Tensor Networks for dissipative systems

Mari-Carmen Bañuls,

with J. I. Cirac (MPQ), J. Cui (Ulm),

I. de Vega (LMU)

PRL 114, 220601 (2015)
PRA 92, 052116 (2015)

MPQ

Max-Planck-Institut
für Quantenoptik
(Garching b. München)

624. WE-Heraeus Seminar
Bad Honnef 19.9.2016

What are TNS?

- TNS = Tensor Network States

Context: quantum many body systems

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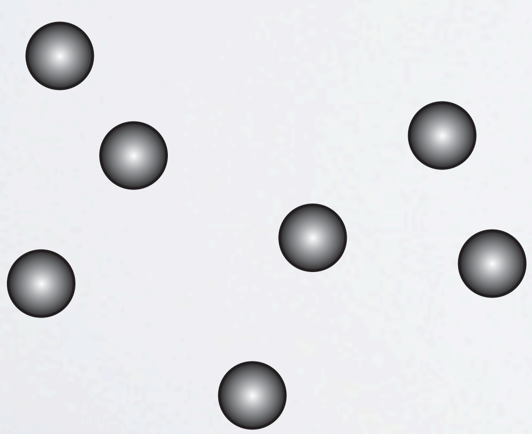
$\{|i\rangle\}_{i=0}^{d-1}$

N



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Context: quantum many body systems

interacting with each other

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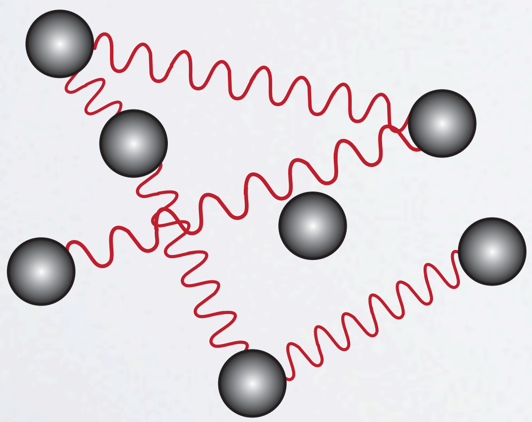

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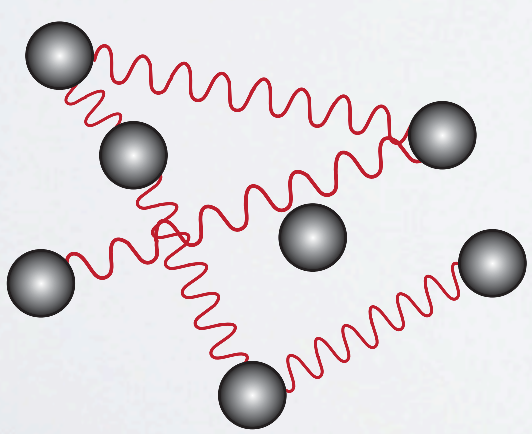

Goal: describe equilibrium states

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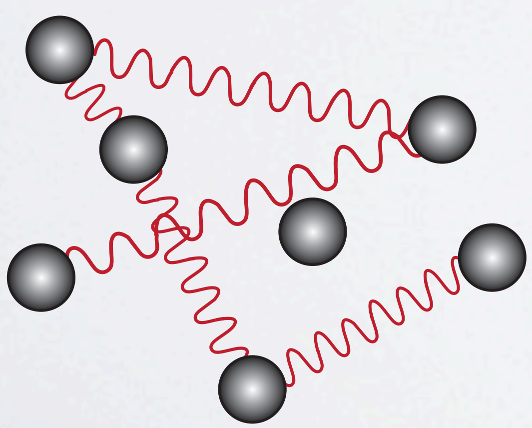

interacting with each other

Goal: describe equilibrium states

ground, thermal states

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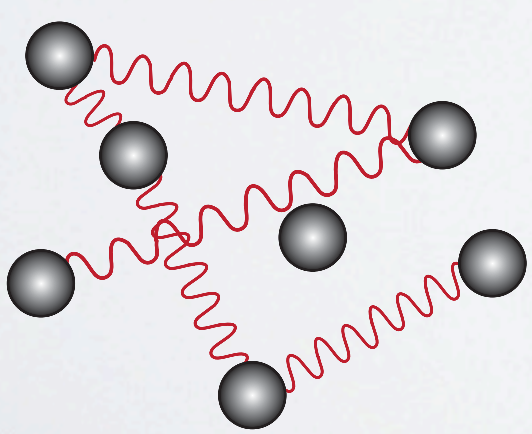

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interesting states?

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A general state of the N-body Hilbert space has exponentially many coefficients

$$|\Psi\rangle = \sum_{i_j} c_{i_1 \dots i_N} |i_1 \dots i_N\rangle$$

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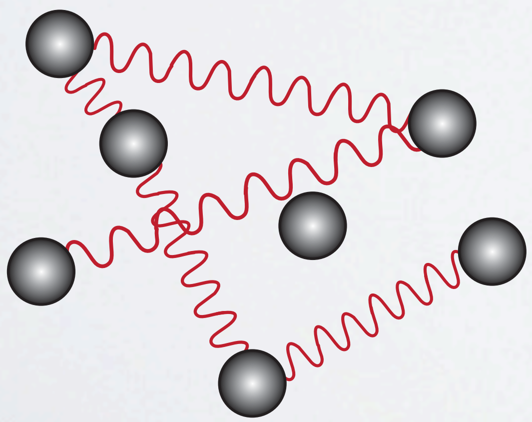

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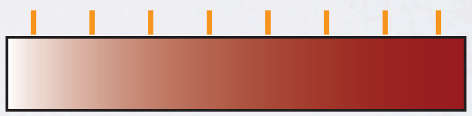
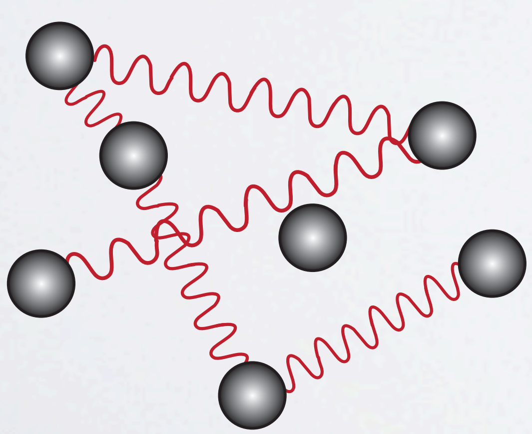


N-legged tensor

d^N

N



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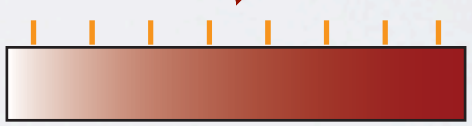

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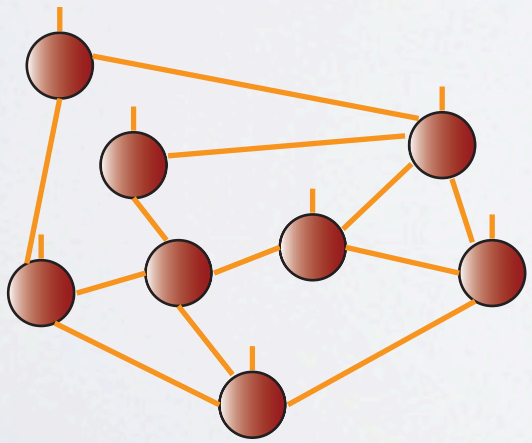
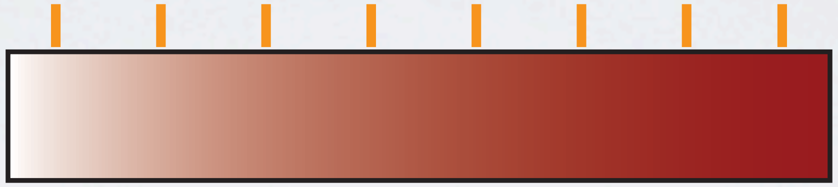


$\mathrm{poly}(N)$

A TNS has only a polynomial number of parameters

1D SYSTEMS: MPS

- MPS = Matrix Product States



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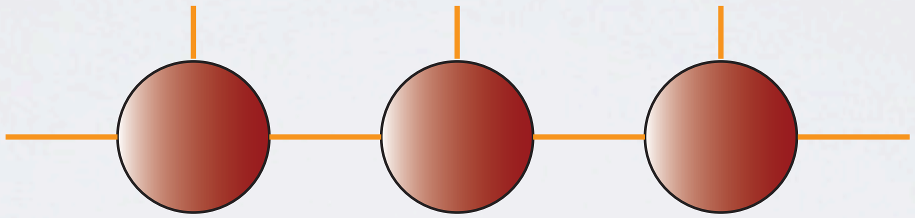

$$|\Psi\rangle = \sum_{i_1 \ldots i_N} \mathrm{tr}(A_1^{i_1} A_2^{i_2} \ldots A_N^{i_N}) |i_1 \ldots i_N\rangle$$

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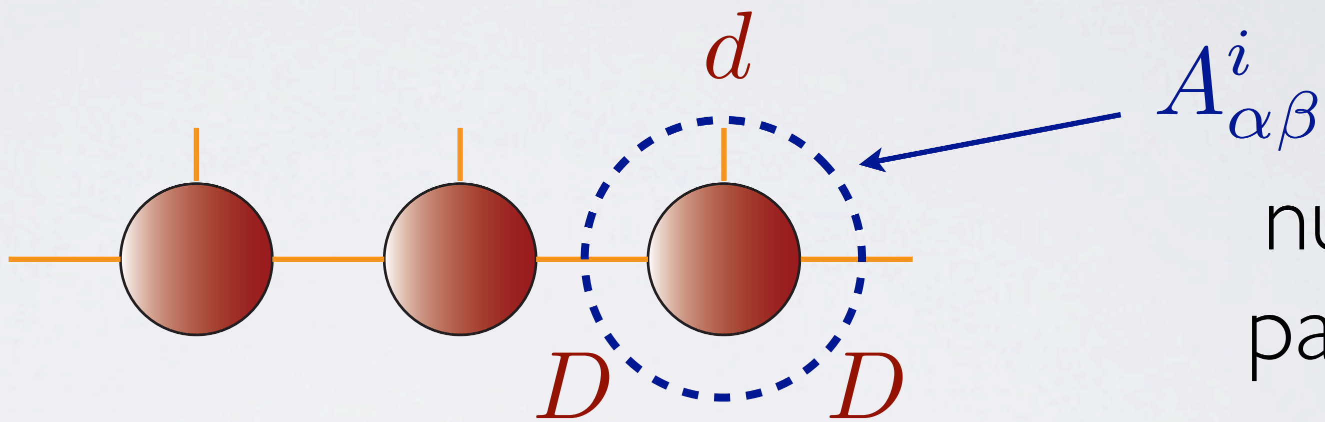

number of parameters

NdD^2

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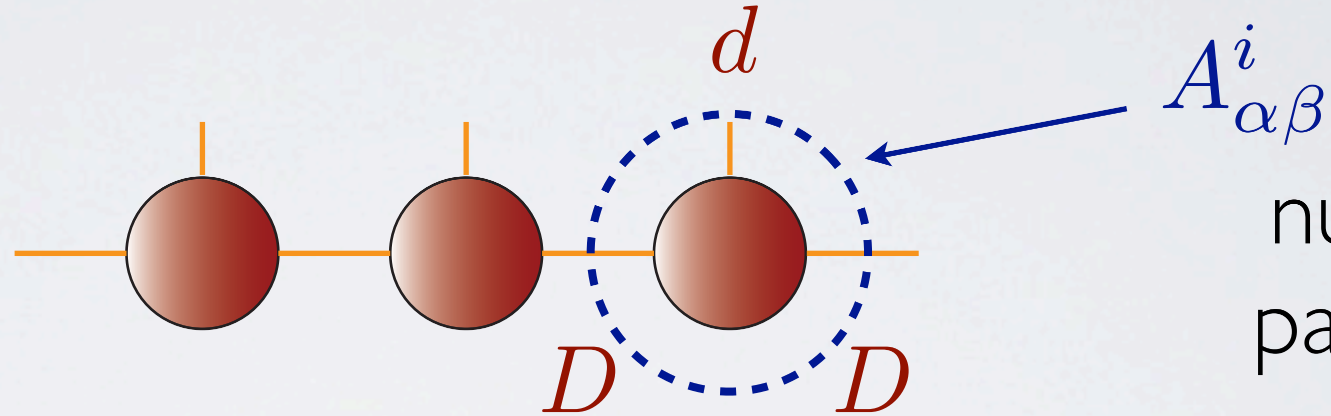

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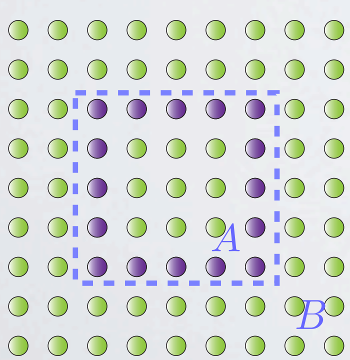

Area law by construction

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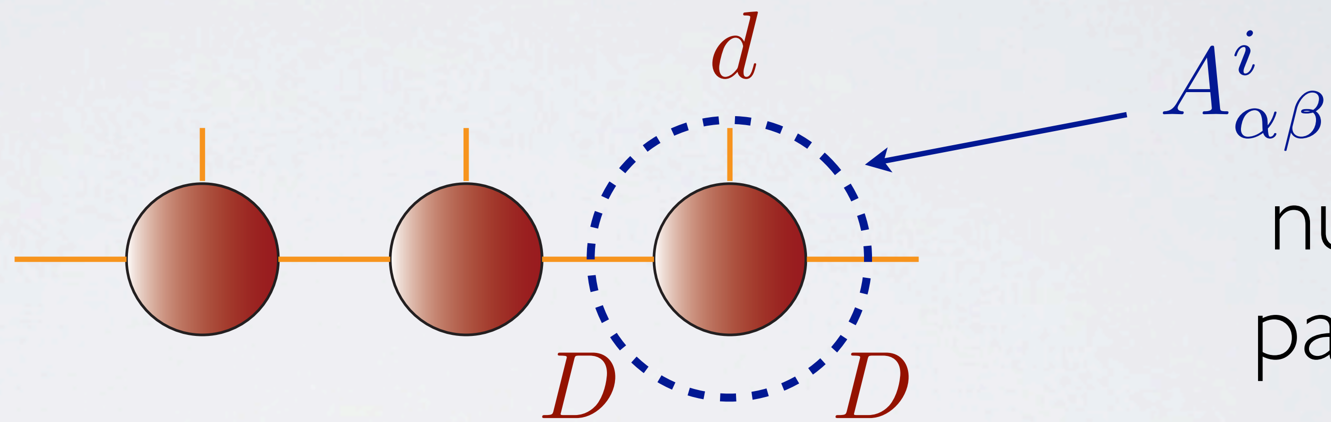

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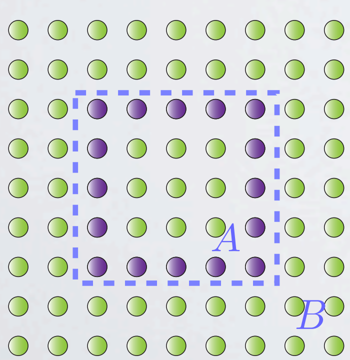
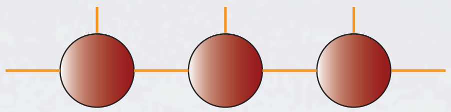


Area law by construction

Bounded entanglement $S(L/2) \leq \log D$

What can MPS be used for?

MPS extremely successful tool



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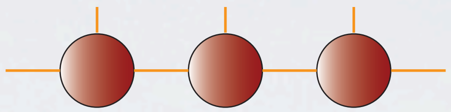
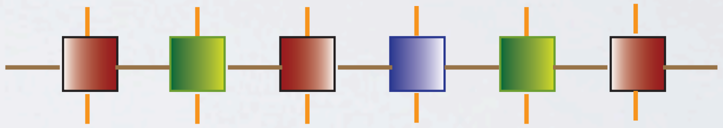


LOCAL HAMILTONIAN 

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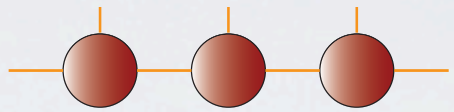
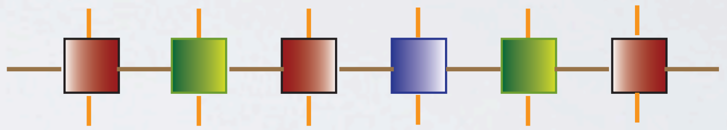


LOCAL HAMILTONIAN 

good approximation of ground states

gapped finite range Hamiltonian
⇒ area law (ground state)

Verstraete, Cirac, PRB 2006
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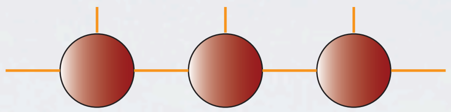
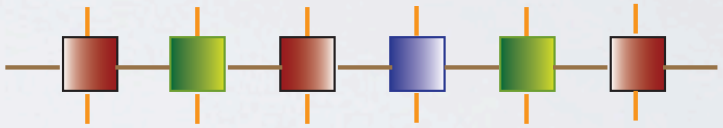


LOCAL HAMILTONIAN 

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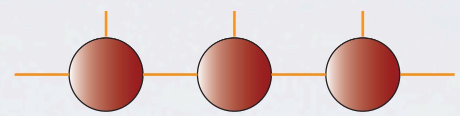
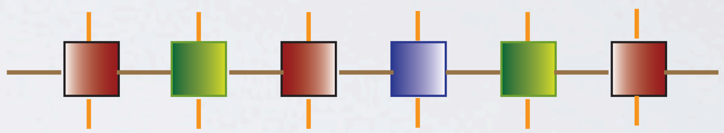


LOCAL HAMILTONIAN 

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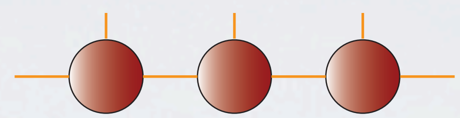
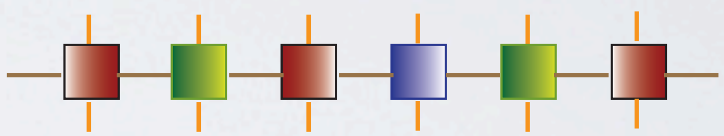


LOCAL HAMILTONIAN 

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time evolution can be simulated too

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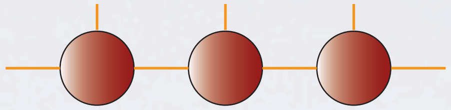

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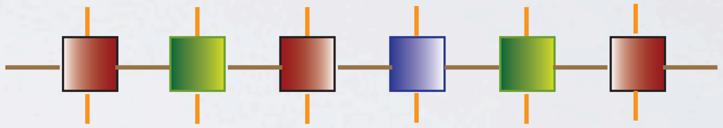

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but entanglement can grow fast!

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FOR MIXED STATES...

MIXED STATES

- MPO = Matrix Product Operator

Same kind of ansatz for operators

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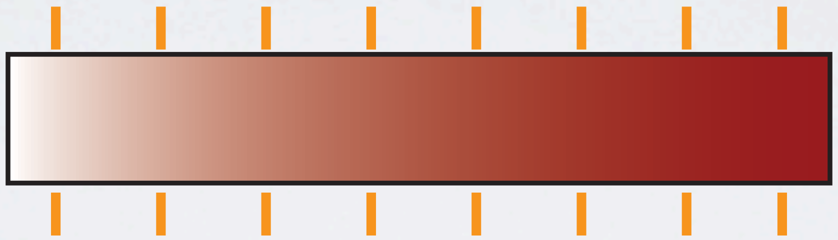

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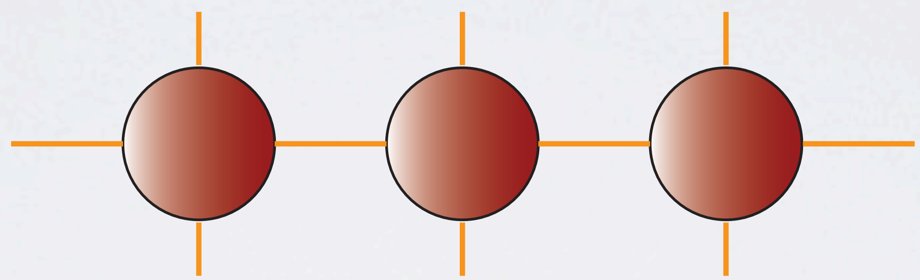
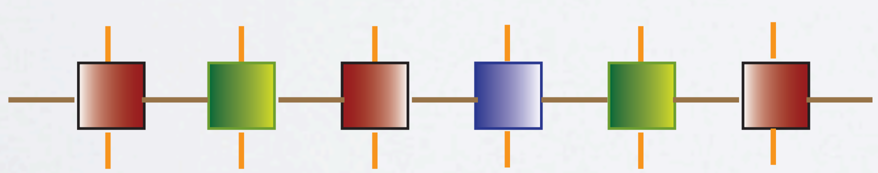


$$\hat{M} = \sum_{i_1, j_1 \dots i_N, j_N} \mathrm{tr}(M_1^{i_1 j_1} M_2^{i_2 j_2} \dots M_N^{i_N j_N}) |i_1 \dots i_N\rangle\langle j_1 \dots j_N|$$



Routinely used for H and $U(t)$

Verstraete, García-Ripoll, Cirac PRL 2004

Pirvu et al., NJP 2010

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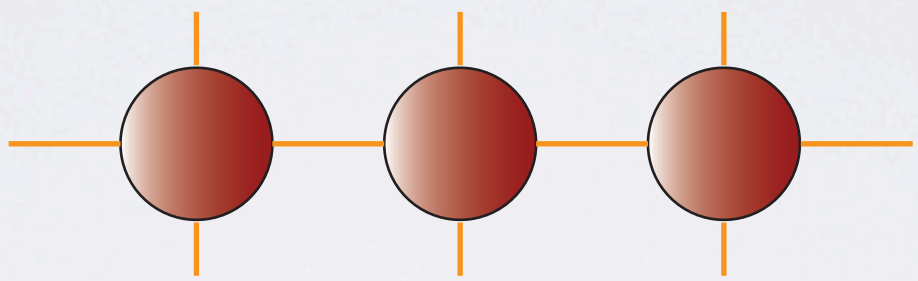

density operators need some properties

$$\rho = \sum_{i_1, j_1 \ldots i_N, j_N} \mathrm{tr}(M_1^{i_1 j_1} M_2^{i_2 j_2} \ldots M_N^{i_N j_N}) |i_1 \ldots i_N\rangle\langle j_1 \ldots j_N|$$

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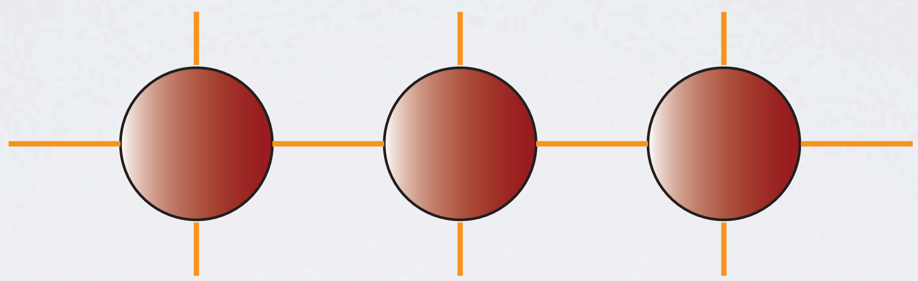

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$$\rho = \rho^\dagger \qquad \mathrm{tr}\rho = 1 \qquad \rho \geq 0$$

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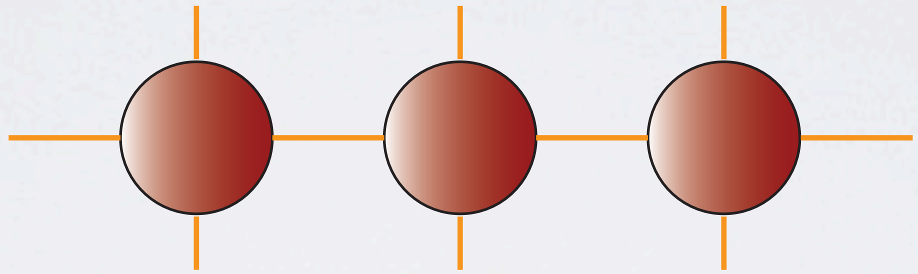

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can we impose them *locally*?

