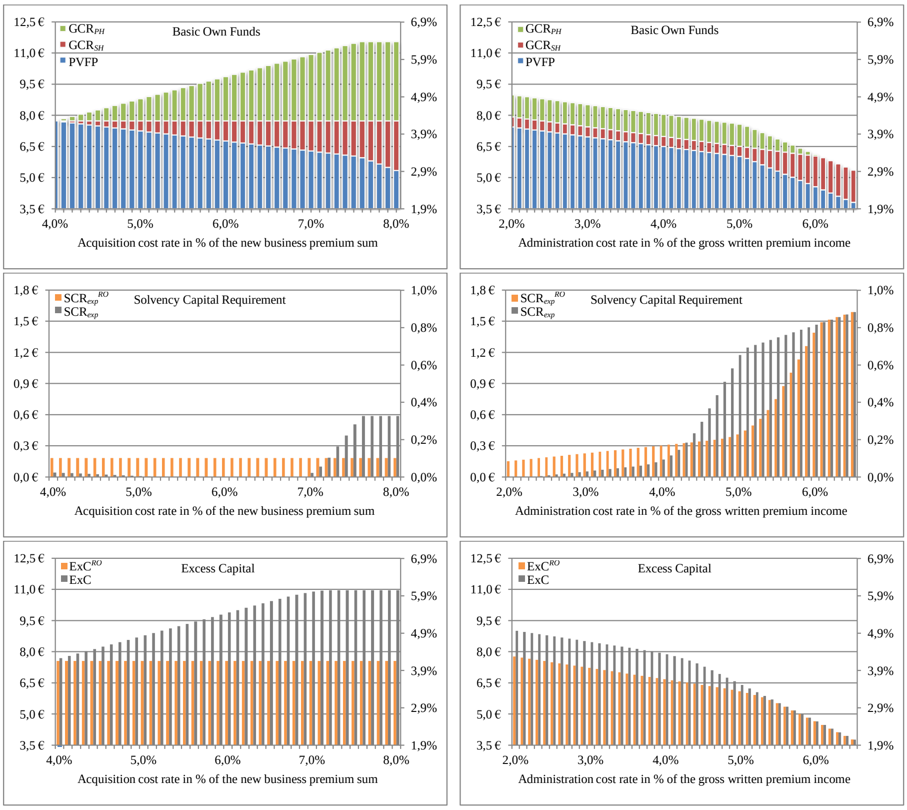
6.1.1 Effects on GCR and BOF

Fig. 3 illustrates the dependency of the GCR and the BOF on the actual expenses. In the upper-left graph, GCR = 0 for an acquisition cost rate of 4.0 %, since initial acquisition charges are sufficient to cover the incurring acquisition costs. An increase of the best estimate acquisition cost rate leads to an increase of the initial funding gap of each new business cohort which is covered by the cost surplus of existing business. Initially, this results in a linear growth of GCR_{PH} and GCR_{SH}, respectively, with the growth rate of the two items being driven by the cost surplus participation rate $p^C = 70\,\%$. As mentioned before, the rise of GCR_{SH} leads to an equivalent decrease of the PVFP (with zero impact on the BOF), since the funds additionally inherited by the company reduce the company's profits. In contrast, the rise of GCR_{PH} results in an increase of the BOF (and a reduction of the FDB). Although counterintuitive at first sight, less profitable new business increases the BOF.

[21] Regarding the variation of the acquisition costs, we assume that the ratio between the commission and the internal acquisition costs remains as given in the base case.

The BOF reach a maximum of 11.5 m€ at an acquisition cost rate of 7.6 %. This corresponds to an increase of 3.8 m€ (49 %) compared to the pure run off perspective. If the acquisition cost rate exceeds 7.6 %, the overall cost surplus Sp_t^C in run 3 becomes negative and additional acquistion costs have to be fully covered by the company. Therefore, GCR_{PH} remains constant and the entire cost surplus of the in-force business is used to cover the acquisition costs of new business. The PVFP under going concern assumptions decreases with a corresponding increase of GCR_{SH} such that the BOF remain constant.