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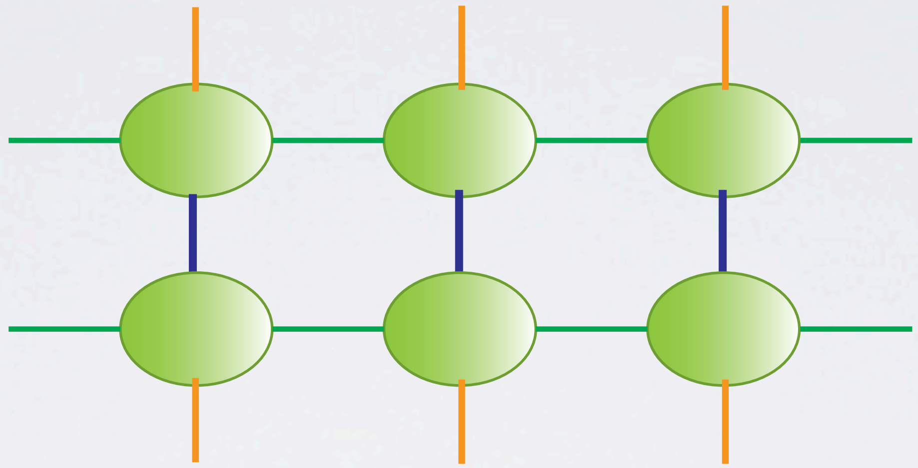
purification



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$\rho = \rho^\dagger$ ✔

$\mathrm{tr}\rho = 1$ ✔

$\rho \geq 0$

in a way

$$\rho_S = \mathrm{tr}_A |\Psi_{SA}\rangle\langle\Psi_{SA}|$$

Werner et al., PRL 2016

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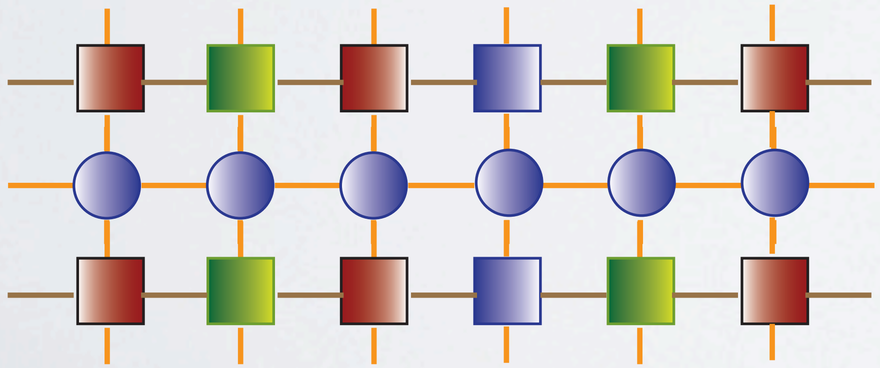

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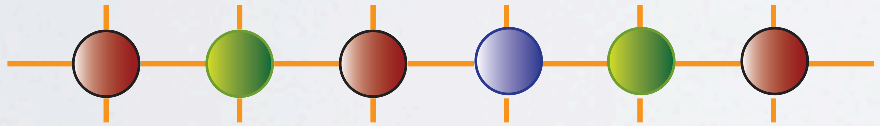

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In this talk...

Variational method to find **MPO** approximations for the **steady states** of Lindblad equations

with J. Cui, J. I. Cirac
PRL 114, 220601 (2015)

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Using **MPS** to describe whole system coupled to a **thermal environment**

with I. de Vega
PRA 92, 052116 (2015)

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A possibility for open systems

Real-time dynamics produces
a steady state

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Real-time dynamics produces a steady state

$$\frac{d\rho(t)}{dt} = \mathcal{L}(\rho) \longrightarrow \mathcal{L}(\rho_S) = 0$$

fixed point of Liouvillian map

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We can approximate it as a MPO

works by García-Ripoll et al., Zwolak, Prosen, ...

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simulating long time evolution

~imaginary time evolution

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A possibility for open systems

Real-time dynamics produces a steady state

$$\frac{d\rho(t)}{dt} = \mathcal{L}(\rho) \longrightarrow \mathcal{L}(\rho_S) = 0$$

fixed point of Liouvillian map

dissipative QC
dissipative QPT

We can approximate it as a MPO

simulating long time evolution
~imaginary time evolution

variationally
~DMRG

VARIATIONAL STEADY STATES

METHOD

VARIATIONAL STEADY STATES

METHOD

Dynamics determined by Liouvillian

VARIATIONAL STEADY STATES

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$$\frac{d\rho}{dt} = \mathcal{L}(\rho)$$

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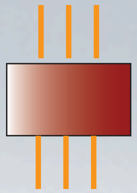

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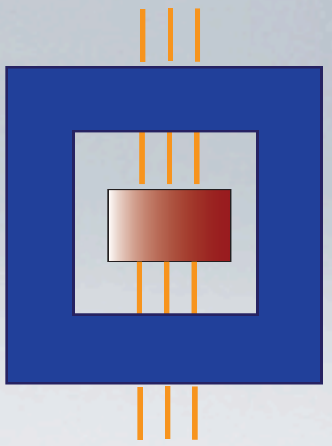

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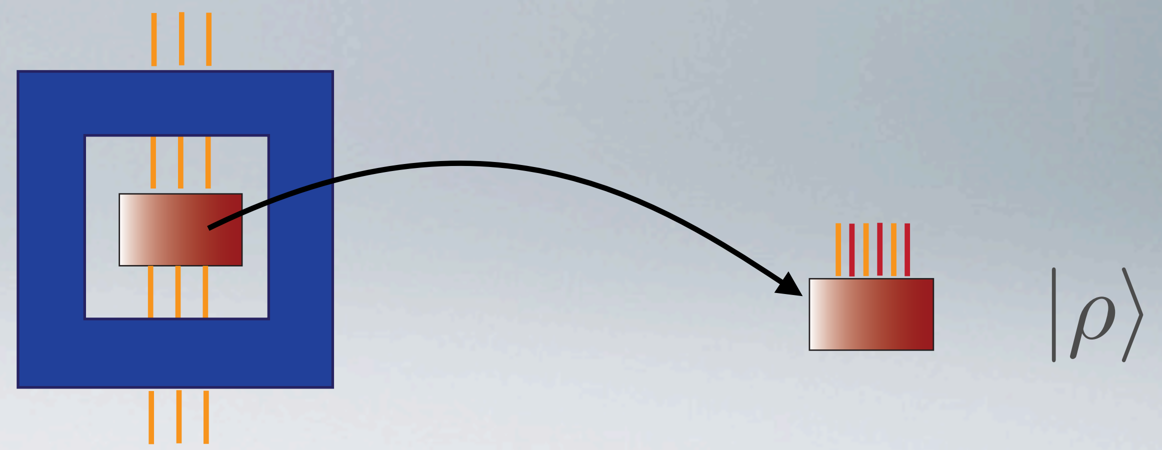

vectorize $|\rho\rangle$

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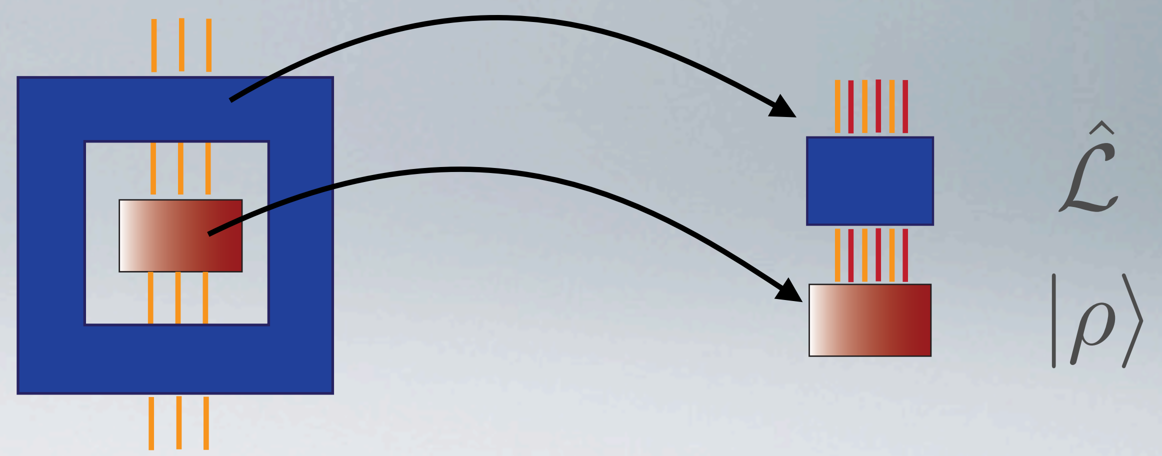

vectorize $|\rho\rangle$

superoperator $\hat{\mathcal{L}}$

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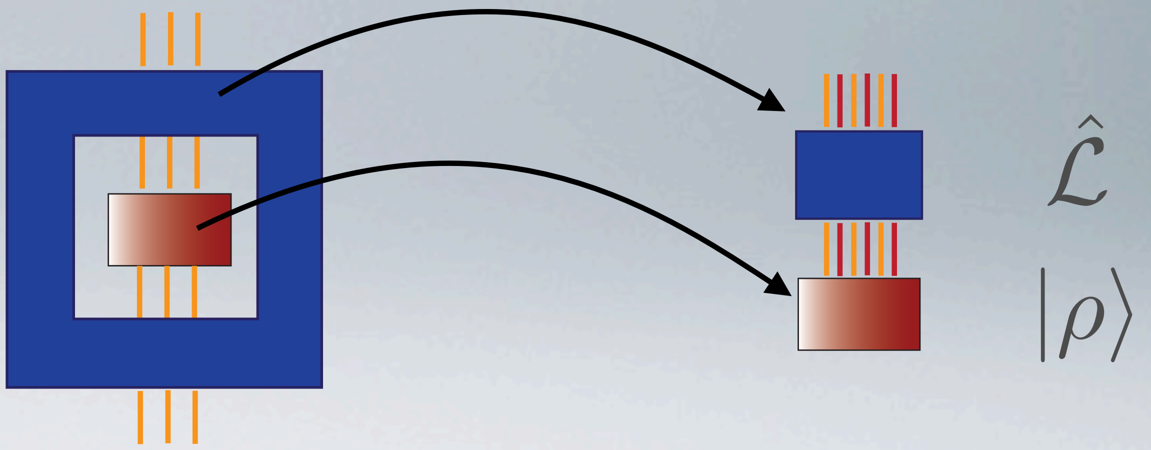

vectorize $|\rho\rangle$

superoperator $\hat{\mathcal{L}}$

WANTED

fixed point of evolution

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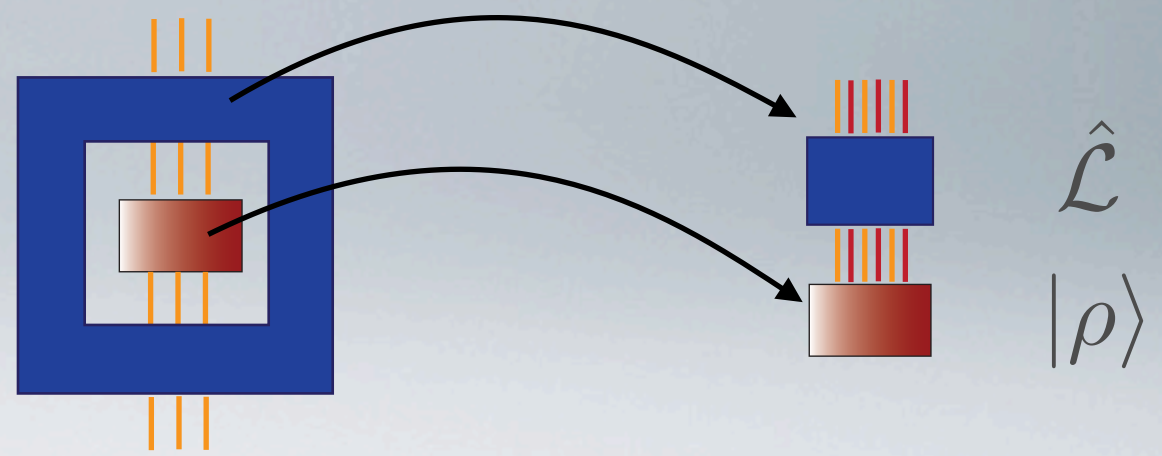

vectorize $|\rho\rangle$

superoperator $\hat{\mathcal{L}}$

WANTED

fixed point of evolution

Search for the null vector

$$\hat{\mathcal{L}}|\rho\rangle = 0$$

Mascarenhas et al., PRA92, 022116 (2015)

VARIATIONAL STEADY STATES

METHOD

VARIATIONAL STEADY STATES

METHOD

Analogy to GS search

VARIATIONAL STEADY STATES

METHOD

Analogy to GS search

H

VARIATIONAL STEADY STATES

METHOD

Analogy to GS search

H

$\min\ \lambda$

VARIATIONAL STEADY STATES

METHOD

Analogy to GS search

H

$\min \ \lambda$

$|\Psi_{\mathrm{GS}}\rangle$

VARIATIONAL STEADY STATES

METHOD

Analogy to GS search

H	$\hat{\mathcal{L}}$
$\min\ \lambda$	
$\lvert\Psi_{\mathrm{GS}}\rangle$	

VARIATIONAL STEADY STATES

METHOD

Analogy to GS search

H	$\hat{\mathcal{L}}$
$\min\ \lambda$	$\lambda = 0$
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VARIATIONAL STEADY STATES

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Analogy to GS search

H	$\hat{\mathcal{L}}$
$\min\ \lambda$	$\lambda = 0$
$\lvert\Psi_{\mathrm{GS}}\rangle$	

$$e^{\hat{\mathcal{L}}}|\rho_S\rangle = |\rho_S\rangle$$

VARIATIONAL STEADY STATES

METHOD

Analogy to GS search

H	$\hat{\mathcal{L}}$
$\min\ \lambda$	$\lambda = 0$
$\lvert\Psi_{\mathrm{GS}}\rangle$	$\lvert\rho_{\mathrm{S}}\rangle$

$$e^{\hat{\mathcal{L}}}|\rho_S\rangle = |\rho_S\rangle$$

VARIATIONAL STEADY STATES

METHOD

Analogy to GS search

Hermitian	H	$\hat{\mathcal{L}}$	non-Hermitian
	$\min\ \lambda$	$\lambda = 0$	$e^{\hat{\mathcal{L}}}\lvert\rho_S\rangle = \lvert\rho_S\rangle$
	$\lvert\Psi_{\mathrm{GS}}\rangle$	$\lvert\rho_{\mathrm{S}}\rangle$	

VARIATIONAL STEADY STATES

METHOD

Analogy to GS search

Hermitian H | $\hat{\mathcal{L}}$ non-Hermitian

$\min \lambda$ | $\lambda = 0$

$e^{\hat{\mathcal{L}}}|\rho_S\rangle = |\rho_S\rangle$

$|\Psi_{\mathrm{GS}}\rangle$ | $|\rho_{\mathrm{S}}\rangle$

$$\hat{\mathcal{L}}^\dagger \hat{\mathcal{L}} \geq 0$$

VARIATIONAL STEADY STATES

METHOD

Analogy to GS search

Hermitian	H	$\hat{\mathcal{L}}$	non-Hermitian
	$\min\ \lambda$	$\lambda = 0$	$e^{\hat{\mathcal{L}}}\|\rho_S\rangle = \|\rho_S\rangle$
	$\|\Psi_{\mathrm{GS}}\rangle$	$\|\rho_{\mathrm{S}}\rangle$	

$$\hat{\mathcal{L}}^\dagger \hat{\mathcal{L}} |\rho_S\rangle = 0$$

lowest eigenvalue

VARIATIONAL STEADY STATES

METHOD

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Master equation of Lindblad form

VARIATIONAL STEADY STATES

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Master equation of Lindblad form

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_k \gamma_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\rho L_k^\dagger L_k - \frac{1}{2} L_k^\dagger L_k \rho \right)$$

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~*local* (MPO)

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$$\frac{d|\rho\rangle}{dt} = \left[-i(H \otimes I - I \otimes H^T) + \sum_k \gamma_k \left(L_k \otimes L_k^* - \frac{1}{2} I \otimes L_k^T L_k^* - \frac{1}{2} L_k^\dagger L_k \otimes I \right) \right] |\rho\rangle$$

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$$\hat{\mathcal{L}}$$

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~*local* (MPO)

local

$\hat{\mathcal{L}}$ MPO $\longrightarrow$ $\hat{\mathcal{L}}^\dagger \hat{\mathcal{L}}$

$$\frac{d|\rho\rangle}{dt} = \left[-i(H \otimes I - I \otimes H^T) + \sum_k \gamma_k \left(L_k \otimes L_k^* - \frac{1}{2} I \otimes L_k^T L_k^* - \frac{1}{2} L_k^\dagger L_k \otimes I \right) \right] |\rho\rangle$$

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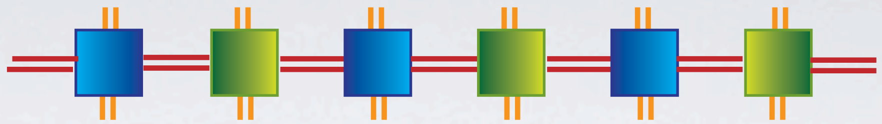

$\hat{\mathcal{L}}$

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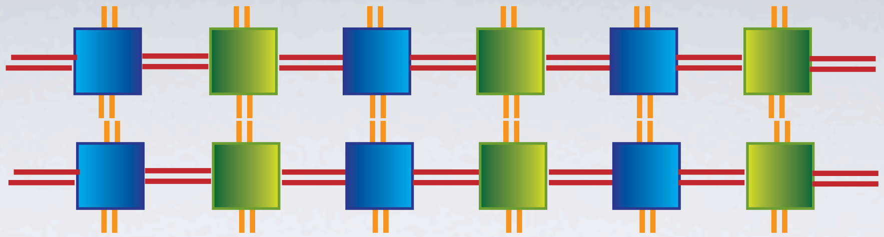

$\hat{\mathcal{L}}^\dagger \hat{\mathcal{L}}$

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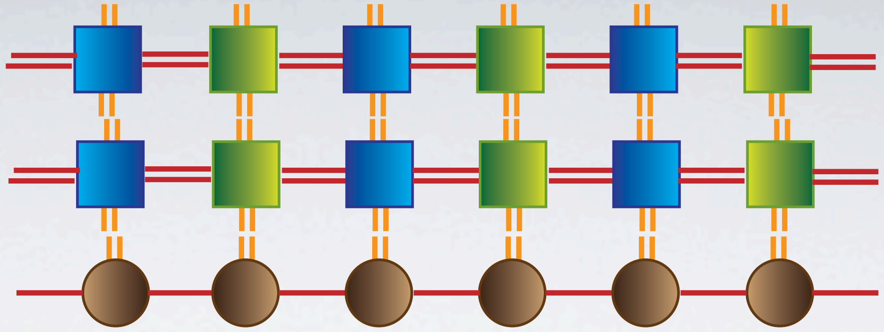

$\hat{\mathcal{L}}^\dagger \hat{\mathcal{L}}$

lowest eigenvalue

BASIC ALGORITHM

Variational minimization of energy

White, PRL 1992

Verstraete, Porras, Cirac, PRL 2004

Schollwöck, RMP 2005, Ann. Phys. 2011

BASIC ALGORITHM

Variational minimization of energy

local
Hamiltonian

$$H = \sum_i$$

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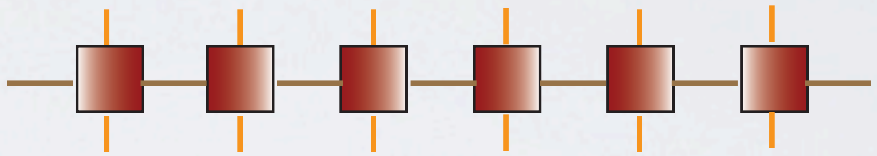
$H =$ 

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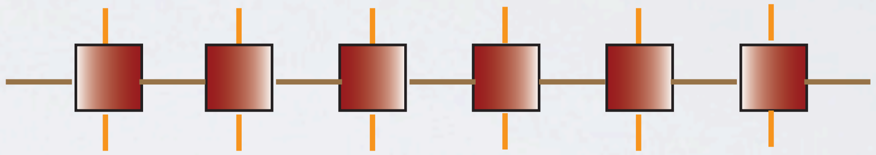
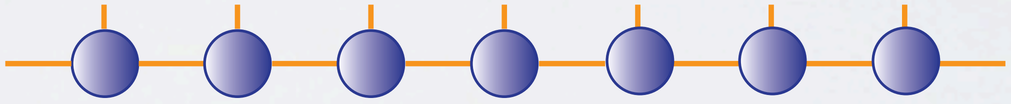
$H =$ 

$|E_0\rangle \simeq$ 

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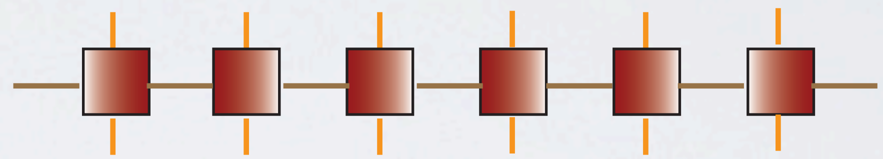
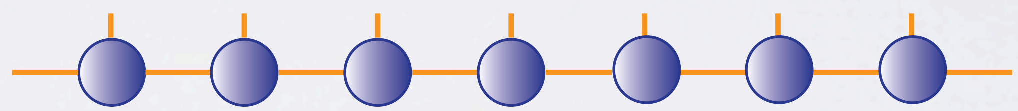
$H =$ 

$|E_0\rangle \simeq$ 

Variational principle

$$\min_{\{A\}} \frac{\langle \Psi | H | \Psi \rangle}{\langle \Psi | \Psi \rangle}$$

White, PRL 1992
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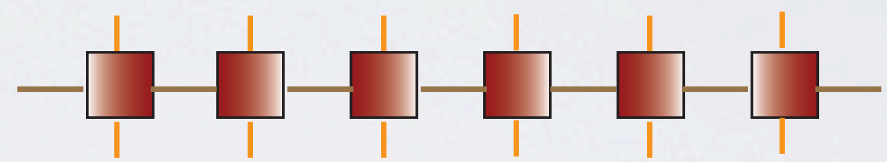
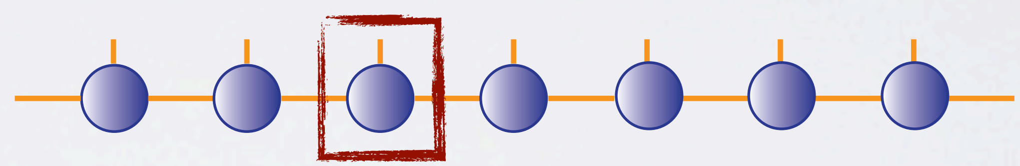
$H =$ 

$|E_0\rangle \simeq$ 

Variational principle

$$\min_{\{A\}} \frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle} \longrightarrow \min_{A} \frac{\bar{A} H_{\text{eff}} A}{\bar{A} N_{\text{eff}} A}$$

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Variational minimization of energy

local
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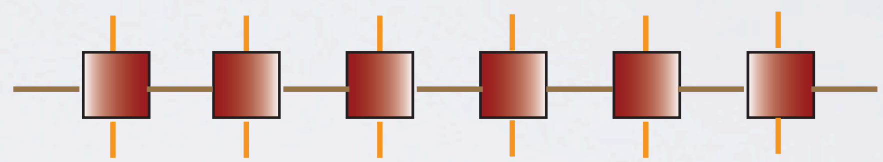
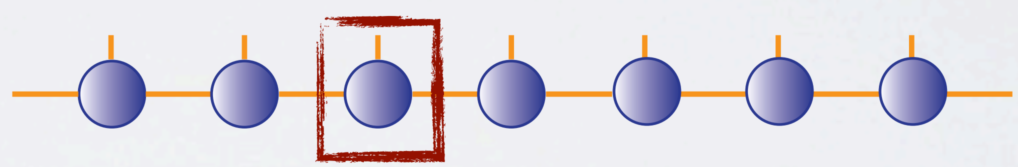
$H =$ 

$|E_0\rangle \simeq$ 

Variational principle

$$\min_{\{A\}} \frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle} \longrightarrow \min_{A} \frac{\bar{A} H_{\text{eff}} A}{\bar{A} N_{\text{eff}} A}$$

sweep back and forth
over tensors

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Schollwöck, RMP 2005, Ann. Phys. 2011

VARIATIONAL STEADY STATES

POTENTIAL ISSUES

de las Cuevas, 2013

VARIATIONAL STEADY STATES

POTENTIAL ISSUES

Positivity

de las Cuevas, 2013

VARIATIONAL STEADY STATES

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fixed point of the evolution

no need to use purification

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Accuracy of MPO approximation

Prosen, Znidaric, 2009
Kastoryano, Eisert, 2013
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maybe smaller gaps? ⇒ metastable states?

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fixed point of the evolution

no need to use purification

de las Cuevas, 2013

Accuracy of MPO approximation

Degeneracies

maybe smaller gaps? ⇒ metastable states?

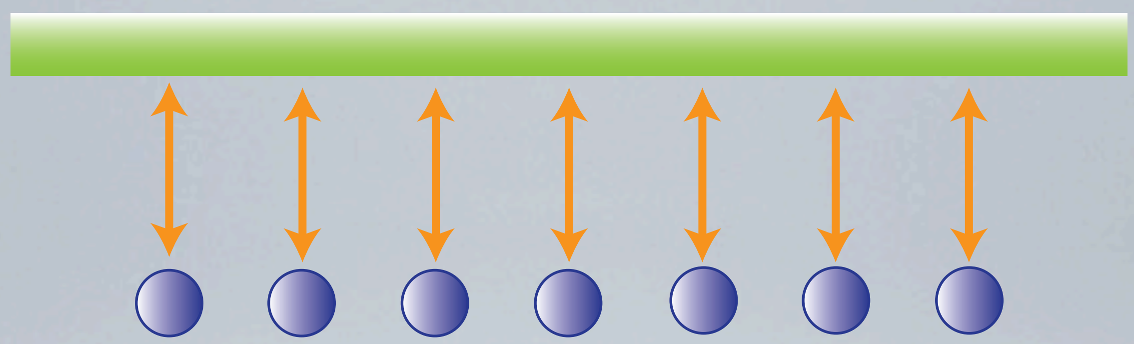
local effective Lindblad operator does not preserve any property ⇒ symmetries?

Prosen, Znidaric, 2009
Kastoryano, Eisert, 2013
Bonnes et al, PRA 2014

Some examples...

DICKE MODEL

N 2-level atoms coupled to same EM mode



Dicke, 1954
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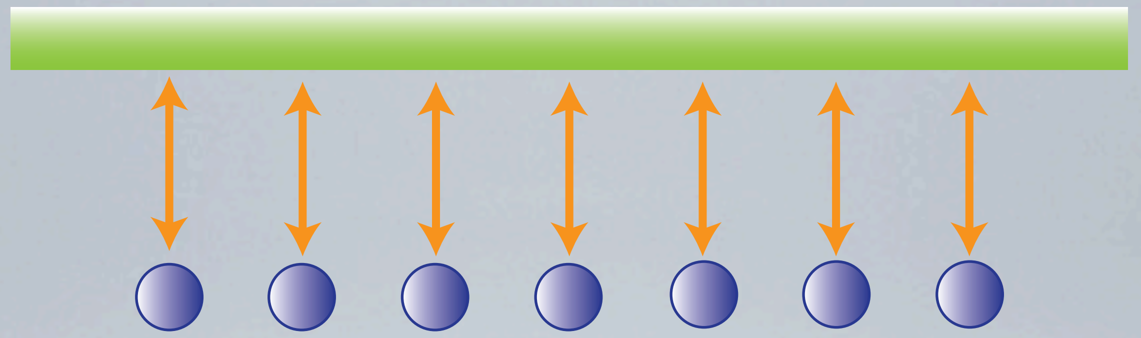

collective coupling

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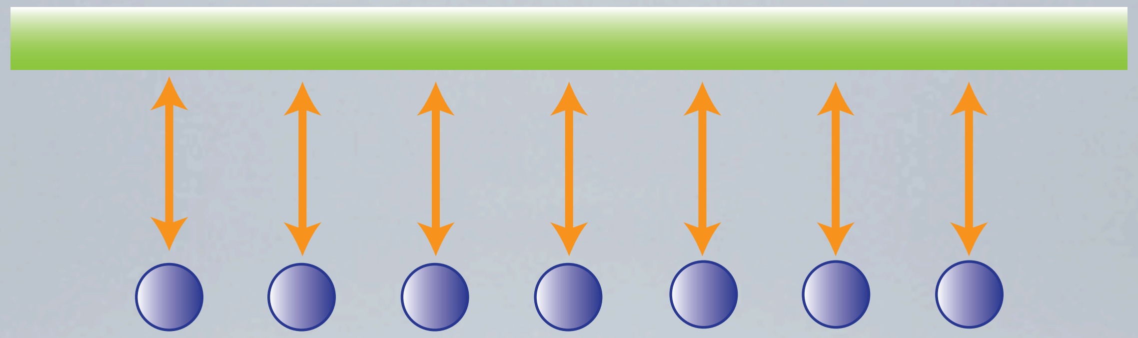

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collective coupling

phase transition to superradiant phase

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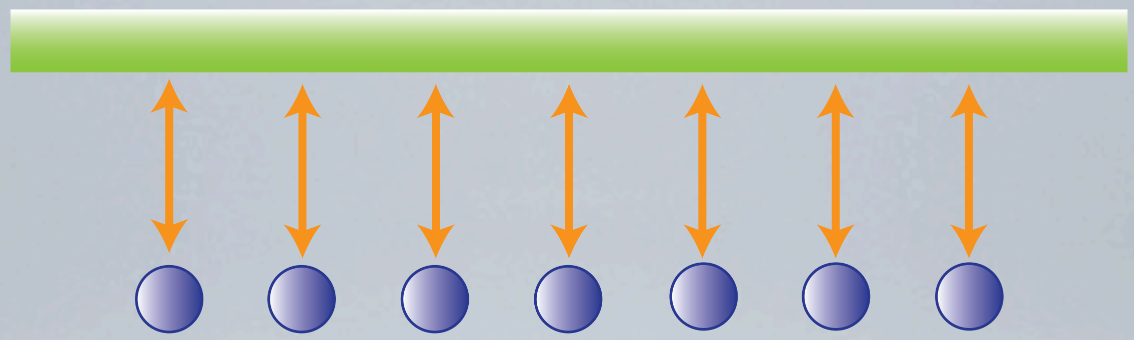

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phase transition to superradiant phase

analytic solution   conserved total spin

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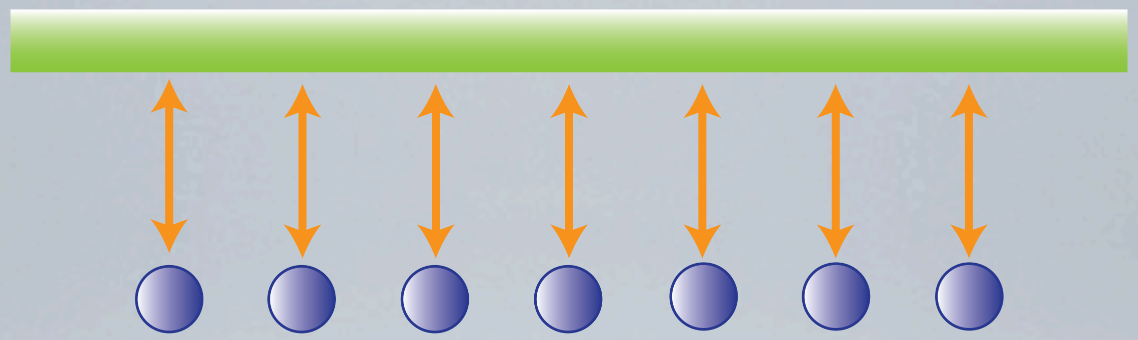


collective coupling

phase transition to superradiant phase

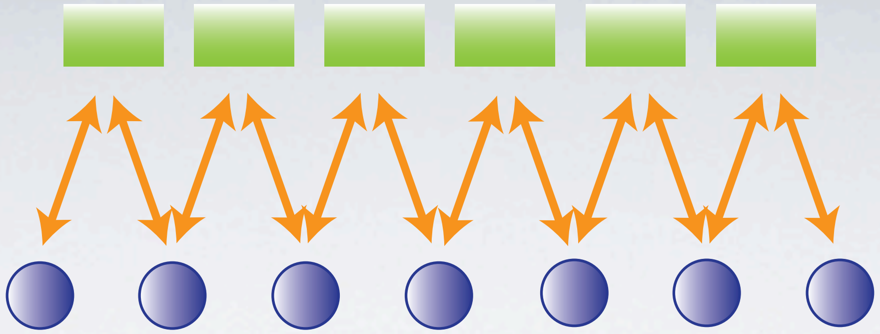
analytic solution conserved total spin

experimentally difficult

Baumann et al., 2010
Hamner et al., 2014
Baden et al., 2014

Do *simpler* models show similar phenomena?

more local



A SIMPLER MODEL

A SIMPLER MODEL

Lower dimensional version of Dicke model

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N 2-level systems with dissipation coupling NN

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Lower dimensional version of Dicke model

N 2-level systems with dissipation coupling NN

$$\frac{d\rho}{dt} = -i\Omega[S_x,\rho] + \Gamma \sum_n \left(S^-_{n\ n+1}\rho S^+_{n\ n+1} - \frac{1}{2}\rho S^+_{n\ n+1} S^-_{n\ n+1} - \frac{1}{2} S^+_{n\ n+1} S^-_{n\ n+1}\rho \right)$$

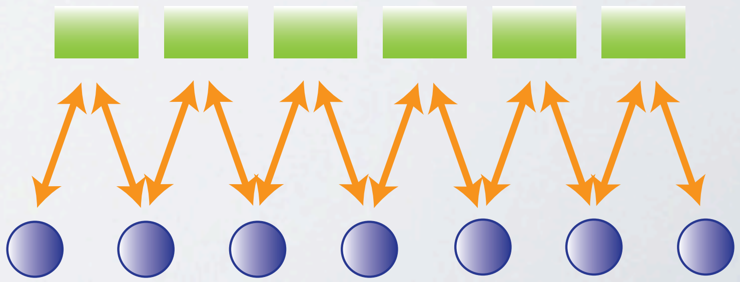
A SIMPLER MODEL

Lower dimensional version of Dicke model

N 2-level systems with dissipation coupling NN

$$\frac{d\rho}{dt} = -i\Omega[S_x, \rho] + \Gamma \sum_n \left(S^-_{n\ n+1} \rho S^+_{n\ n+1} - \frac{1}{2} \rho S^+_{n\ n+1} S^-_{n\ n+1} - \frac{1}{2} S^+_{n\ n+1} S^-_{n\ n+1} \rho \right)$$

$$S^+_{n\ n+1} = \sigma^+_{n+1} \otimes I + I \otimes \sigma^+_{n+1}$$



LOW DIM DICKE MODEL

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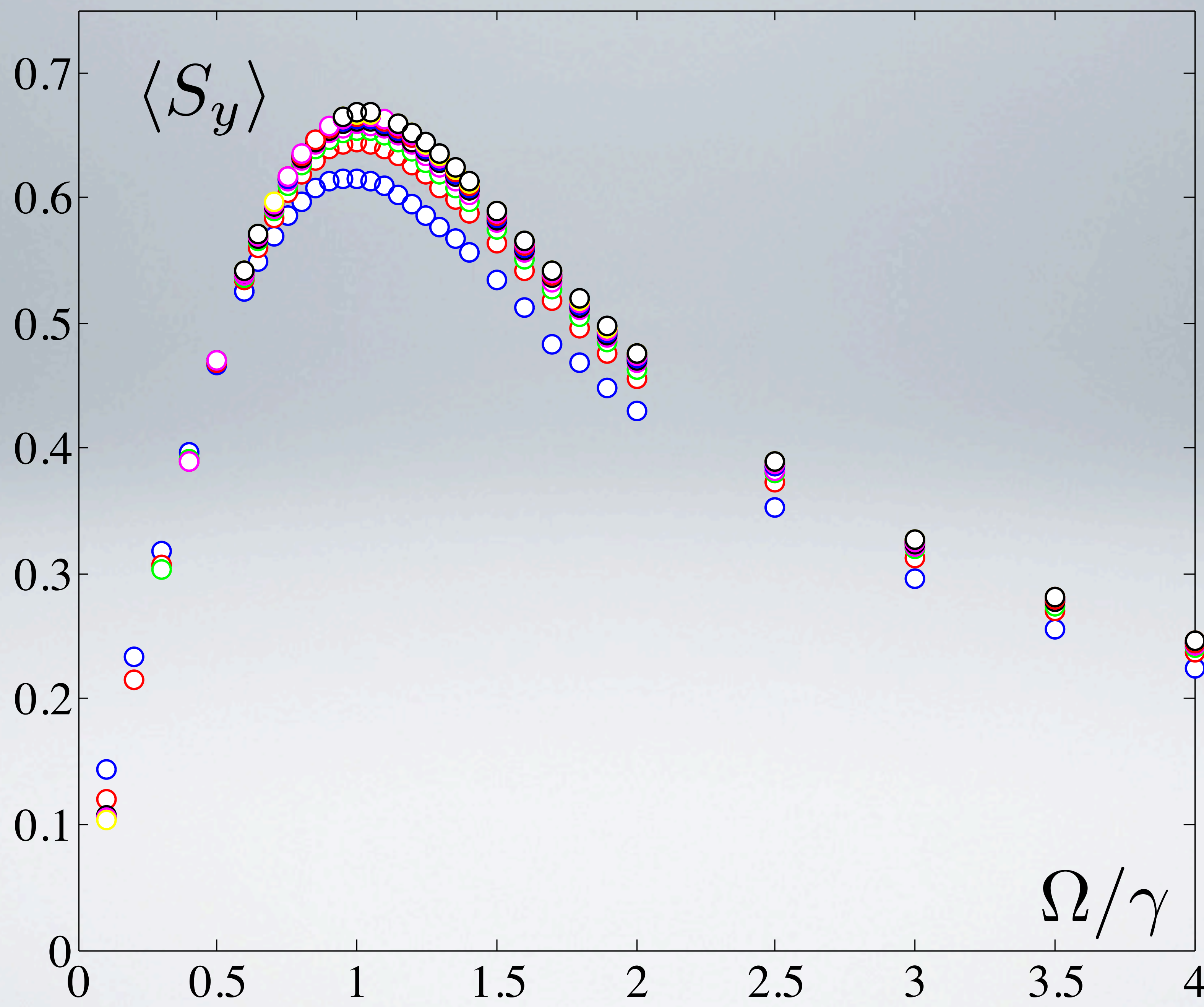