Note that for acquisition cost rates above 5.7 %, actual acquisition costs of new business contracts are not covered by corresponding charges over the lifetime of the new business contract. As discussed in section 2.3, this type of cross-subsidization should not be reflected in the FDB. This implies an upper limit for the GCR and thus the BOF.



Notes: The primary vertical axis is given in millions. The secondary vertical axis represents the percentage of the balance sheet total.

Fig. 3 Influence of the best estimate cost parameters on the BOF, SCR and ExC (CE values)

In the upper-right graph of Fig. 3, the value of the total GCR remains constant. This is caused by the fact that the same acquisition cost rate (5.0 % as in the base case) is applied for all runs and only the administration cost rate varies. Obviously, the amount of cost surplus required to cover the acquisition expenses of the future new business is not affected by the level of the administration cost rate. However, the administration cost rate has an impact on the composition of the GCR and the overall amount of BOF since acquisition and administration cost surplus are balanced on the first stage of the surplus participation process, as described in section 2.2.3.

If the administration cost rate increases, less cost surplus is available to finance new business. However, as long as the cost surplus remains positive, the policyholders and the company share the financing of the acquisition costs in line with the cost surplus participation rate p^C. Therefore, an increase of the administration cost rate reduces the PVFP and FDB proportionally which leads to an corresponding decrease of the BOF.

Once the administration cost rate reaches 5.1 %, the administration cost surplus in run 3 becomes negative. Hence, the cost surplus is not sufficient any more to cover the entire acquisition costs of new business. Consequently, GCR_{PH} decreases and eventually drops to zero whereas GCR_{SH} increases. At the same time, the decrease of PVFP is accelerated since the deficit in the cost surplus has to be covered by the company. Overall, the BOF decrease at a higher rate.

At an administration cost rate of 6.1 %, the cost surplus becomes negative in run 2 as well. Therefore, $\text{GCR}_{PH} = 0$ and acquistion costs of new business are fully borne by the company. Any further increase of the administration costs one-to-one reduces the BOF.

6.1.2 Effects on SCR and ExC

As we have seen in the base case, allowance for a GCR significantly reduces SCR_{exp} compared to a pure run off perspective (denoted by SCR_{exp}^{RO}). Since the stress scenario includes a combined stress of internal acquisition costs and administration costs, the SCR is a combination of the change of the BOF due to increasing acquisition and administration cost rates. Therefore, the results presented in the first row of Fig. 3 clearly affect the SCR results shown in the second row of Fig. 3.

Obviously, SCR_{exp}^{RO} does not depend on the acquisition cost rate (cf. left-hand graph). As the stress scenario is defined by a relative shock of the best estimate expenses and an increase of the cost inflation rate, the absolute change of expenses due to stress increases with the best estimate acquisition/administration cost rate. Consequently, SCR_{exp}^{RO} increases with increasing administration cost rate before stress (cf. right-hand graph).

Overall, the behavior of the SCR curve is driven by the asymmetry regarding the splitting of surplus between the policyholders and the insurance company (cf. section 2.2.2). If cost surplus turns from positive to negative due to the cost stress, the insurer's profits are reduced disproportionately since the losses incurred under the stress scenario are fully borne by the insurer (whereas profits before stress are split with policyholders).

For acquisition cost rates below 7.2 %, we observe that $\text{SCR}_{exp} < \text{SCR}_{exp}^{RO}$ (cf. left-hand graph). This is due to the fact that the cost surplus in run 3 (and consequently in run 2 as well) remains positive after stress. As described for the base case in section 5.2, the reduced PVFP caused by the administration cost stress is partly compensated by the rise of the GCR due to increased internal acquisition costs. This results in an SCR_{exp} that is almost neglible and even decreases slightly. This changes for acquistion cost rates above 7.0 %: SCR_{exp} increases and reaches a maximum that is three times higher than SCR_{exp}^{RO}. The increase is again due to the fact that cost surplus becomes negative after stress. For acquisition cost rates above 7.5 %, SCR_{exp} remains constant since losses due to the stress have to be fully covered by the insurance company.