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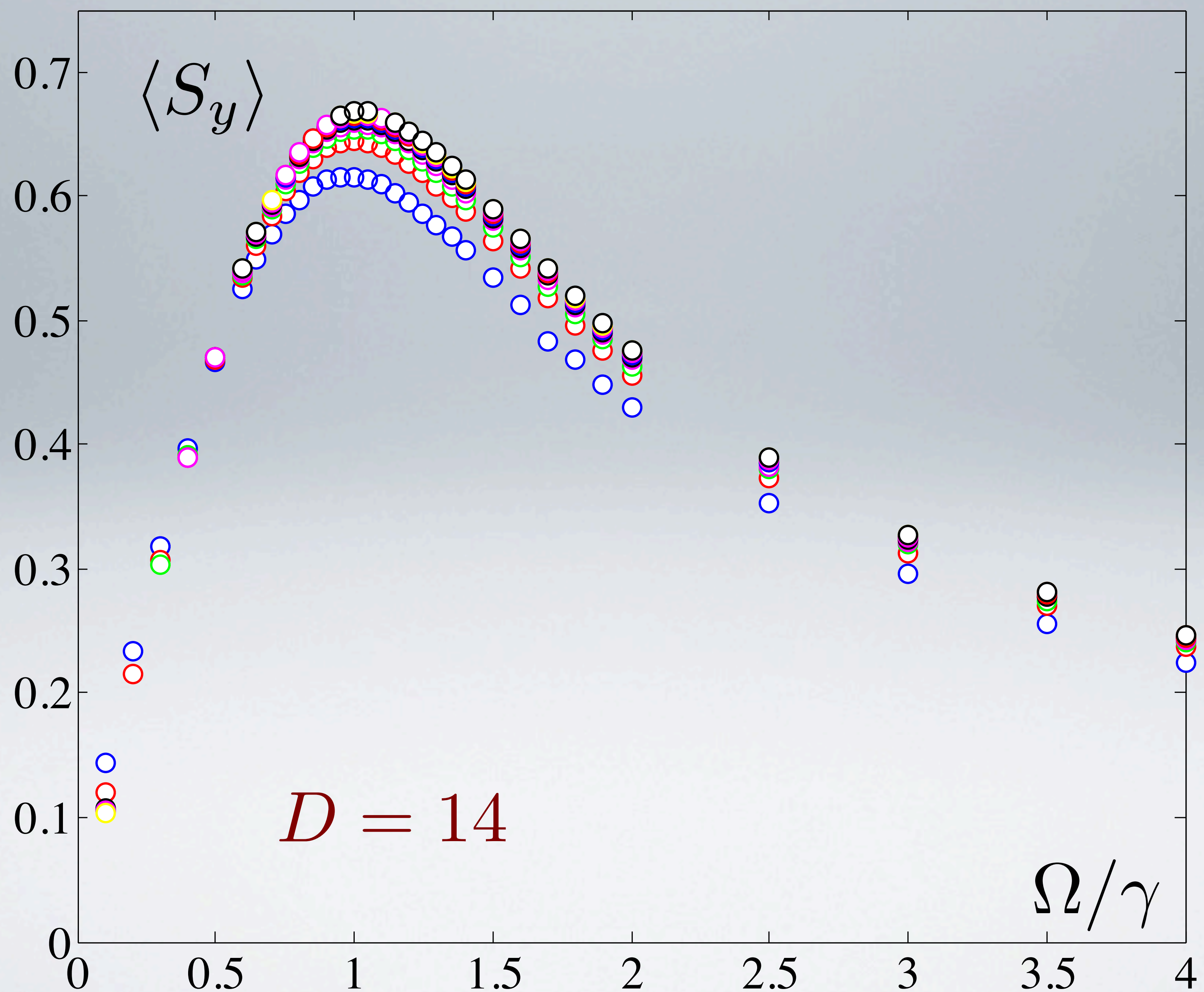
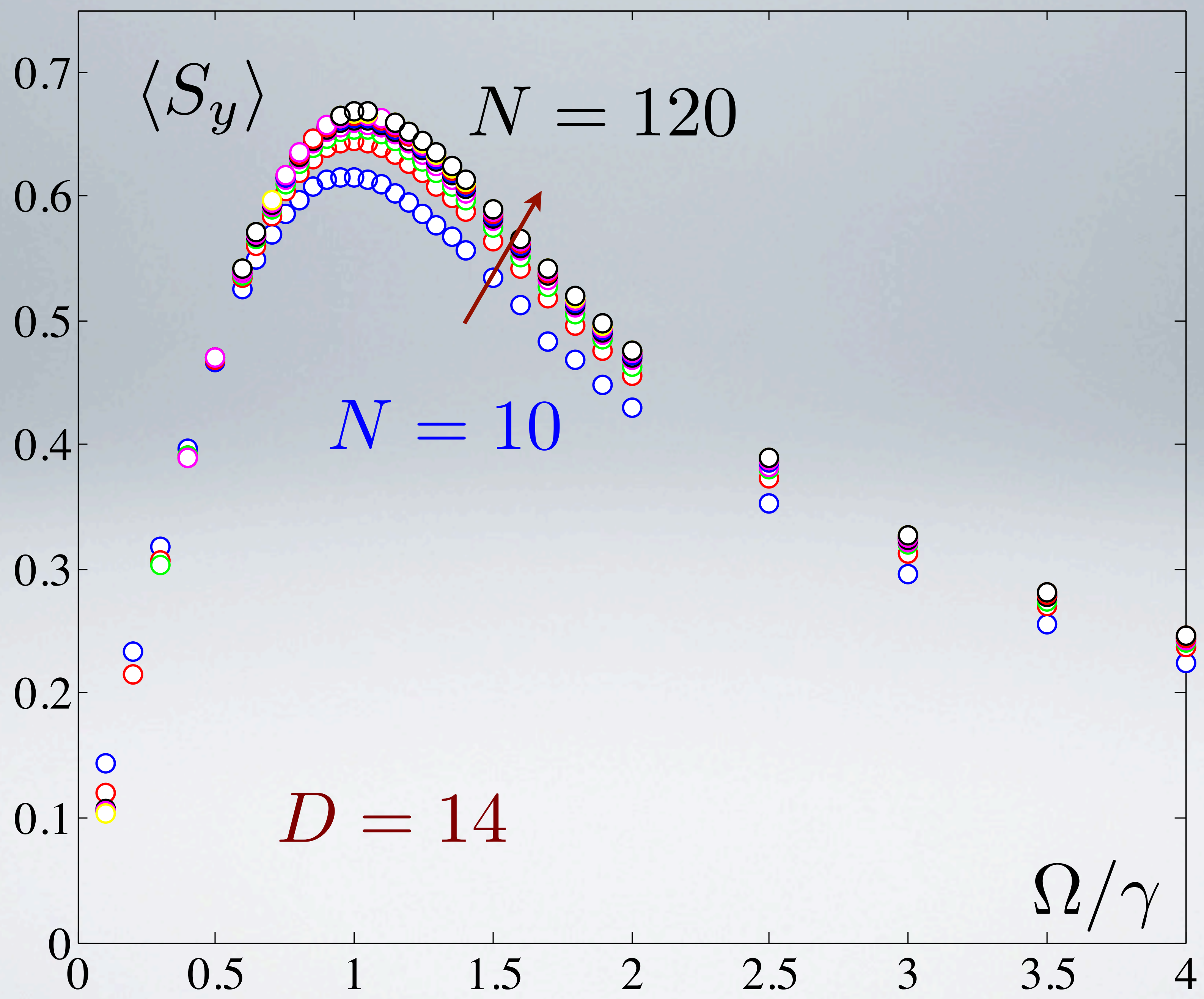


LOW DIM DICKE MODEL



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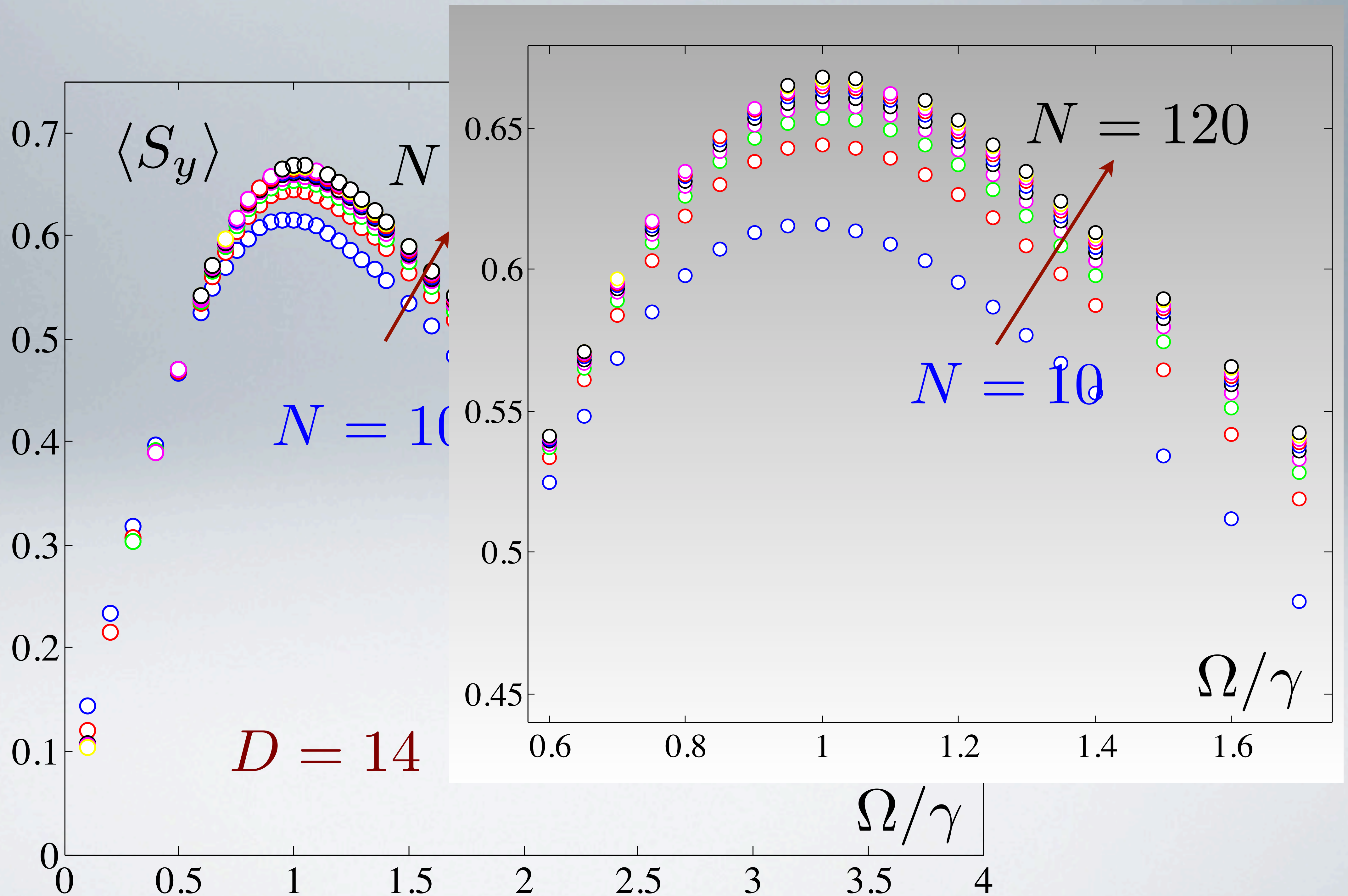

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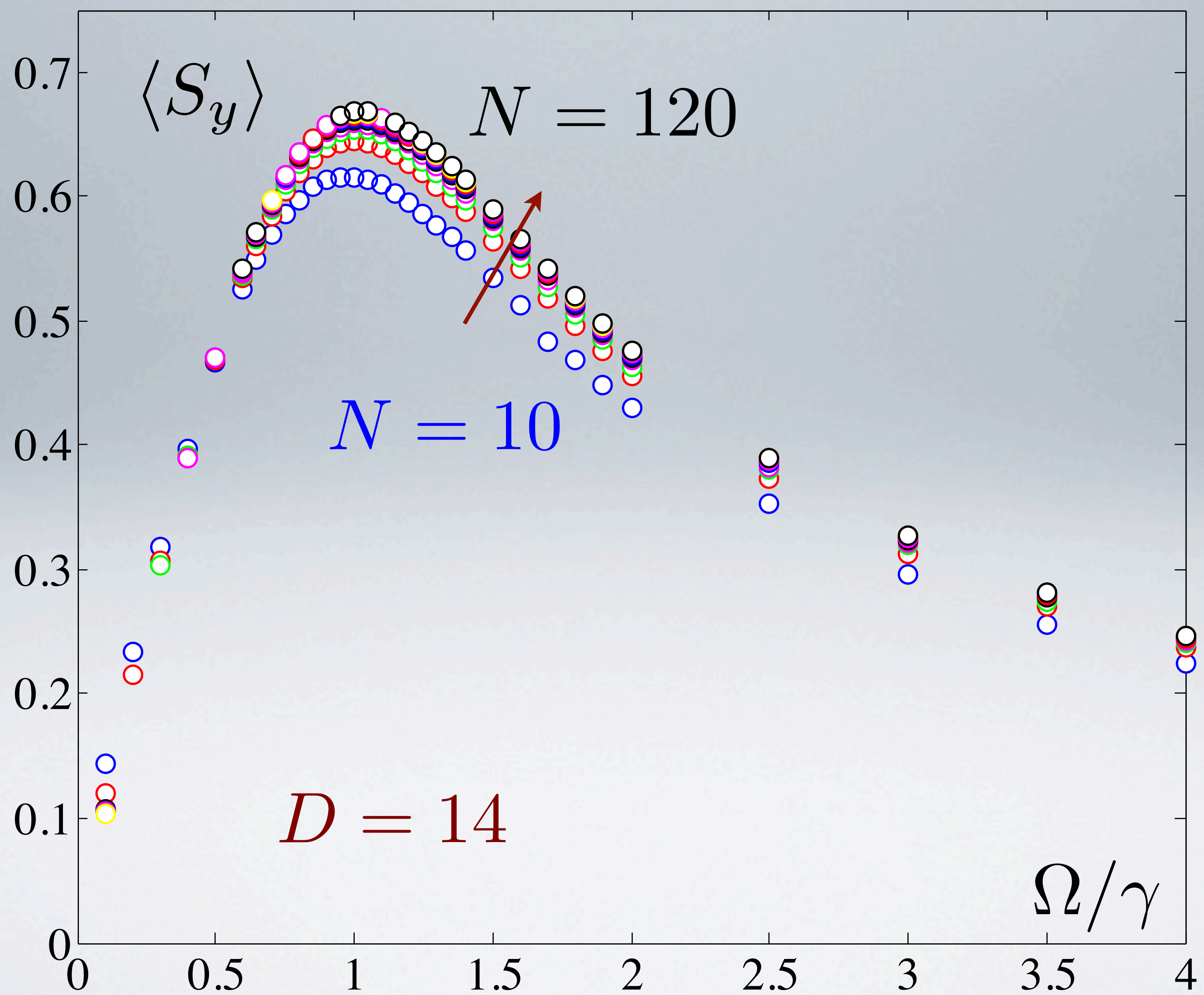

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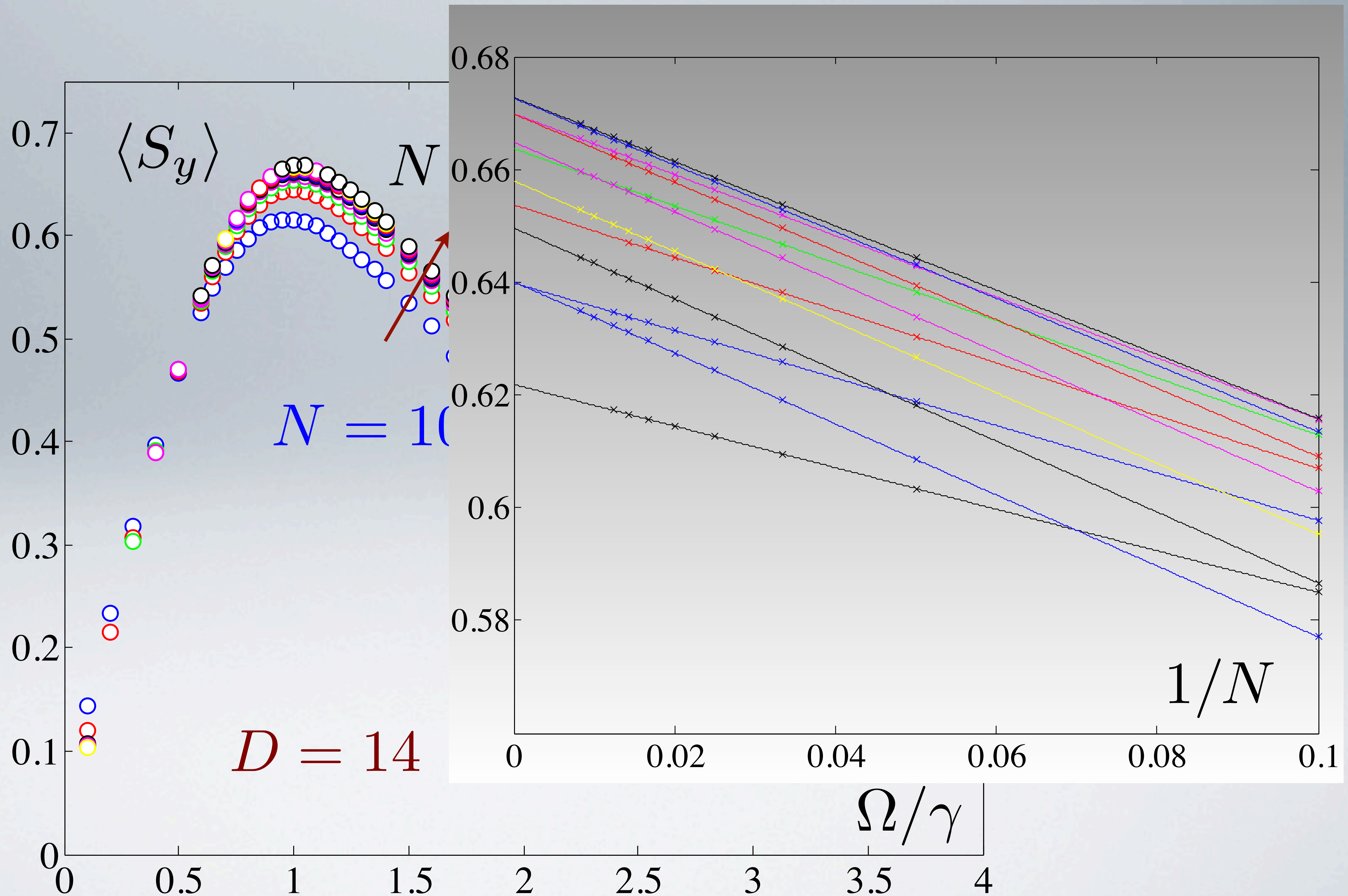

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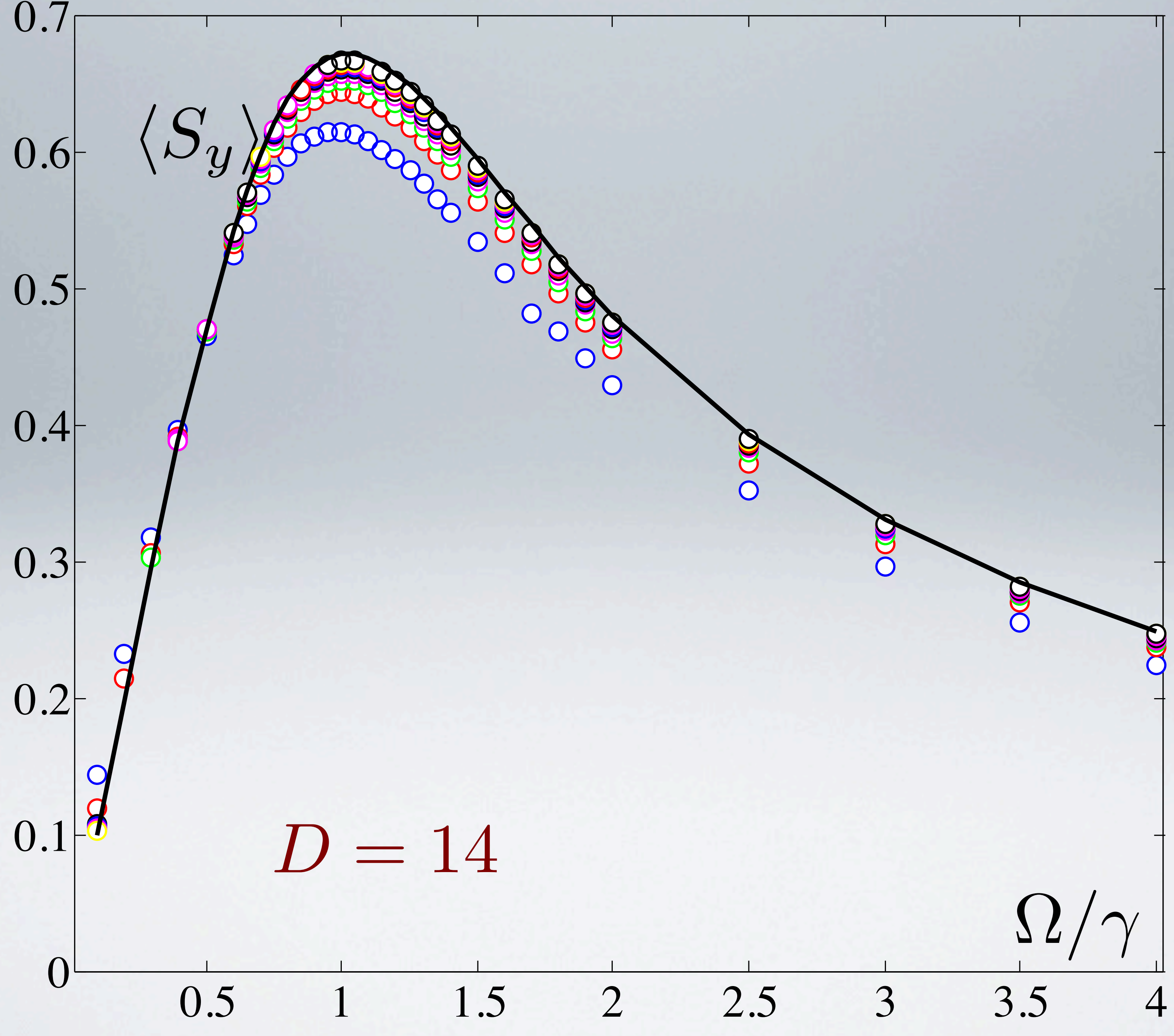

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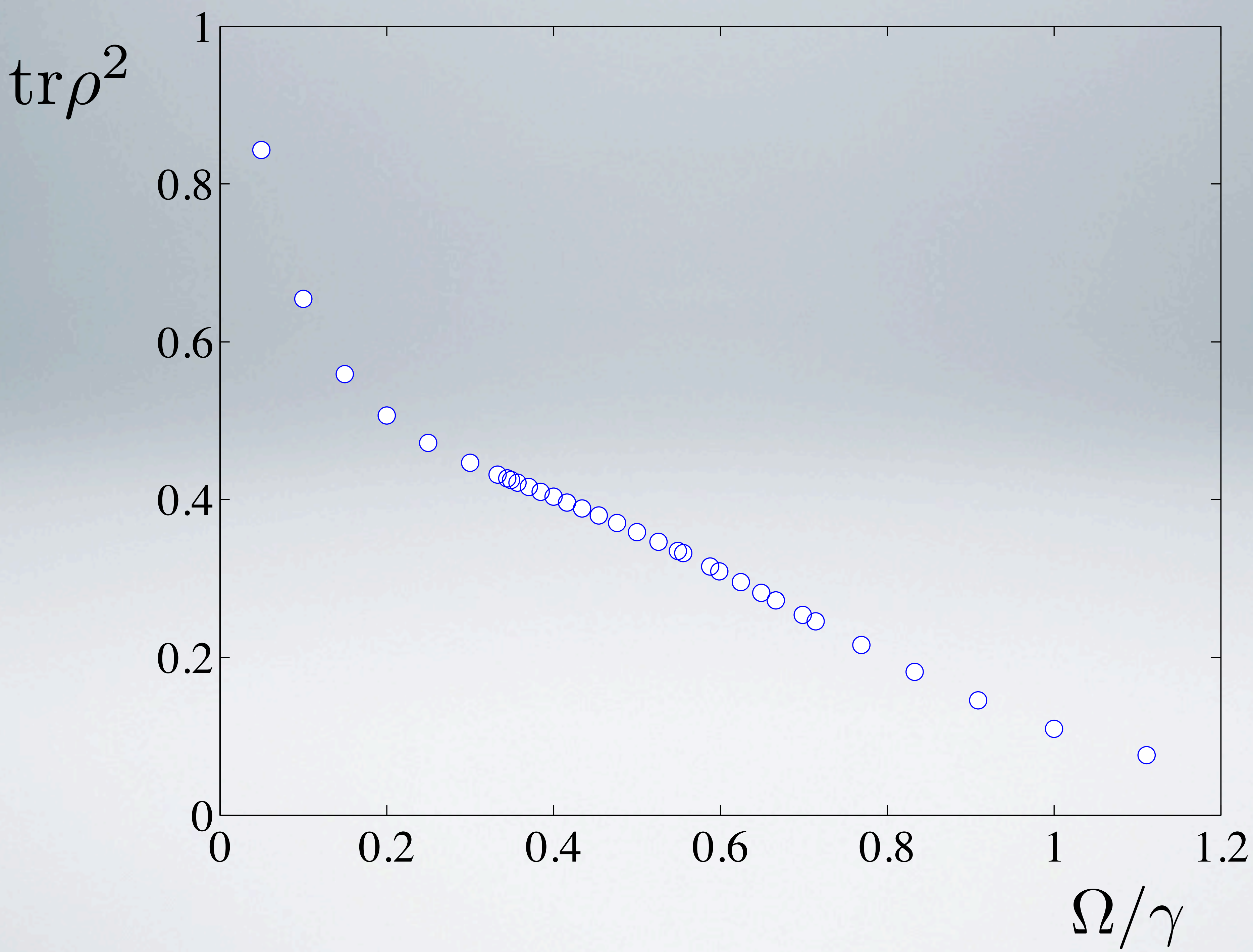

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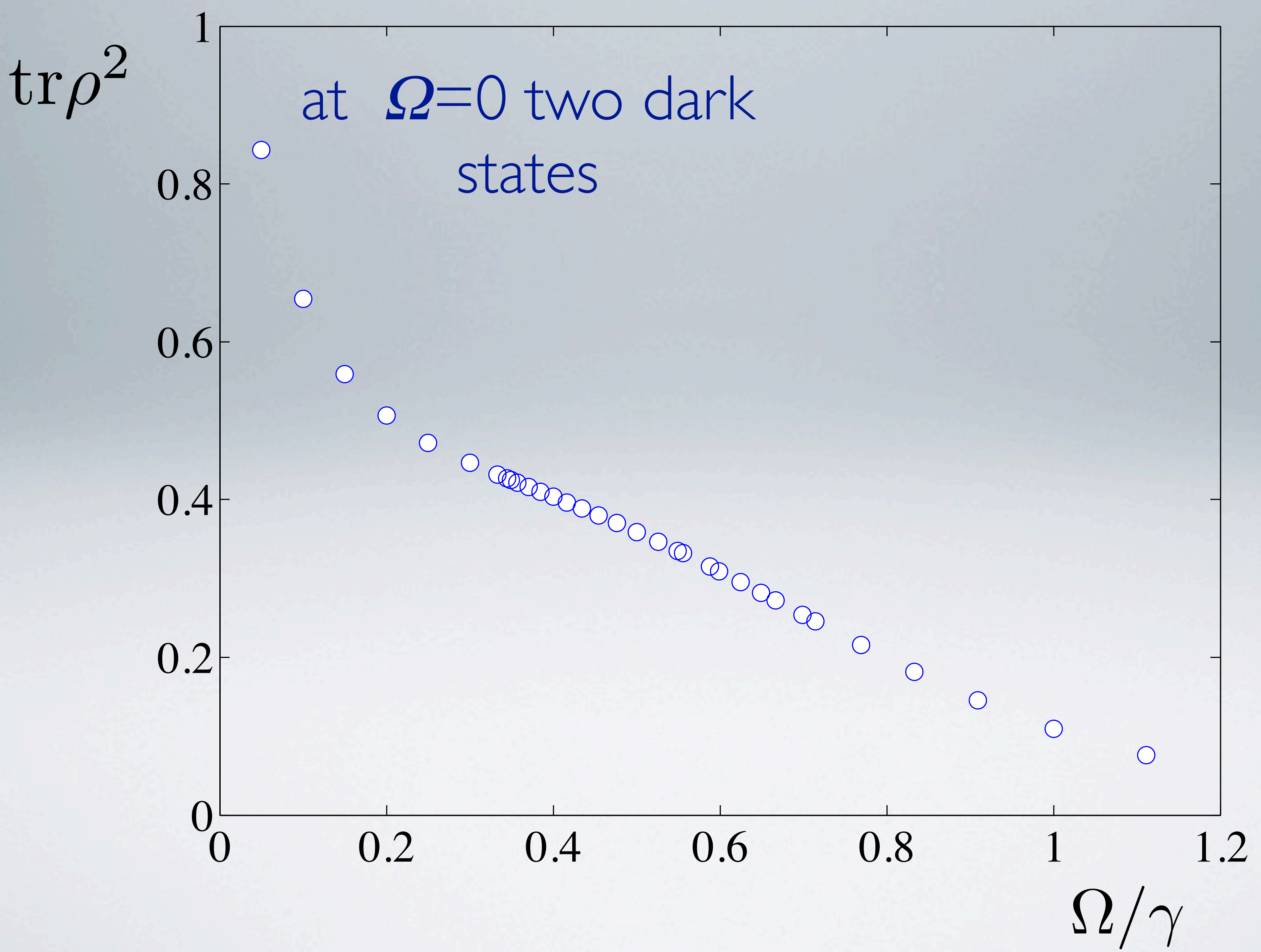

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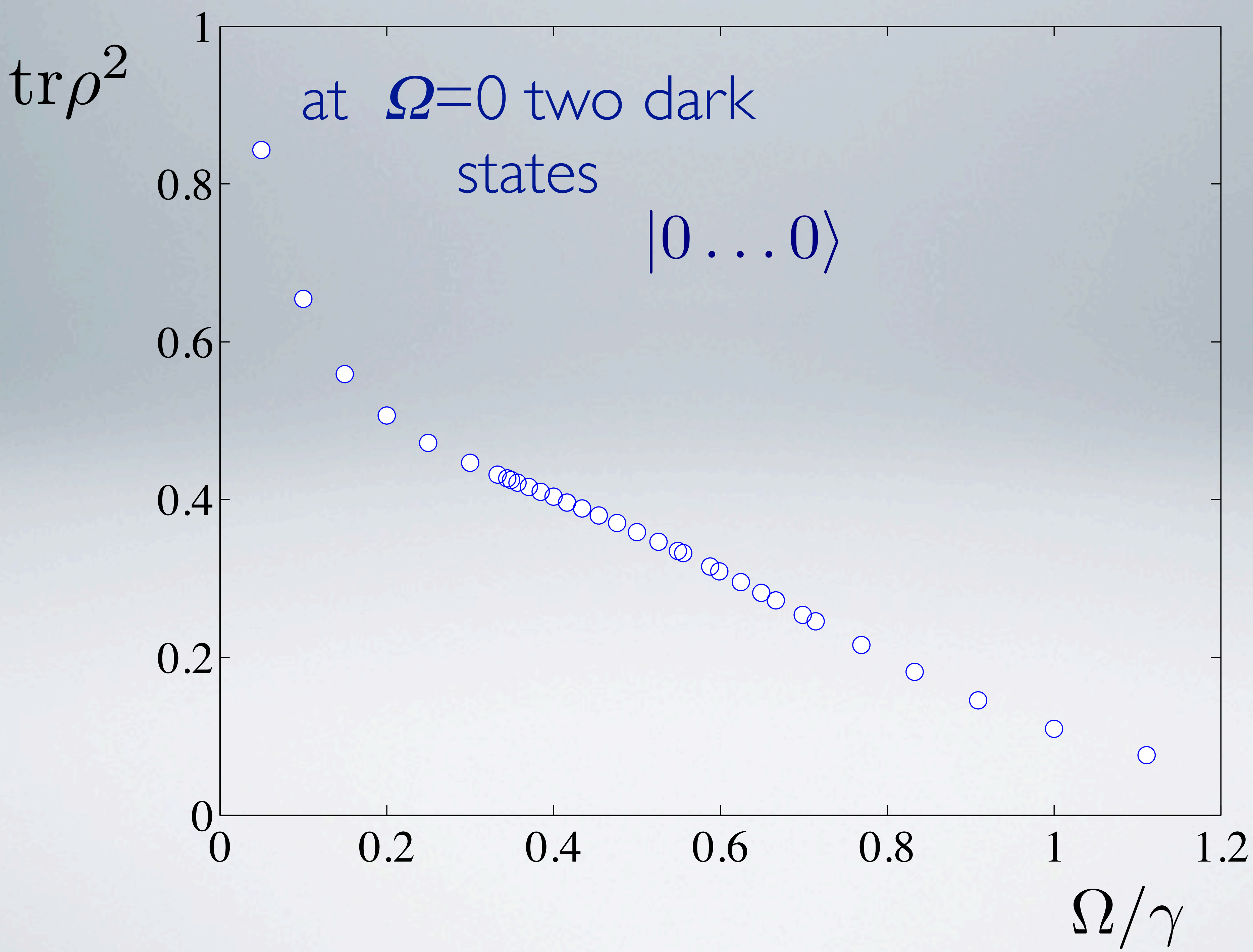

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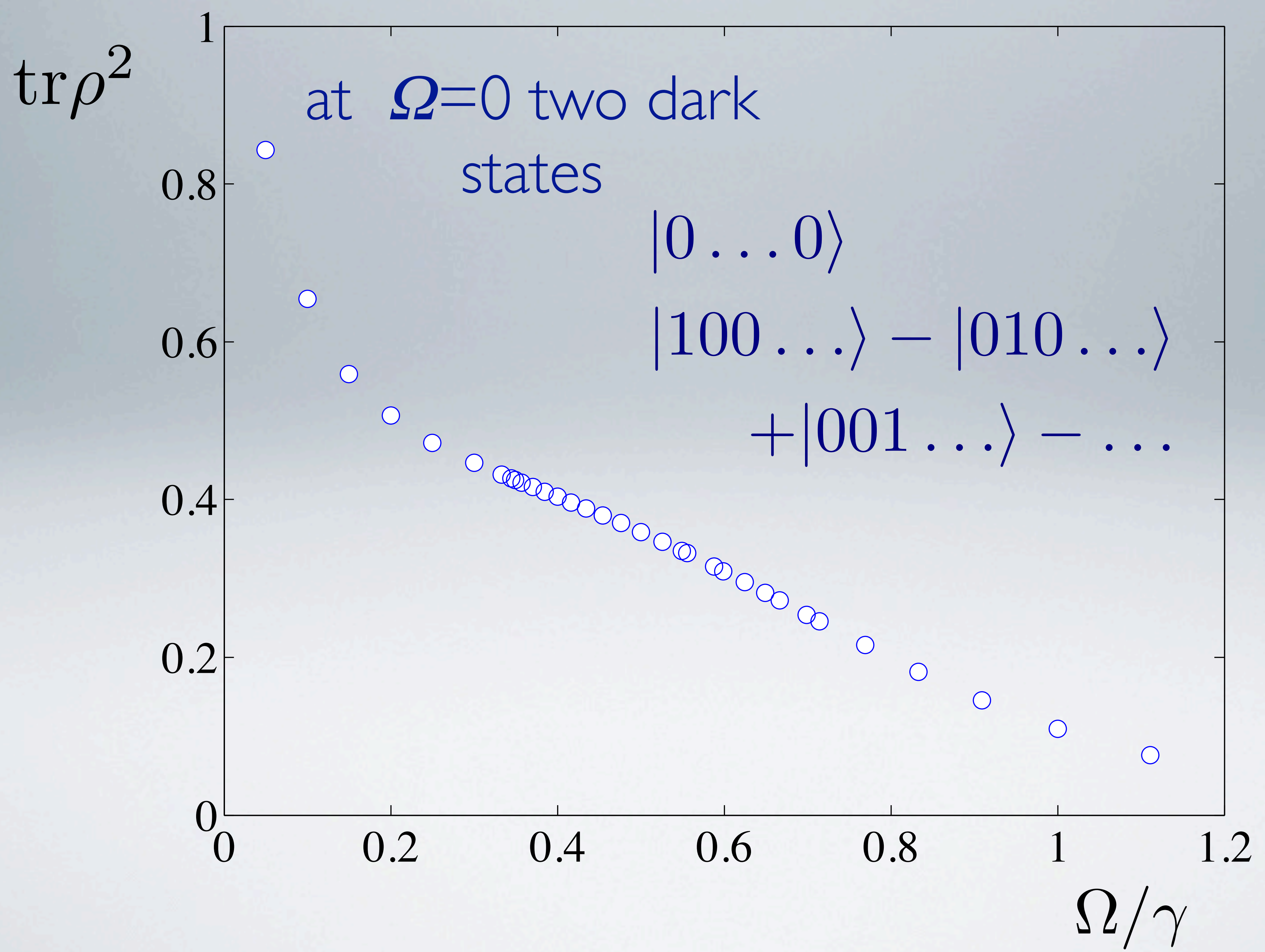

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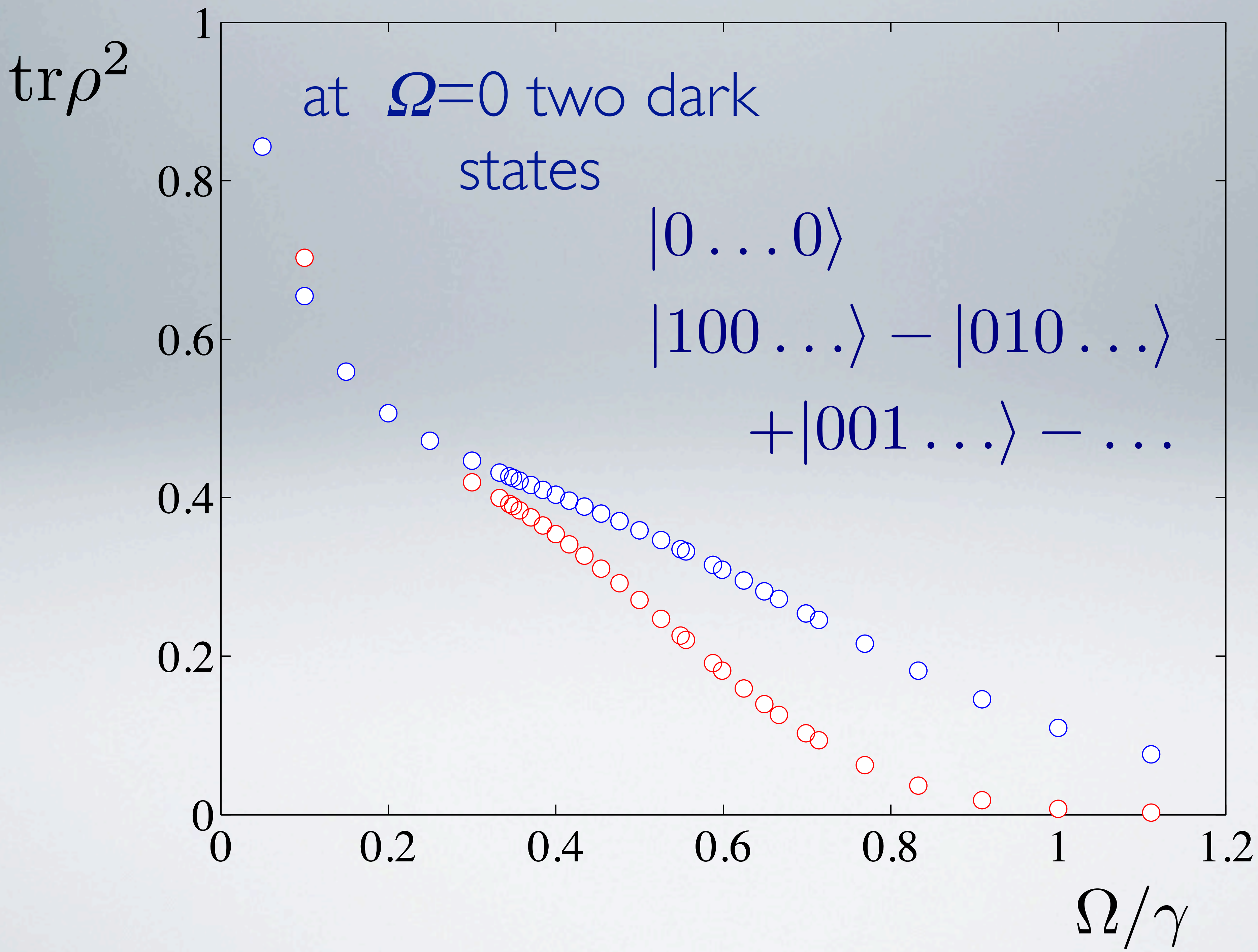

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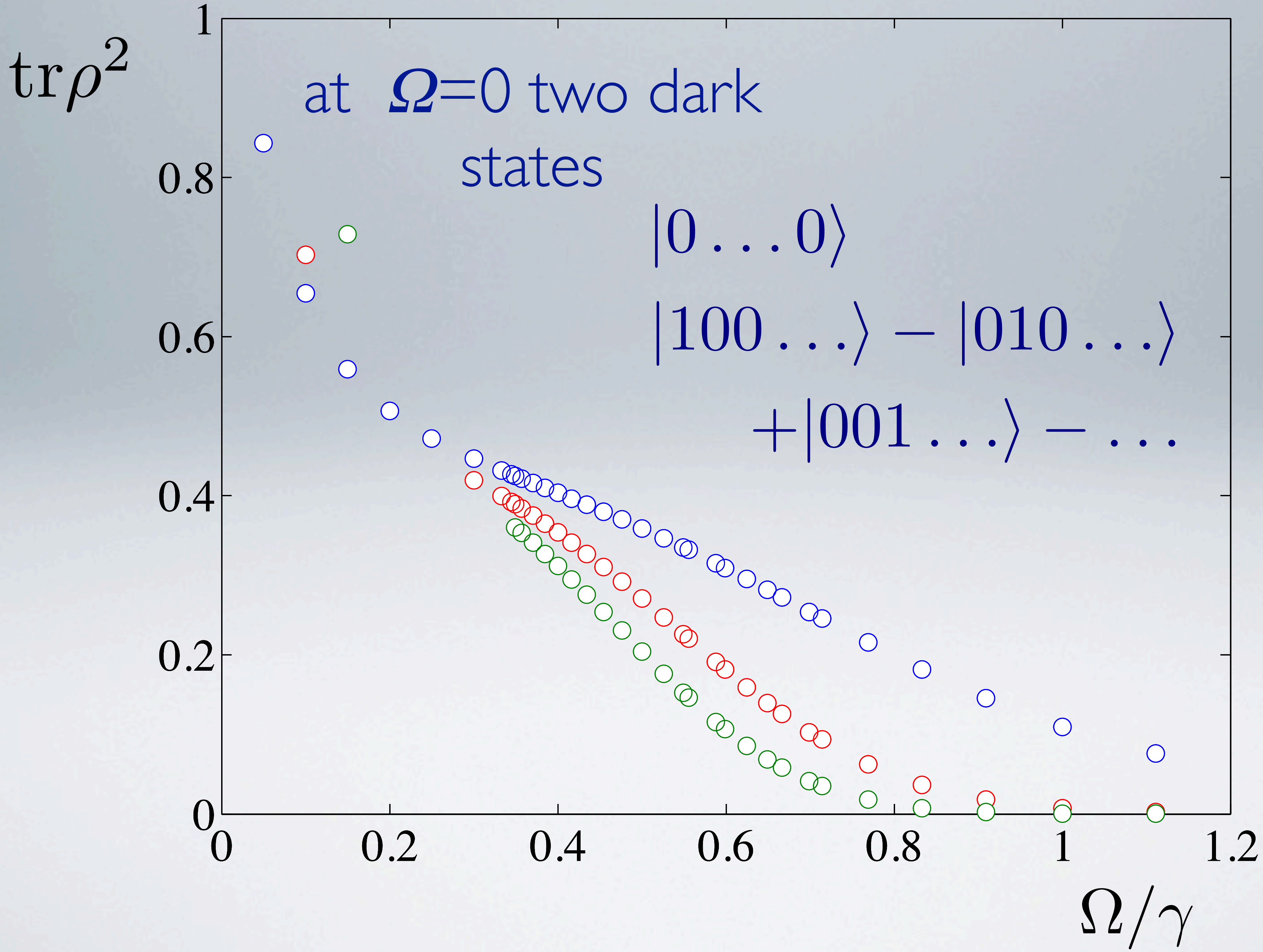

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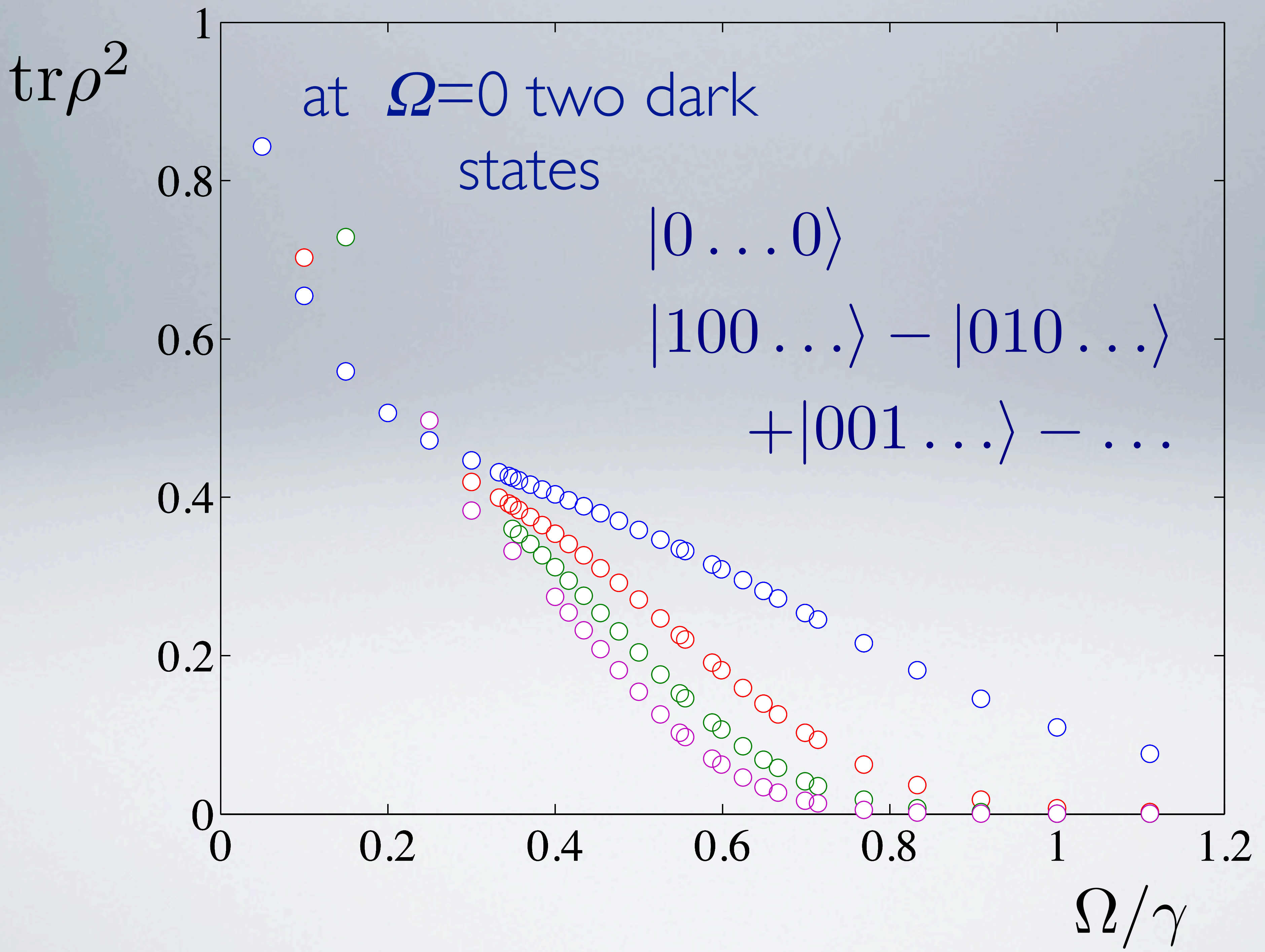