An increase of the administration cost rate before stress leads to a linear increase of SCR_{exp} for small administration cost rates (cf. right-hand graph). For an administration cost rate of 4.1 %, the shock of the expenses already leads to a negative cost surplus in run 3, whereas in run 2 the stress can still be partly absorbed by a reduction of the policyholders' cost surplus. This causes a strong increase of SCR_{exp}. Once the administration cost rate exceeds 5.2 %, the stress also results in a negative cost surplus in run 2 and SCR_{exp} again increases linearly. Finally, SCR_{exp} and SCR_{exp}^{RO} coincide for administration cost rates above 6.0 % since in both runs the cost surplus is already negative in the best estimate scenario.

Combining the changes in the BOF and the SCR caused by the GCR, the last row of Fig. 3 shows the impact on the ExC, again compared to a pure run off perspective. An increase of the acquisition cost rate increases ExC linearly until a maximum is reached for an acquisition cost rate of 7.2 %. At this point, the additional ExC amounts to 3.4 m€ (which corresponds to an increase of 45 %). For a low level of administration costs, the difference between the ExC under a going concern and under a pure run off perspective in the right-hand graph remains at its maximum of 1.2 m€ (+14 %). With increasing administration cost rate, the ExC under going concern assumptions converges towards the corresponding run off value.

6.2 Cost model

As demonstrated in section 4.2, the GCR is influenced by the modeling of overhead costs and cost inflation. Therefore, we now investigate the impact of these aspects of the cost model on the results.

6.2.1 Cost inflation

In contrast to the base case, we now consider inflation that affects both sum insured and costs; we choose $ir_t^G = ir_t^C = 2.0\,\%$ for illustrating the effects. We keep the steady state assumption (on a nominal basis) and therefore assume that the same inflation rate applied also in the past.

This implies that premiums differ between the cohorts and that charges increase in line with the inflation rate. Since $ir_t^C = ir_t^G$, the increase of the administration costs over time is leveled by the additional premium income due to new written business. Therefore, in a steady state, the administration cost surplus (as a percentage of premium income) remains constant over time under going concern assumptions (i. e. in run 1 and run 3). In contrast, in a pure run off perspective (run 2), the increase of costs due to inflation is not compensated by a corresponding growth of the premium income. This results in a decreasing administration cost surplus over time.

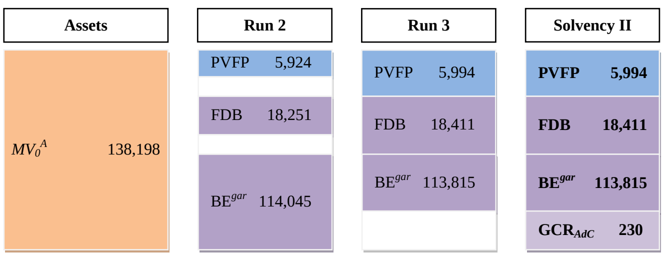


Fig. 4 Solvency II balance sheet at time $t = 0$ with ${}^{inh}AcC_t^* = 0$ (in 1,000 €)

To illustrate this effect, Fig. 4 shows results for a scenario in which the actual acquisition costs are equal to the initial acquisition charges. Thus, ${}^{inh}AcC_t^* = 0$ and there is no inheritance regarding acquisition costs. However, the PVFP and FDB of run 3 exceed the corresponding items of run 2, which corresponds to an inheritance of cost surplus from future new business to in-force business. This can be interpreted as an administration cost relief provided by future new business. Therefore, the (negative) inheritance of administration cost surplus can be determined as

$$\mathrm{GCR}_{AdC} = \mathrm{BE}^{gar}_{(\mathrm{run}\,2)} - \mathrm{BE}^{gar}_{(\mathrm{run}\,3)}\,.$$

Note that GCR_{AdC} represents funds that are used to cover acquisition costs of future new business under going concern assumptions, but are needed to cover additional administration costs (caused by inflation) in case of a run off. As discussed in section 4.3, these funds cannot be counted as BOF but have to be considered as part of the BE.