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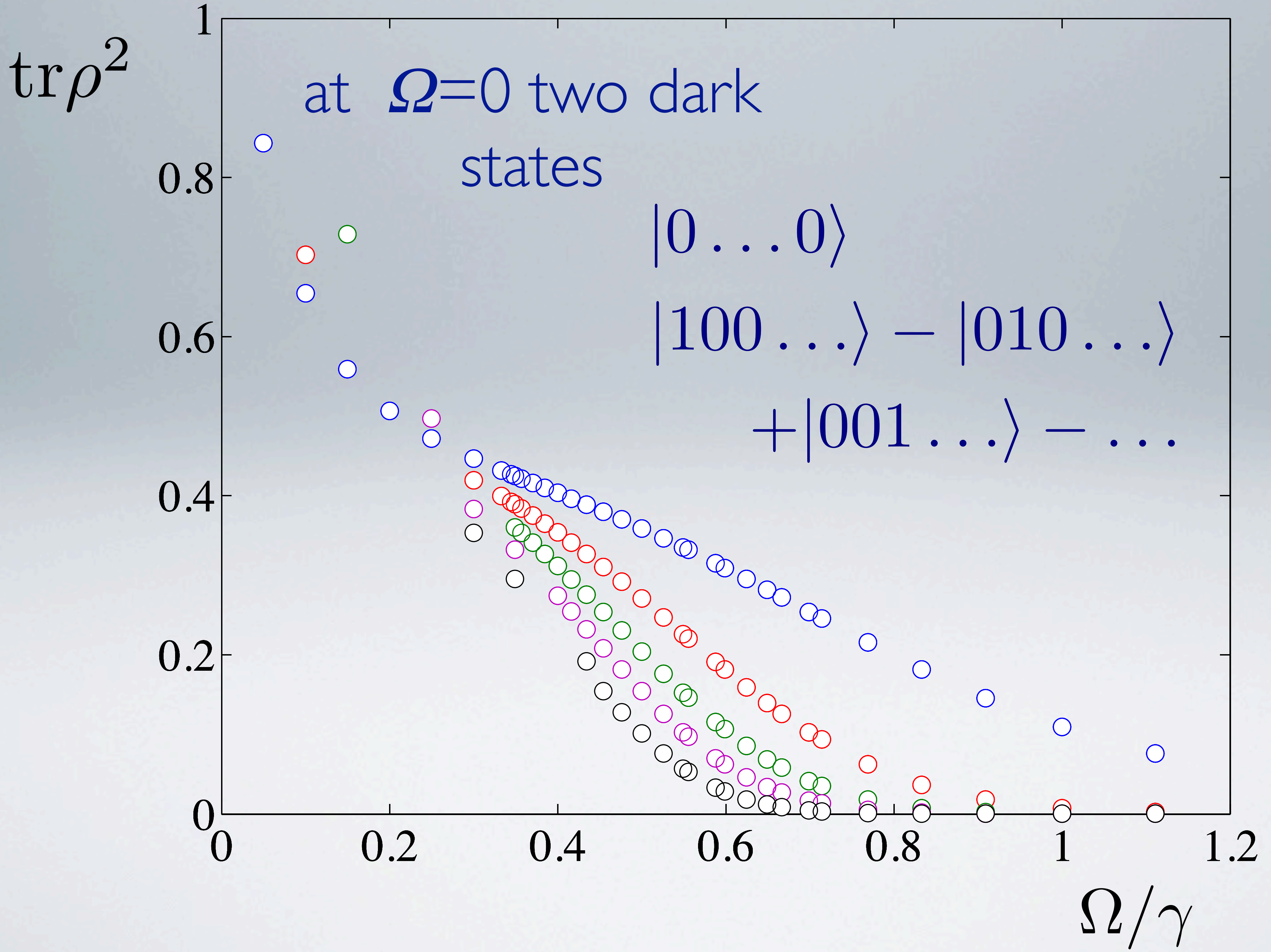

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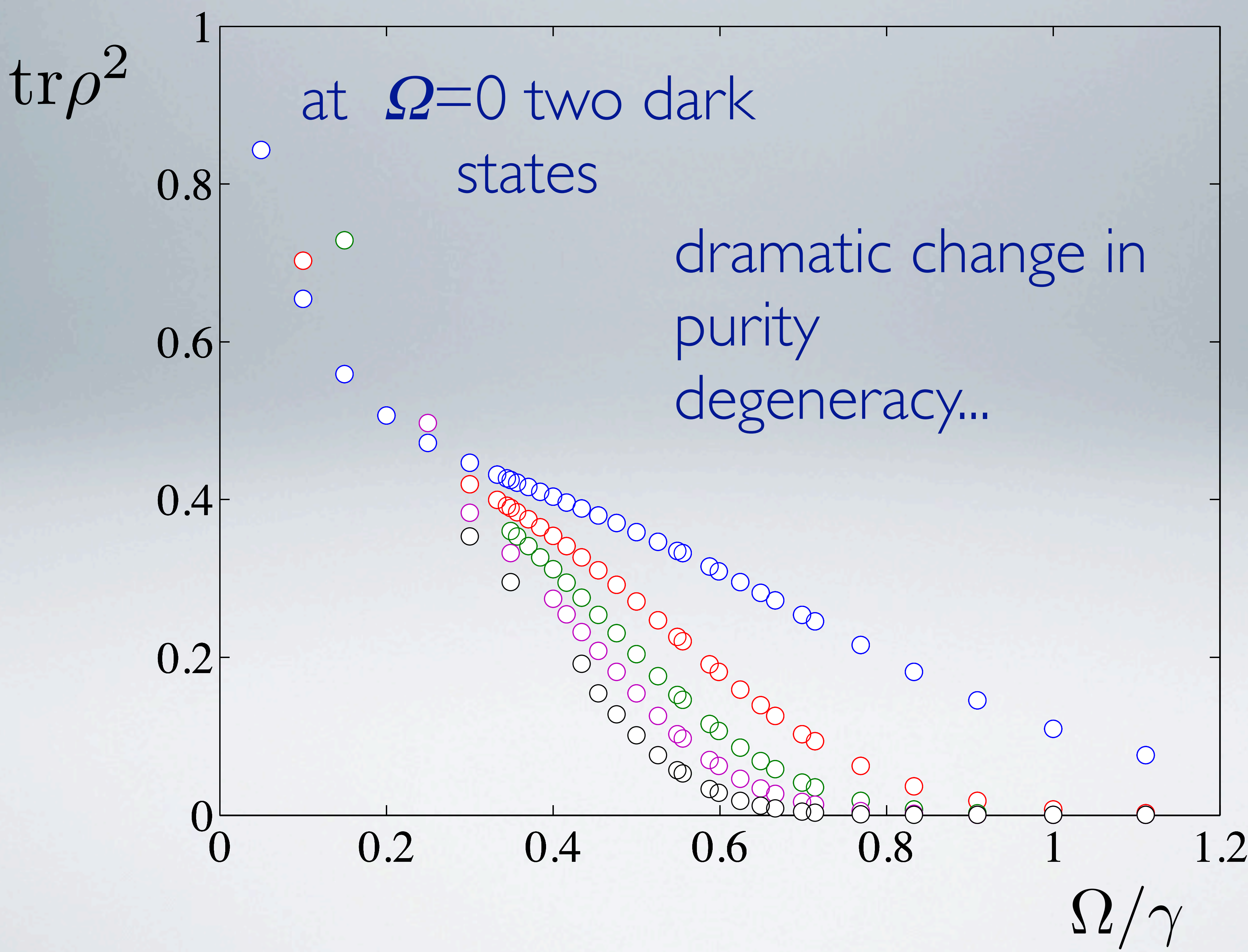

EXAMPLE MODEL: DISSIPATIVE ISING CHAIN

$$H = \sum_n \sigma_z^{[n]} \sigma_z^{[n+1]} + g\sigma_x^{[n]}$$

PRL 114, 220601 (2015)

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steady state can show AFM ordering

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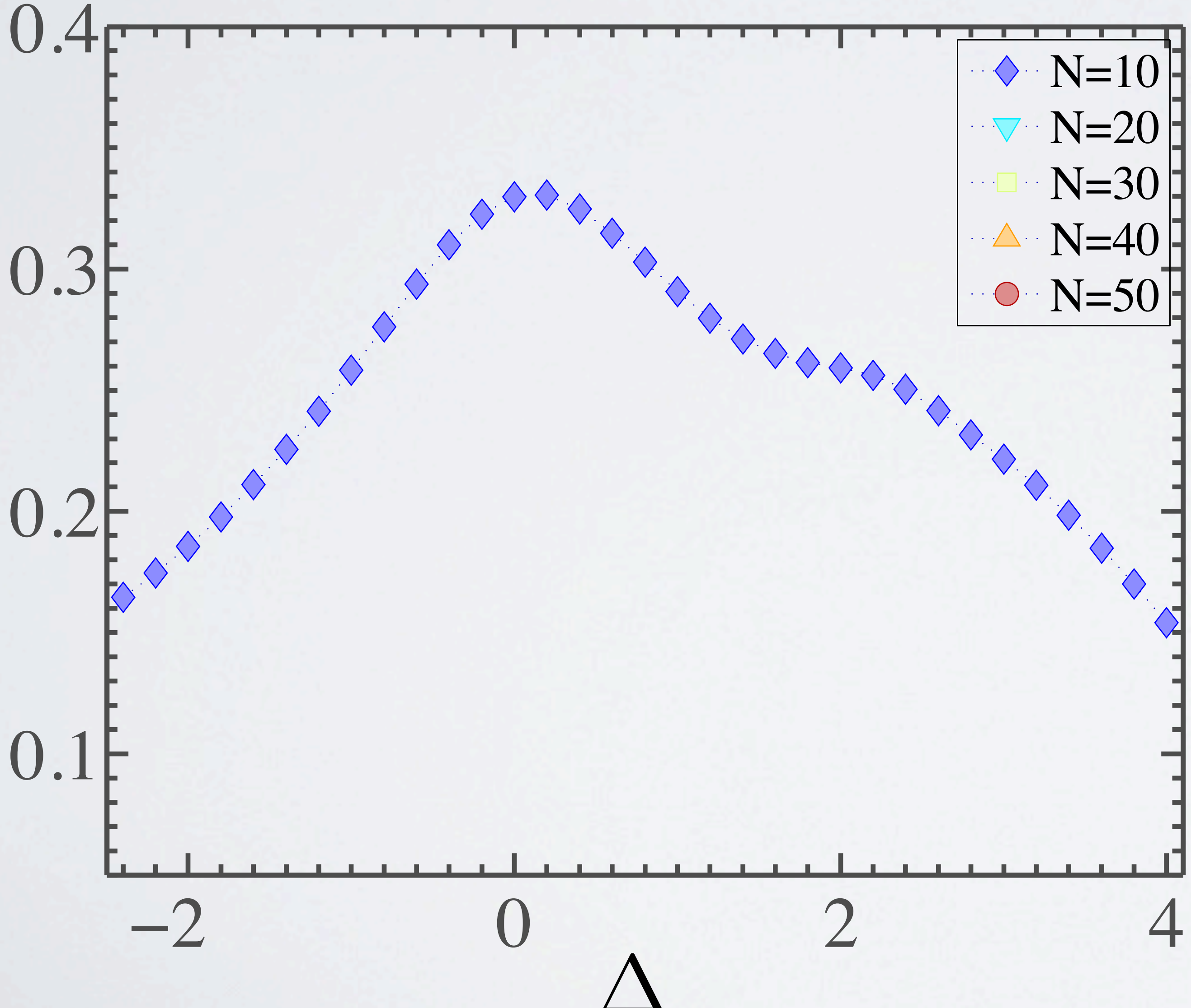
DISSIPATIVE ISING CHAIN

AF order
(staggered magnetization)



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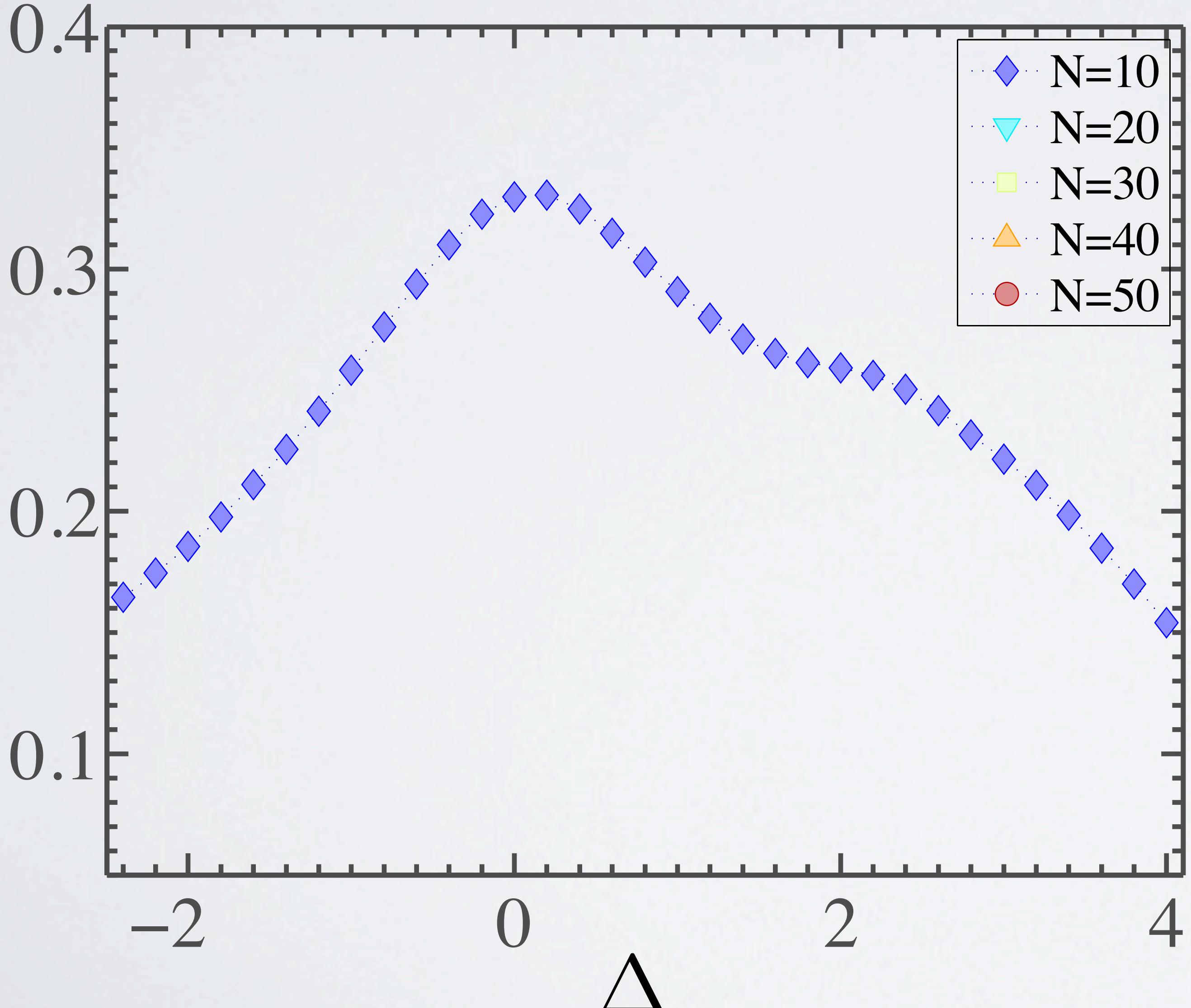

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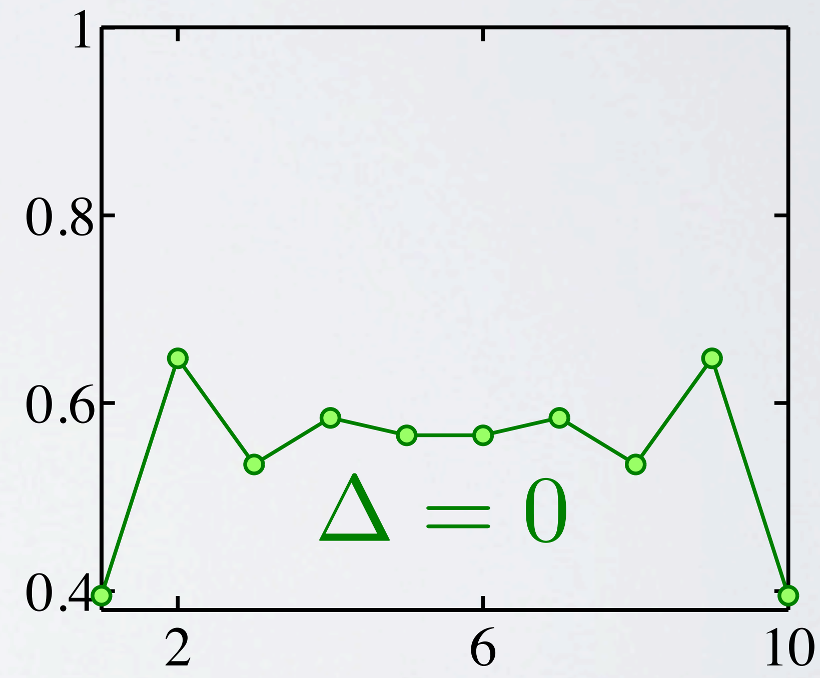

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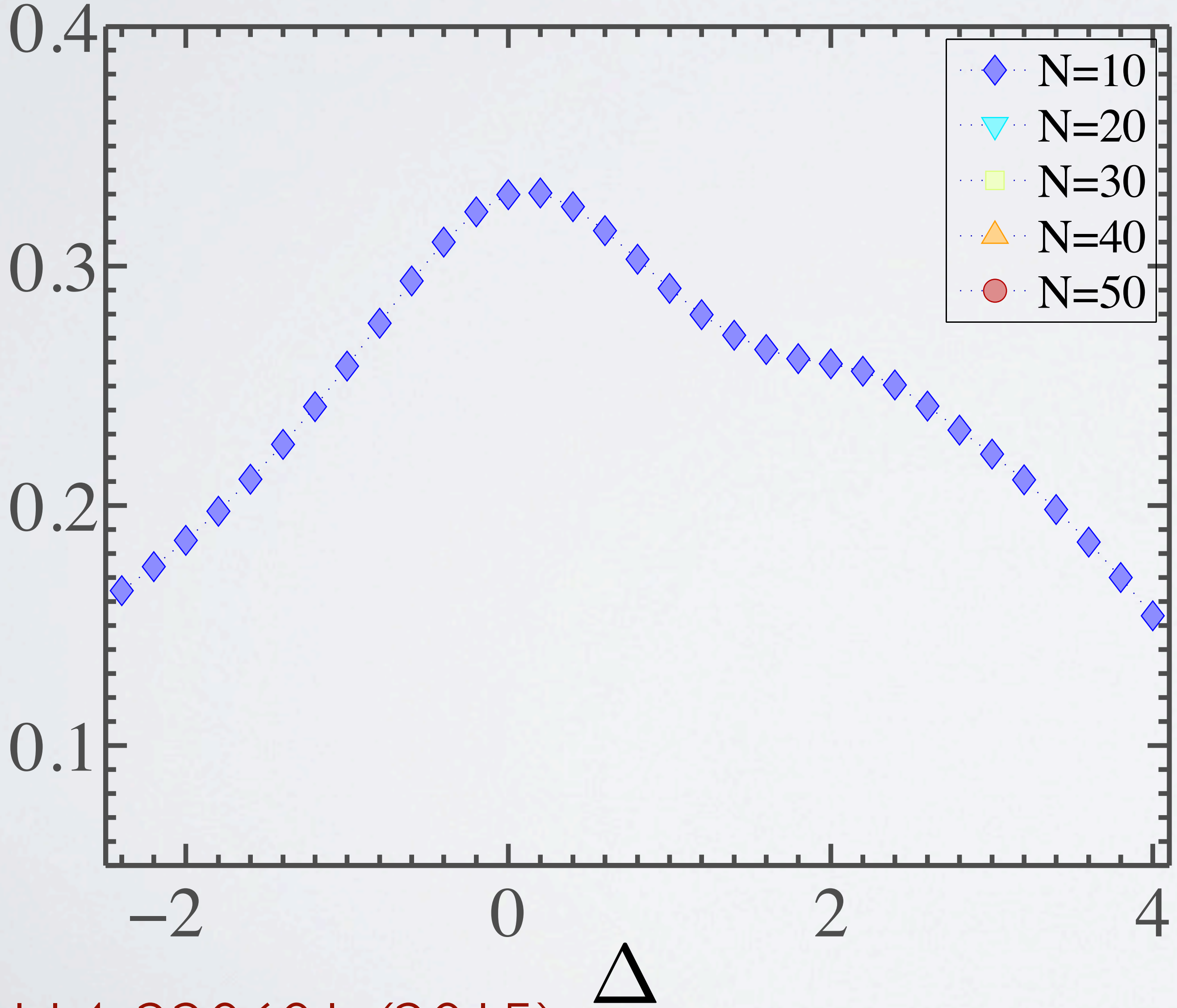