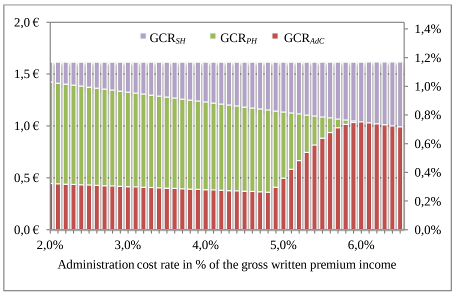


Fig. 5 Influence of inflation on the inheritance of cost surplus (CE values)

We now assume again that new business exhibits an initial funding gap that may be covered by the cost surplus of existing business (as in the base case). Taking into account the effects regarding the administration cost surplus described above, the GCR as well as the overall level of the BOF is reduced. Fig. 5 illustrates this effect.[22] It shows that GCR_{AdC} grows linearly with the administration cost rate, proportionally reducing GCR_{SH} and GCR_{PH} and consequently the additional BOF caused by the inheritance of cost surplus. Therefore, the value of the overall GCR remains constant but its composition changes.

GCR_{AdC} only depends on the level of the administration costs but not on the acquisition costs. If the acquisition cost rate is varied, GCR_{PH} and GCR_{SH} change accordingly, but GCR_{AdC} remains unchanged. Therefore, we limited the analyses to the level of the administration cost rate.

6.2.2 Overhead costs

Starting from the base case, we again assume the administration cost rate under going concern assumptions to equal 2.4 % of the premium sum from new business. However, in order to illustrate the effects, we decompose the total administration costs into a fixed part (10%) and a variable part (90%) as shown in Table 12. Due to the fixed administration costs, the

[22] Note that again the results of Fig. 5 are based on a single projection in the CE scenario. As analyzed in section 5.2, this simplification has no material impact on the size and composition of the GCR.

Table 12 Best estimate administration cost parameters

${}^{fix}AdC_t^*$	${}^{var}AdC_t^*$	AdC_t^{pP}
20.83 €	45,031.62 €	2.4 %

administration cost rate in a run off situation is increasing (as a percentage of premium income). This leads to a higher administration cost burden per policy and consequently to a reduced cost surplus in run 2 (resulting in higher BE^{gar} and a reduced PVFP and FDB in run 2). In consequence, similar to the previous analysis, GCR_{AdC} reduces the inheritance of cost surplus by policyholders and the insurer (cf. Fig. 6) and thus reduces the BOF from 8.160 m € to 7.844 m €. Note that results in run 3 are not affected by this change of the cost model.

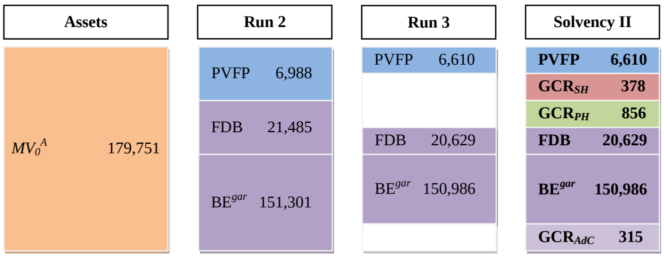


Fig. 6 Influence of overhead costs on the Solvency II balance sheet (in 1,000 €)

6.3 Volume of future new business

The volume of future new business is characterized by a high degree of uncertainty. We therefore vary the volume of future new business by +/- 20% and analyze the impact on the Solvency II balance sheet. In Table 13, the results are compared with corresponding sensitivities regarding the inherited acquisition costs.

Table 13 Liability side of the Solvency II balance sheet for different scenarios

(in 1,000 €)	BOF	PVFP	GCR_{SH}	GCR_{PH}	FDB	BE^{gar}
Best Estimate	8,160	6,610	479	1,071	20,629	150,986
New business +20 %	8,297	6,548	541	1,208	20,492	150,986
${}^{inh}AcC_t^*$ +20 %	8,374	6,514	575	1,285	20,415	150,986
New business −20 %	8,008	6,678	411	919	20,781	150,986
${}^{inh}AcC_t^*$ −20 %	7,946	6,706	383	857	20,843	150,986

As expected, an increase/decrease of the acquisition cost rate by 20 % leads to an increase/decrease of the GCR by 20 %. Opposed to that, for a 20 % increase of the volume of new business, the GCR increases by less than 20 %.

This can be explained by the fact that the volume of future new business not only affects the acquisition costs but also the premium income. E. g., a 20 % increase of the number of new policyholders increases the (uncovered) acquisition costs and the premium income of new business by 20 %. Since, starting from the second year, the first year's new business itself helps to finance the acquisition costs of future new business, a lower share of ${}^{inh}AcC_t^*$ is allocated to the initial in-force business. Thus, the increase of GCR is smaller. In case of a decrease of the volume of new business reverse effects can be observed.

We conclude that the assumptions regarding acquisition costs have a stronger impact on GCR than assumptions regarding the volume of new business.

6.4 Cost surplus participation rate

The insurance company has some discretion regarding the cost surplus participation rate since the MindZV only specifies a minimum rate of $p^C \geq 50\,\%$. The choice of p^C clearly affects the GCR and the BOF as well as the SCR. We therefore consider alternative choices $p^C = 90\,\%$ (same rate as for investment income) and $p^C = 50\,\%$ (minimum rate).

Table 14 Influence of the participation rate p^C on the Solvency II balance sheet

(in 1,000 €)	BOF	PVFP	GCR_{SH}	GCR_{PH}	FDB	BE^{gar}
Best Estimate	8,160	6,610	479	1,071	20,629	150,986
$p^C = 50\,\%$	8,949	7,399	786	764	19,840	150,986
$p^C = 90\,\%$	7,370	5,820	173	1,377	21,418	150,986

Note that $GCR_{SH} > GCR_{PH}$ (cf. second row of Table 14), although $p^C = 50\,\%$, i. e. cost surplus is split evenly between policyholders and the insurer. This is caused by the fact that the policyholder's share of the surplus (including cost surplus) is assigned to the bonus reserve and not paid out until maturity or death. In the meantime, investment return on these additional funds is split between policyholders and the insurer as part of the surplus distribution process. Therefore, the insurer benefits from investment returns on past cost surpluses already allocated to policyholders (but not paid out yet). In consequence, under going concern assumptions, the insurer not only suffers from the reduction of his own share of cost surplus due to acquisition costs of new business, but also from the reduced investment returns in subsequent years due to lower bonus reserves.

In general, a lower participation rate p^C means that the insurer keeps a higher portion of the cost surplus, but also has to finance a higher share of the acquisition costs of new business. Overall, the BOF increase if p^C is decreased. Given that, the higher the participation rate p^C, the stronger is the impact of an increase of the acquisition cost rate on the BOF, since a higher proportion of the acquisition costs is financed by the policyholders (cf. left-hand graph in the first row of Fig. 7).[23] Concerning the administration costs, the reverse effect can be observed. The lower the participation rate p^C, the stronger the impact of an increase of the administration costs rate on the BOF.

A lower participation rate also increases the expense risk for the insurance company since the policyholders absorb a smaller share of the additional costs in the stress scenario.

[23] Again, for the variation of the best estimate cost parameters, Fig. 7 shows the CE values, which differ in scale from the values resulting from a full stochastic valuation, as given in Table 14.