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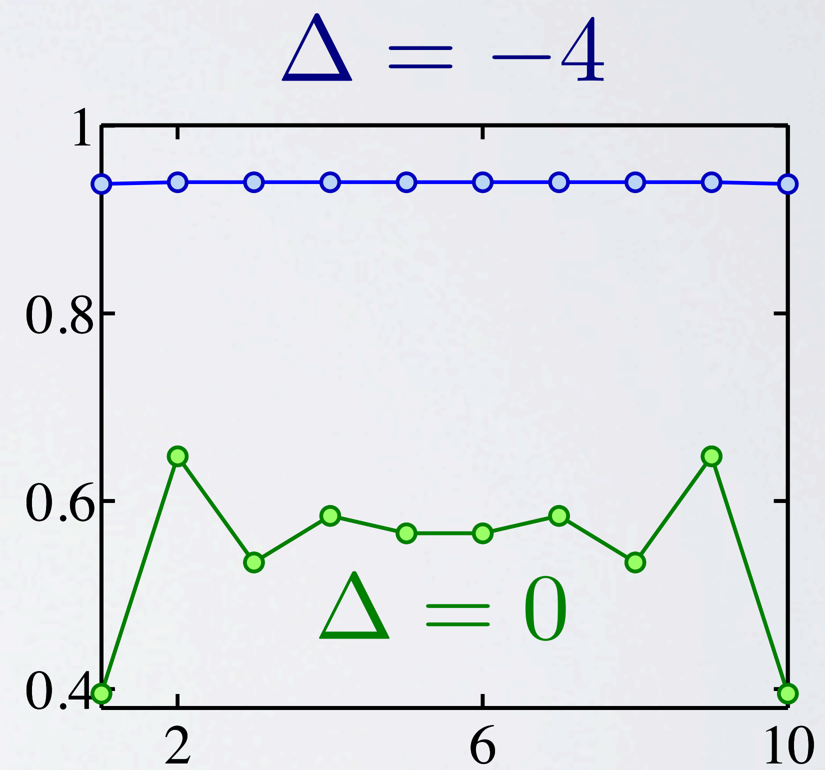

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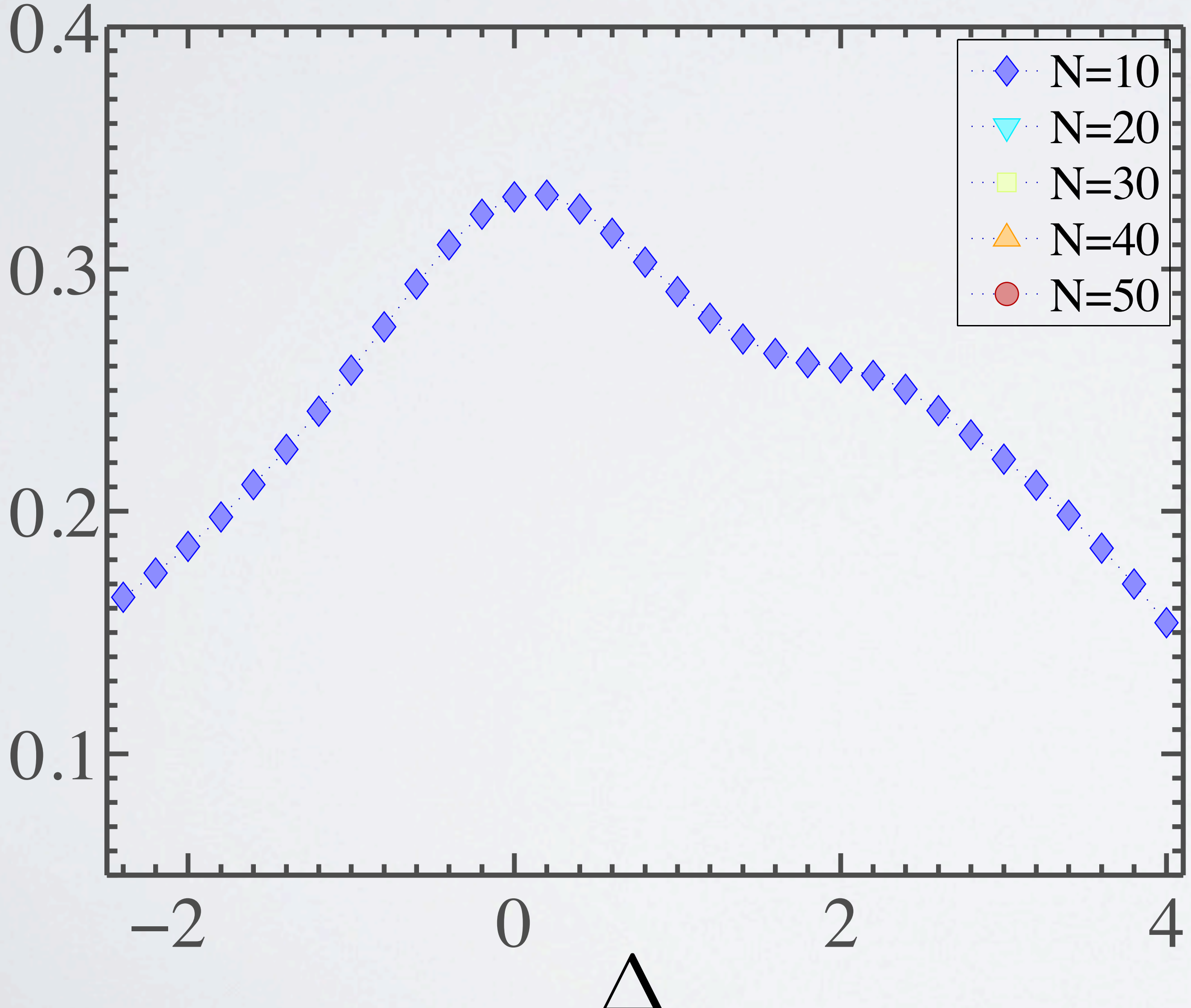

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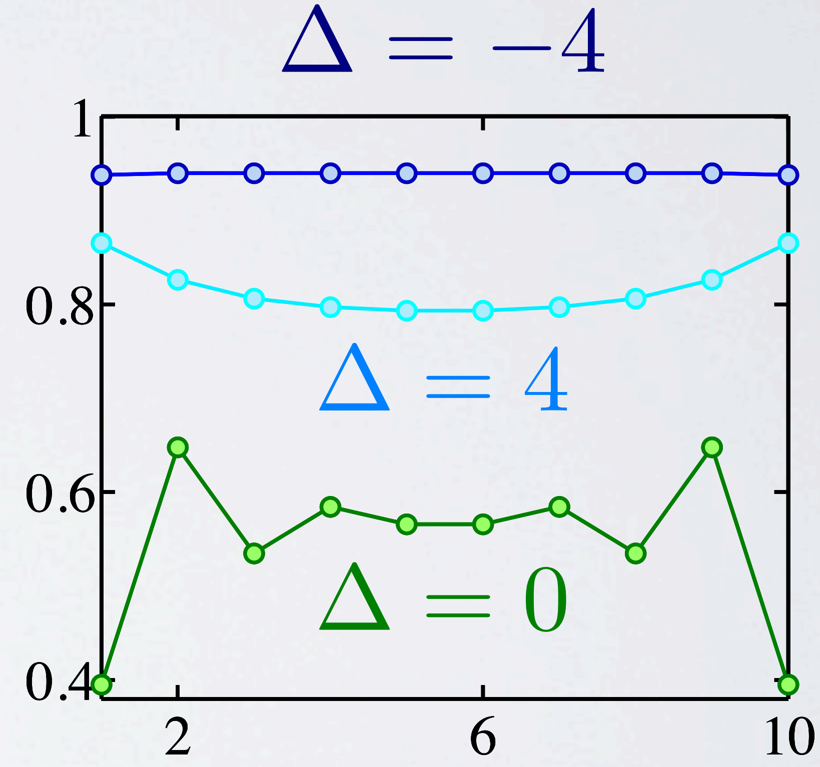

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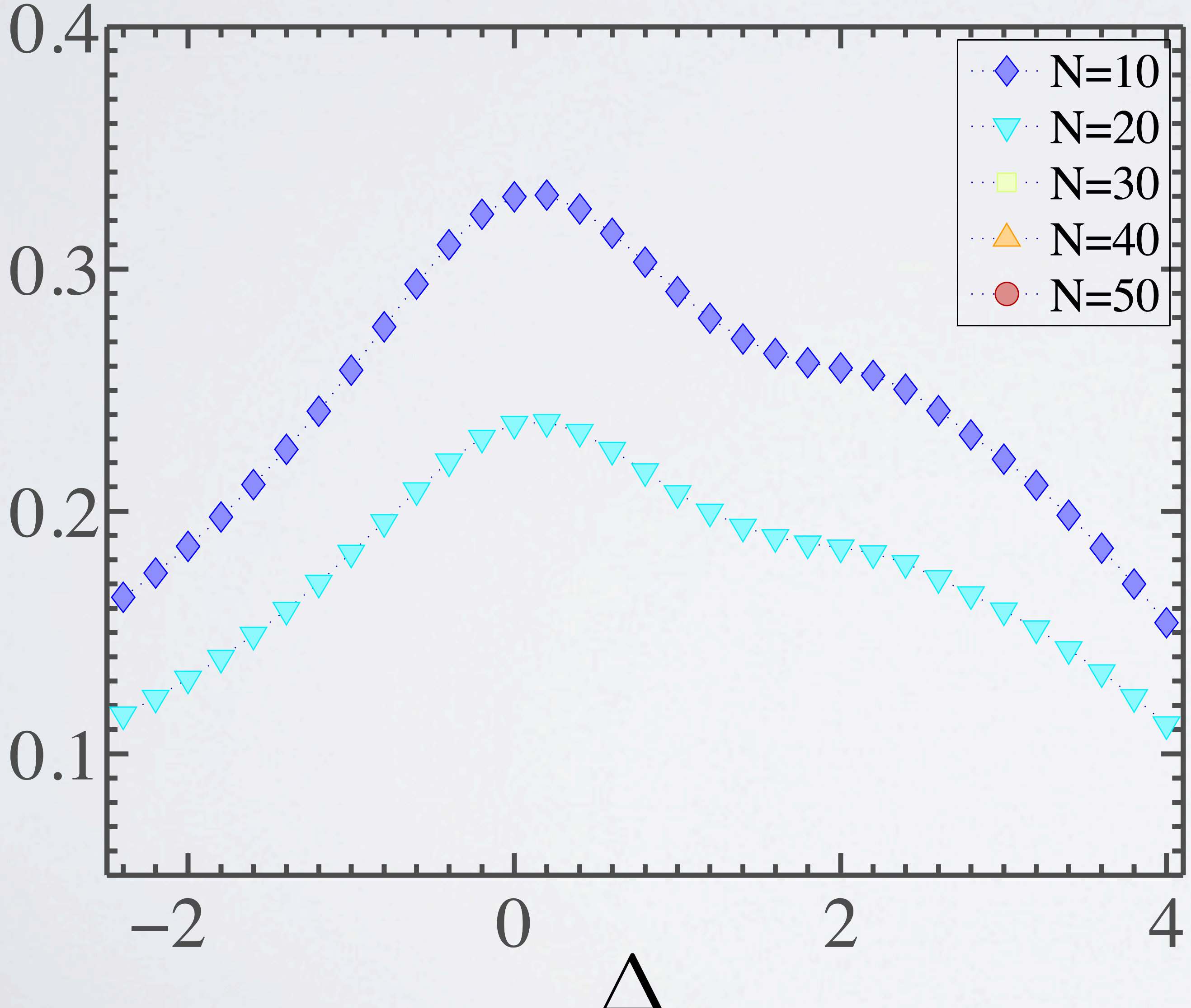

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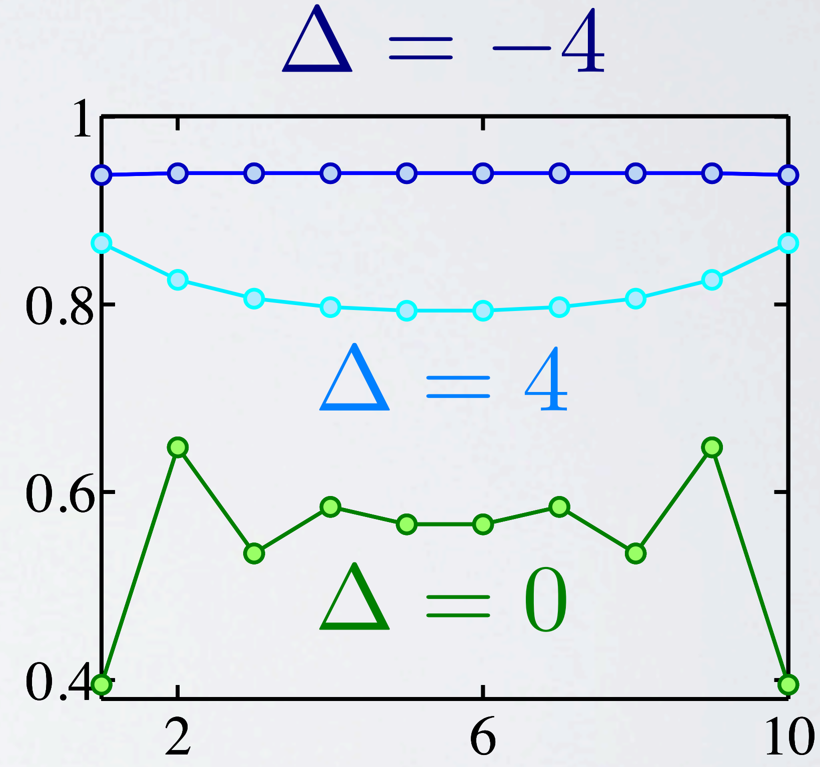

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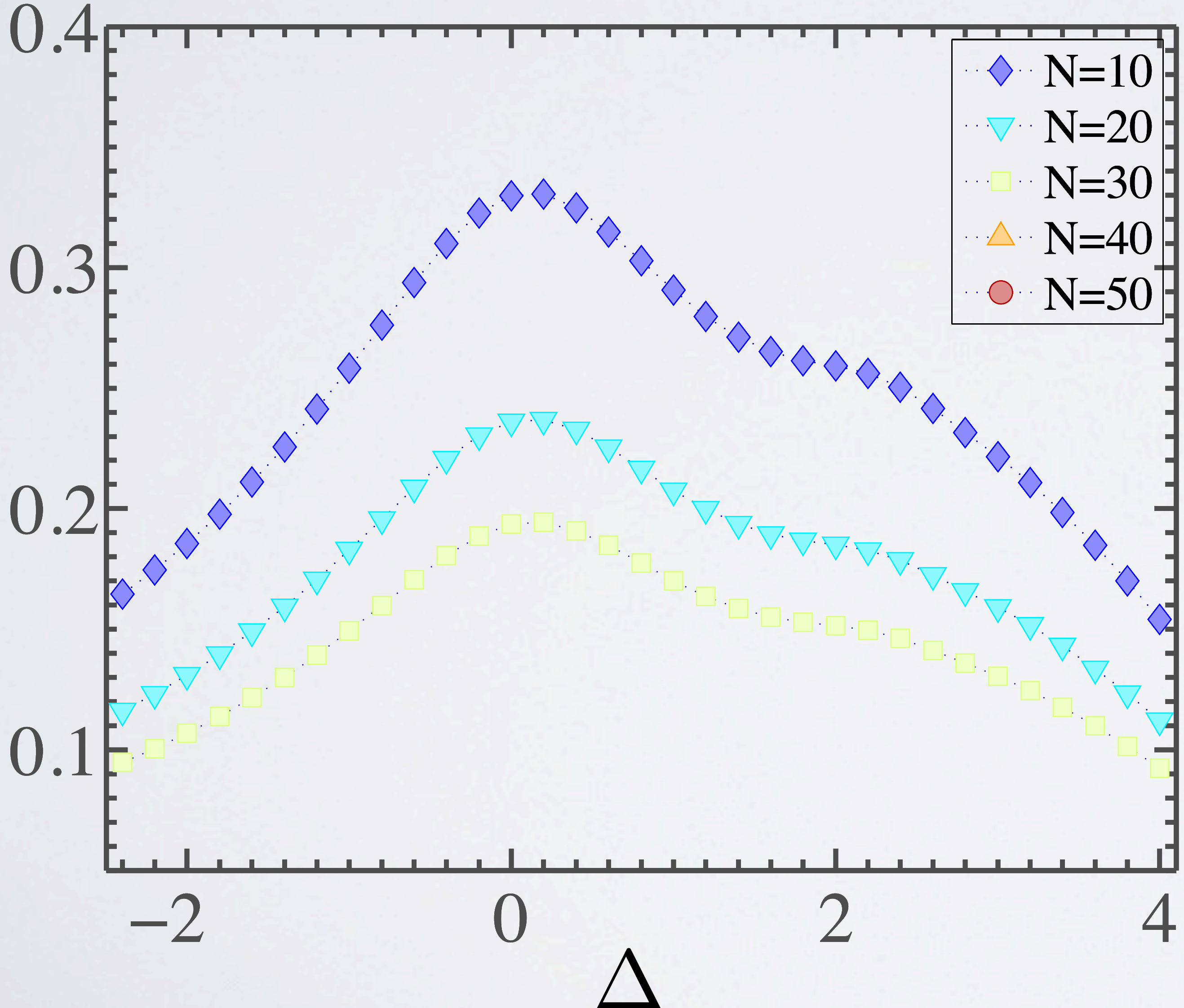

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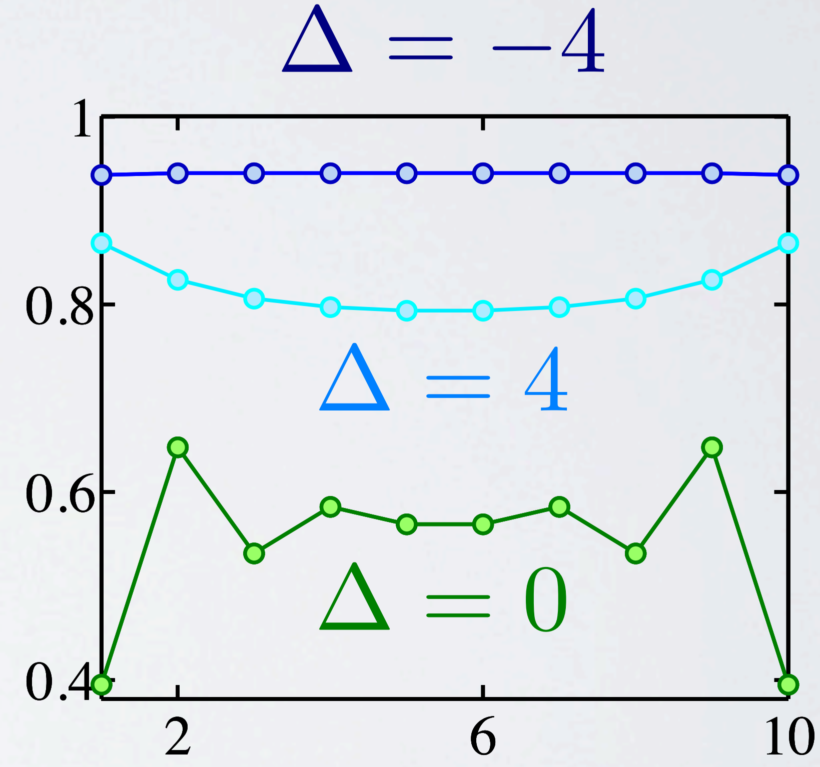

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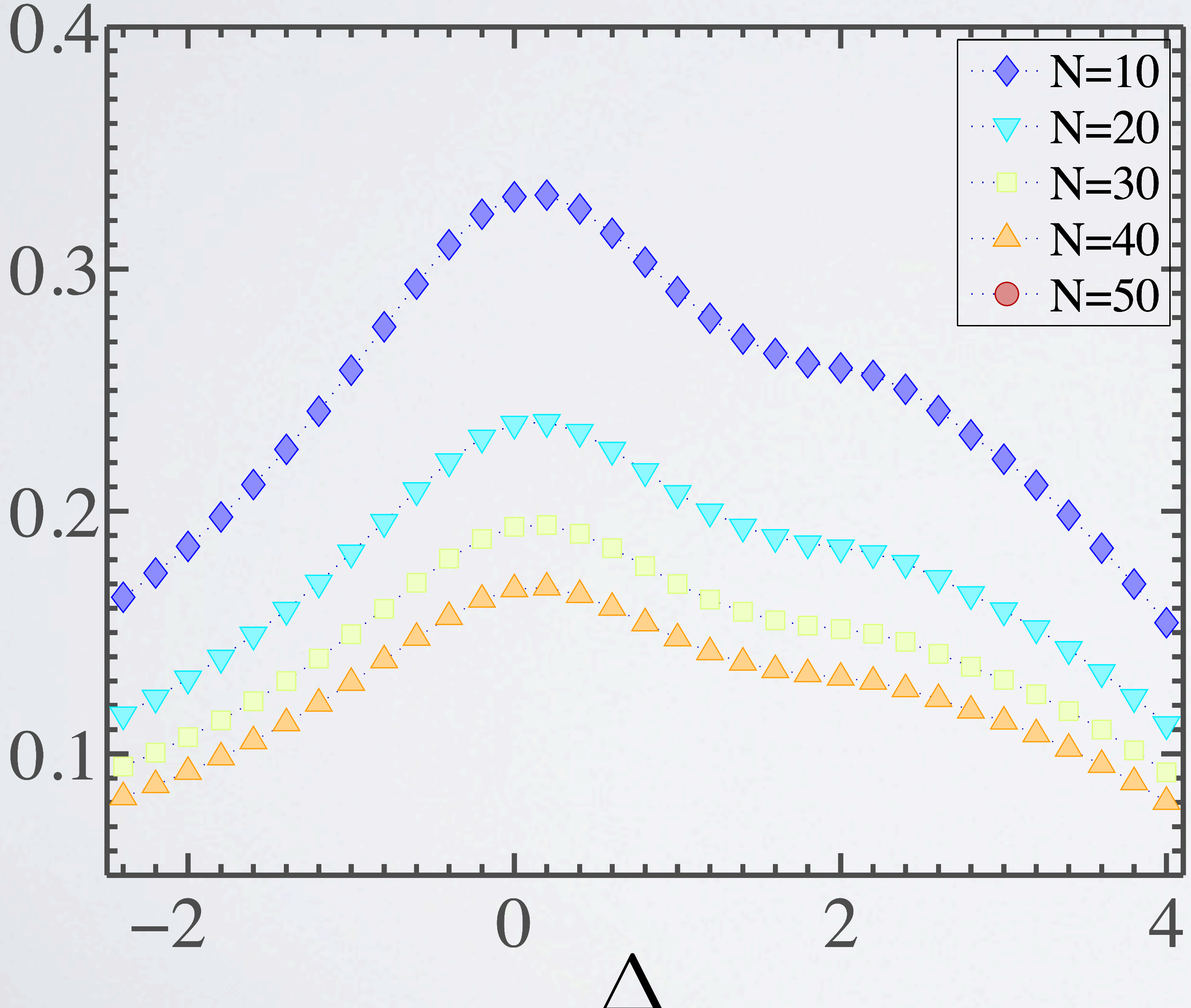