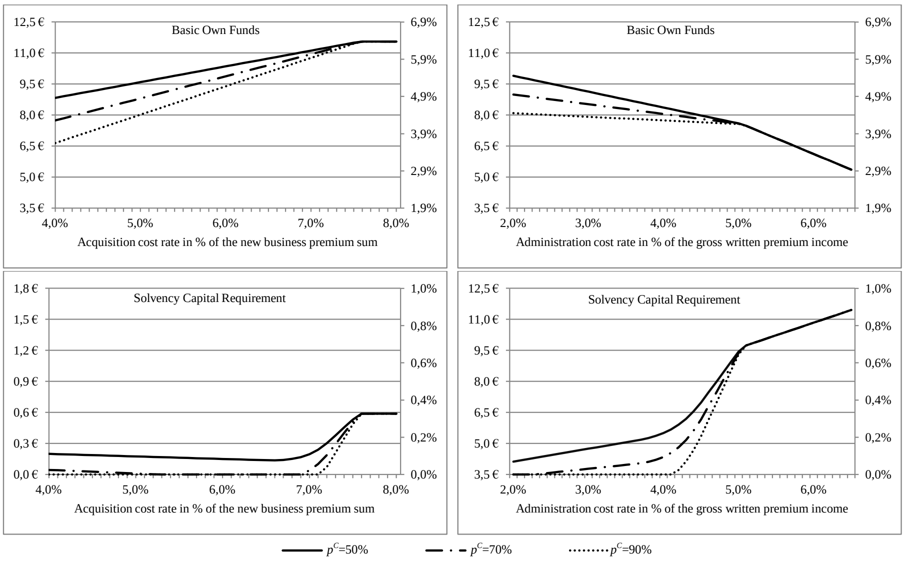


Fig. 7 Influence of the cost participation rate p^C on BOF and SCR (CE values)

This is illustrated in the second row of Fig. 7. However, if the cost surplus becomes negative, the insurer has to fully cover the losses, regardless of the participation rate p^C. In consequence, the BOF of all three scenarios converge to the same amount; the same holds for the SCR.

6.5 Financial market situation

Our final sensitivity analysis illustrates how the inheritance effects depend on the financial market situation. Table 15 shows the company's BOF in a financial stress scenario that corresponds to an immediate drop of interest rates by 100 basis points.

In this stress scenario (which is similar to the stress scenario used for determining the SCR for interest rate risk), the insurer's profits from investment surplus decreases by 76 % and the TVFOG increases by a factor of five. In contrast, PVFP^C and the GCR are rather stable, since the cost surplus does not depend on the financial situation. The values even rise due to lower discounting of future cost surpluses which implies that the interest rate risk is lower if allowance is made for a GCR.

This result is of high relevance, as it shows that the GCR can help to stabilize the BOF, in particular in a situation of low interest rates.

Table 15 Financial Stress

(in 1,000 €)	BOF	PVFP^I	PVFP^C	TVFOG	GCR$_{SH}$	GCR$_{PH}$
Best Estimate	8,160	5,410	1,200	640	479	1,071
Interest rate stress	4,186	1,283	1,265	3,238	482	1,156

7 Conclusion

In this paper, we analyzed the inheritance effects caused by the pre-financing of acquisition costs of new business by the cost surplus of existing business and discussed the impact on the valuation of participating life insurance contracts in the context of Solvency II. Since these effects are inherent in traditional German life insurance, they have to be considered to fulfill the requirements of the Solvency II regime for a run off valuation of future obligations from existing business under going concern assumptions. We further showed that, based on the regulatory framework for surplus participation, the classification of the inherited funds – denoted as GCR – as BOF is justified in general and in line with the Solvency II valuation principles. Note that (reverse) inheritance from future new business to existing business is possible and has to be reflected in the valuation approach as well.

One of the preconditions for the consideration of inheritance effects in the Solvency II balance sheet is the availability of an appropriate and reliable methodology for determining the GCR. So far no such methodology has been proposed in the scientific literature. We solve this problem with an approach that is based on a stochastic projection model and used to perform two separate projection runs with and without allowance for expected future new business. From these runs, an artificial run off projection under going concern assumptions is derived that allows a reliable valuation of the GCR in the Solvency II valuation framework.

Our base case results show that the GCR has a significant impact on the BOF. It has to be pointed out that the inheritance of surplus not only affects the policyholders' surplus participation (GCR_{PH}) – as the Cash Flow Model of the GDV may indicate – but also the company's profits (GCR_{SH}). Whereas the former leads to additional BOF, the latter does not change the overall BOF. The overall GCR mainly consists of inherited cost surplus, but also includes side-effects on investment surplus.

Sensitivity analyses reveal the main factors affecting the size and composition of the GCR and consequently the amount of additional BOF related to inheritance effects. As expected, the amount of acquisition costs of future new business is highly relevant. In general, the inheritance of cost surplus increases with a growing funding gap in future periods which implies that the BOF increase if acquisition costs increase (which may be counterintuitive at first sight). Thereby, the composition of the overall GCR and the increase of the BOF is driven by the split of cost surplus between policyholders and the insurer. Whereas policyholders (partially) contribute to the pre-financing only to the point until the cost surplus is depleted, losses are fully borne by the insurer. In consequence, there is an upper limit for the additional BOF with respect to the acquisition costs.

The surplus participation mechanism in Germany implies that the actual administration costs also affect the GCR: if the administration cost rate increases, the combined acquisition and administration cost surplus is reduced and therefore less cost surplus is available to finance new business. We observe a decrease of the BOF that is accelerated once the (administration) cost surplus becomes negative. Further sensitivities confirm the importance of expected new business volume and corresponding (acquisition) costs for the quantification of the GCR.

Besides the positive impact on the BOF, we also considered (reverse) inheritance from future new business to existing business. As an example, we showed how charges factored in the premium of future new business can compensate future cost inflation. Furthermore, new contracts are required to avoid increasing per policy overhead costs. Therefore, new business helps to keep the cost surplus (allocated to the in-force business) stable in future years. In our approach, this is reflected by a separate component of the GCR that is not classified as BOF.

Our analysis is completed by an assessment of the impact on the SCR for life expense risk and thus on the overall capital position. Depending on the parameter combination, allowance for inheritance effects may increase SCR_{exp} (calculated based on the Solvency II standard formula). However, since this increase is lower than the increase of the BOF, excess capital still increases.

The results presented are based on a simplified valuation of a stylized life insurance company and consequently are rather of illustrative nature. Hence, the ratio of the balance sheet items might change if the methodology is applied to a specific insurance company. However, our approach reflects the main characteristics affecting the GCR, in line with current practice in German life insurance. For example, market data of BaFin (BaFin, 2013) confirm the size of the funding gap caused by acquisition costs assumed in our model. Therefore, we conclude that in practice the GCR significantly affects the BOF under Solvency II. Our analysis show that the GCR can help to stabilize the BOF, in particular in a situation of low interest rates.

The present paper focused on the inheritance effects regarding cost surplus. In general, the balancing and inheritance of surplus extends to all surplus sources. We therefore feel that there is room for additional research. It would be valuable to analyze inheritance effects regarding the investment and risk surplus in the context of Solvency II. Such an analysis would need to discuss restrictions mentioned in the latest explanatory notes of BaFin (cf. BaFin, 2014). Such analyses would also be relevant for other valuation approaches, such as MCEV (in particular regarding the quantification of the new business value).

The Solvency II framework recognizes additional inheritance effects by so-called surplus funds (cf. *Directive 2009/138/EC*, art. 91). These effects relate to accumulated profits that have not been allocated to individual policyholders. Similar to the GCR, a sound economic justification for classification of the surplus funds as tier 1 BOF as well as a reliable methodology for their quantification is missing in the scientific literature.

From a conceptual point of view, the calculation of the Risk Margin under Solvency II may need to be reconsidered, taking into account inheritance effects reflected in the BE and the SCR. Since the reference undertaking underlying the Risk Margin methodology does not write new business, the corresponding SCR may need to be adjusted (considering both GCR and surplus funds). Overall, this discussion illustrates the need for a proper regulatory framework that reflects balancing effects between different cohorts of policyholders as well as balancing effects over time inherent in participating life insurance across Europe.

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