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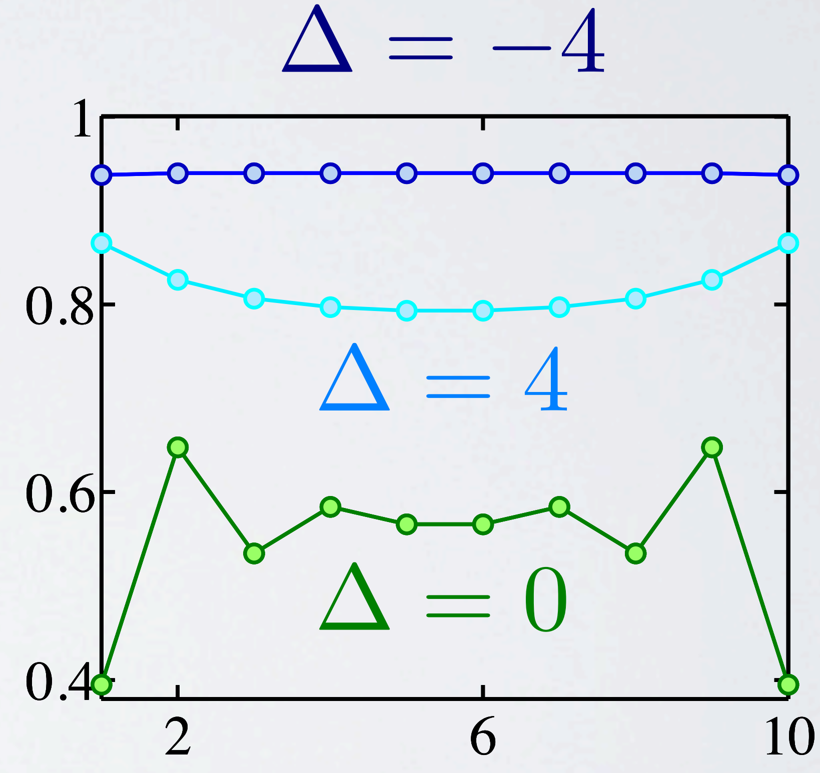

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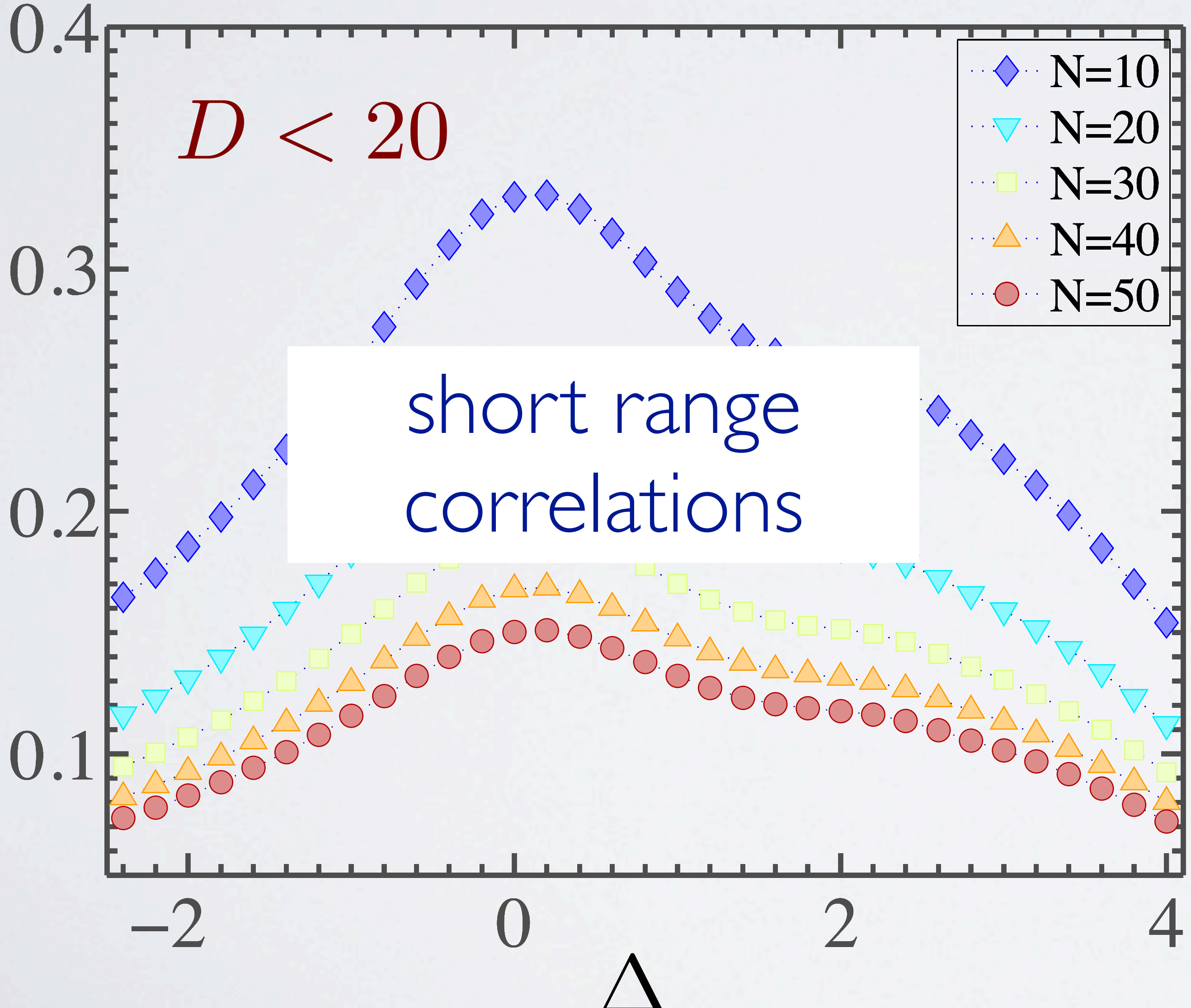

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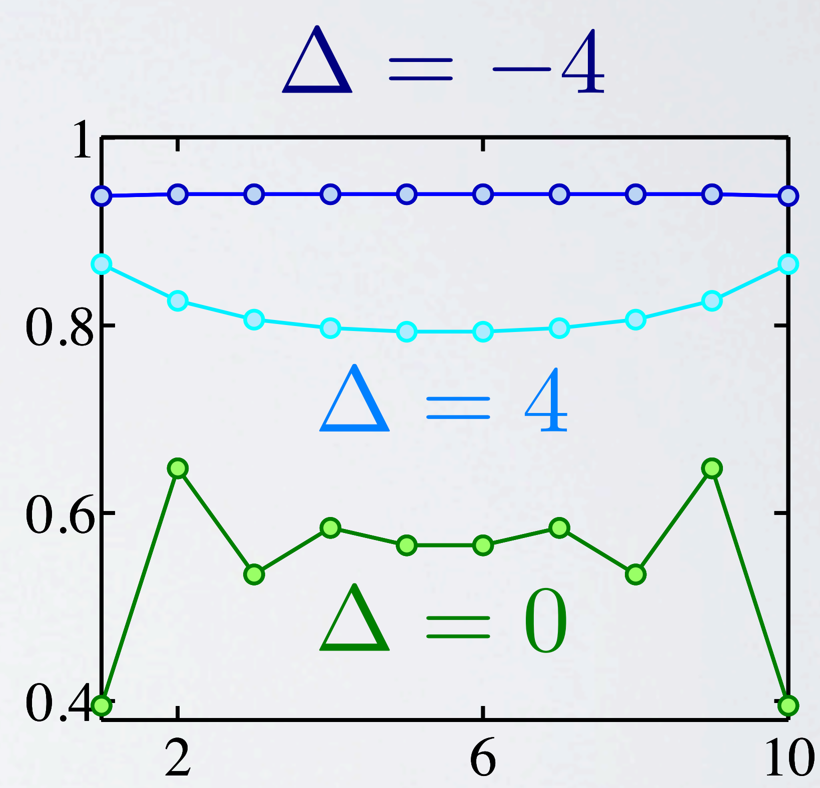

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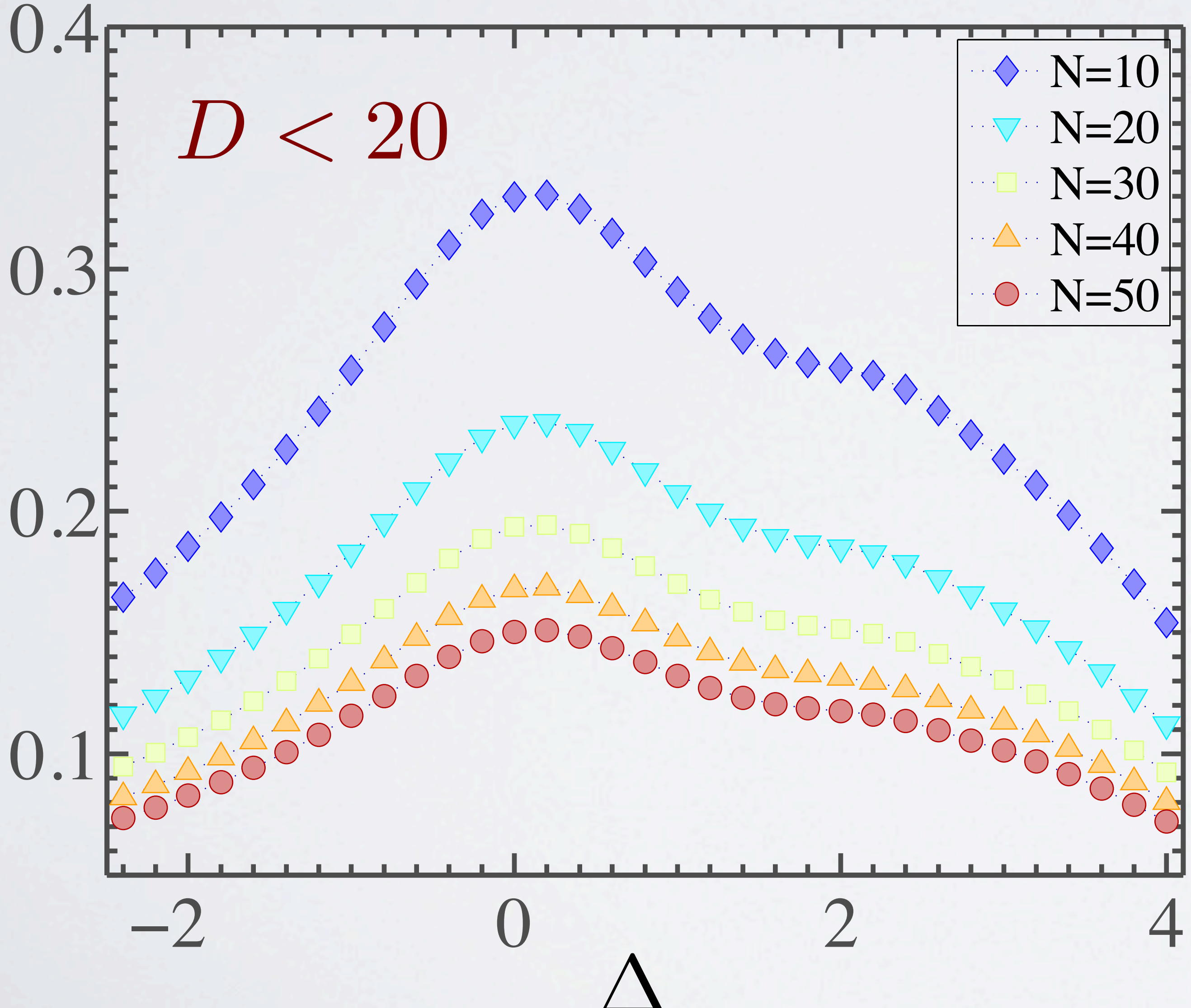
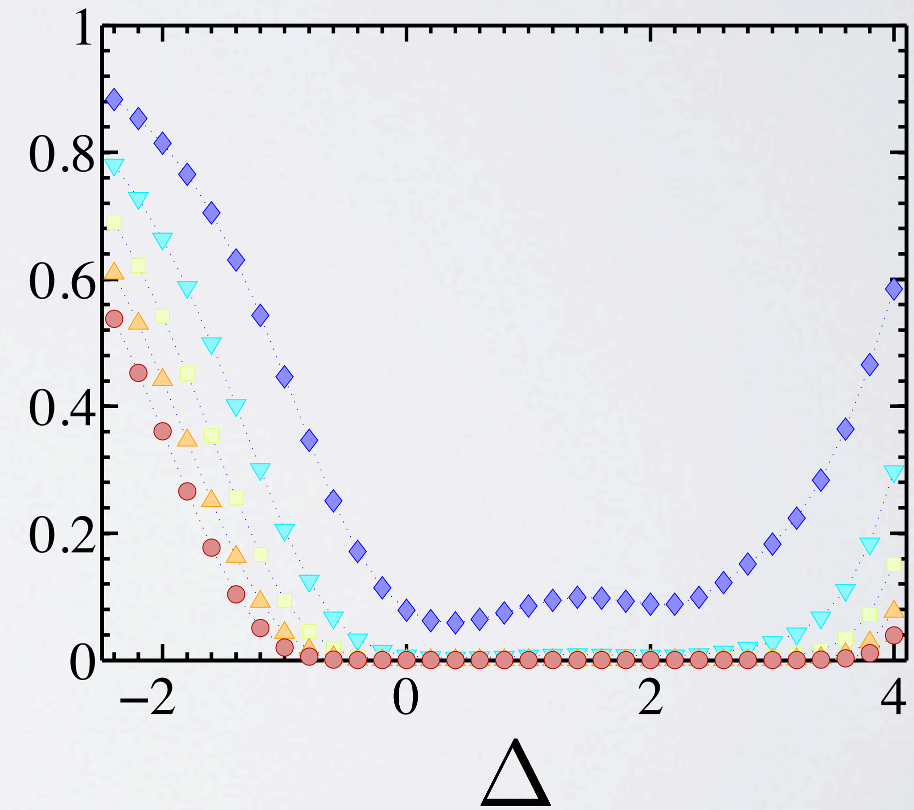


purity



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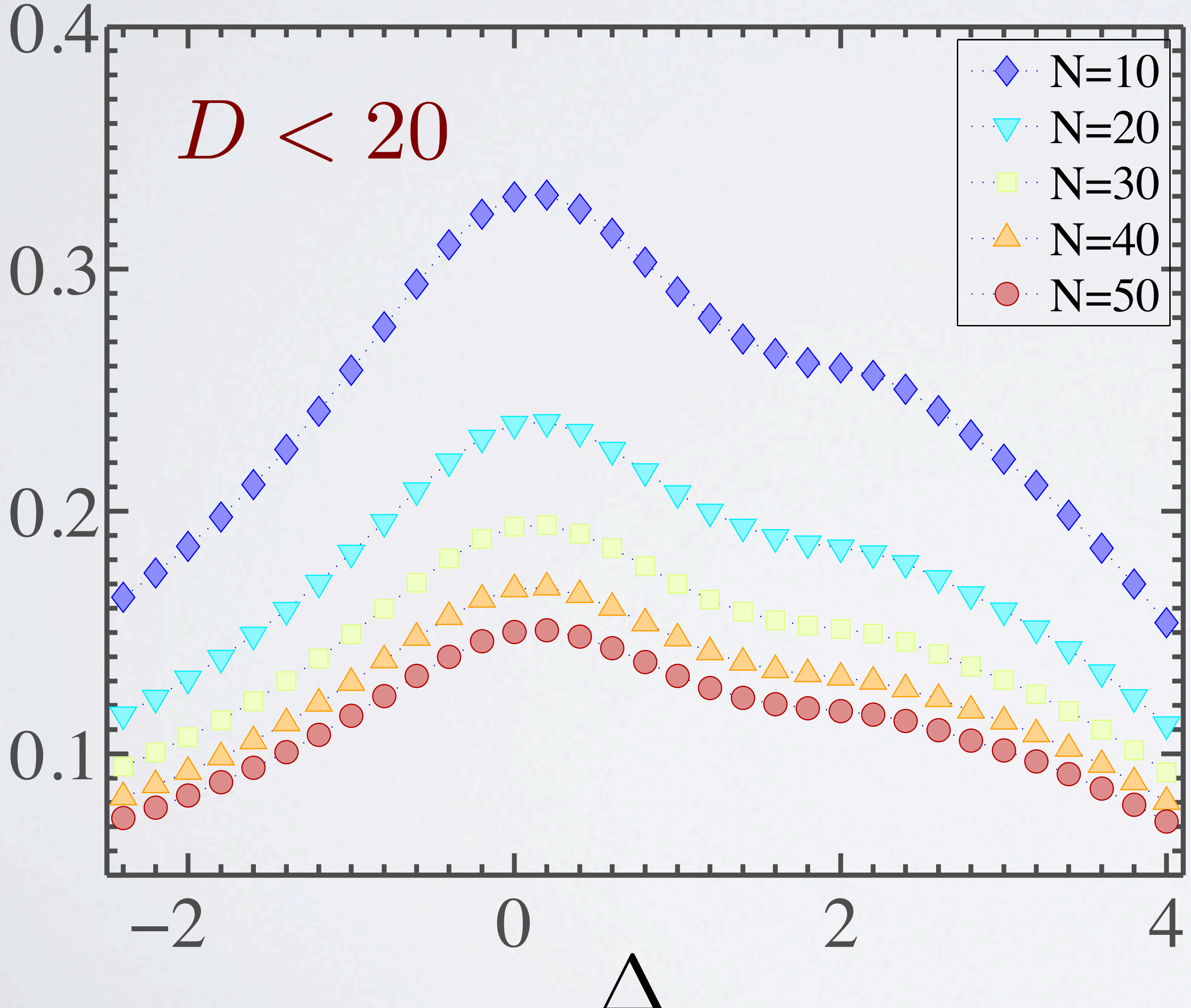
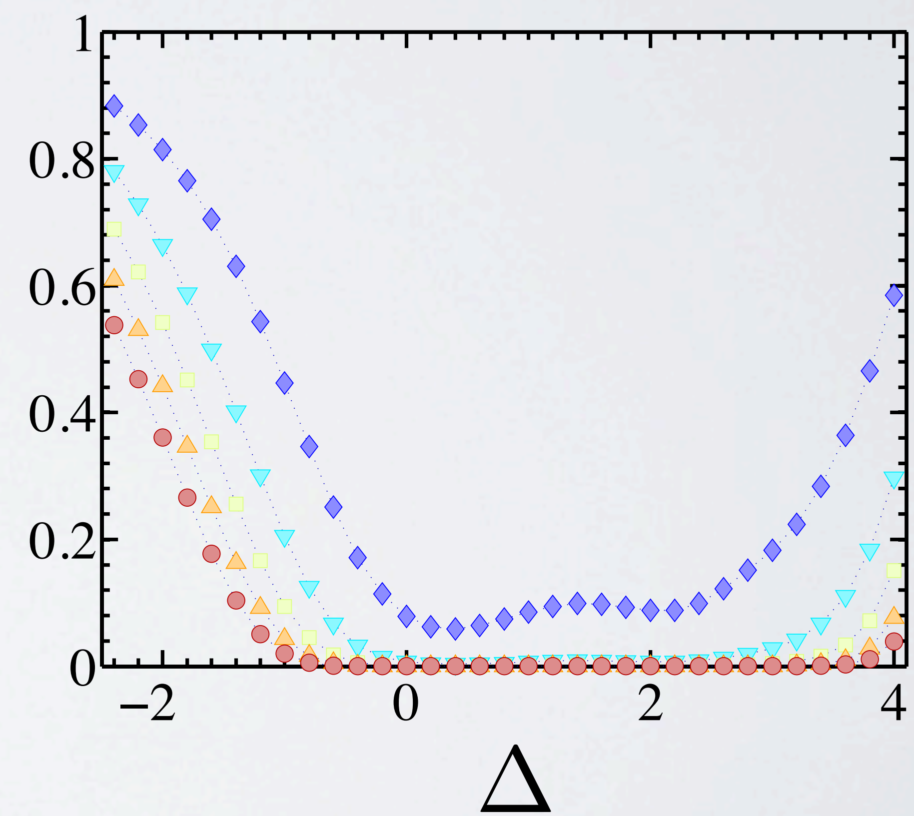


$|\Delta| \to \infty$
completely polarized
steady state

purity



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PRL 114, 220601 (2015)

SUMMARY

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Warm-up phase needed!

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SUMMARY

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Very good convergence (varying models, parameters)

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Stability can be delicate

Warm-up phase needed!

Symmetries can be included, trace one, degeneracies...

things to be understood about MPDOs representations

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Other scenarios for MPO/MPS...

Master equation may not be enough

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- strong system-environment coupling
- correlations between system and environment
- similar time scales for both

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- strong system-environment coupling
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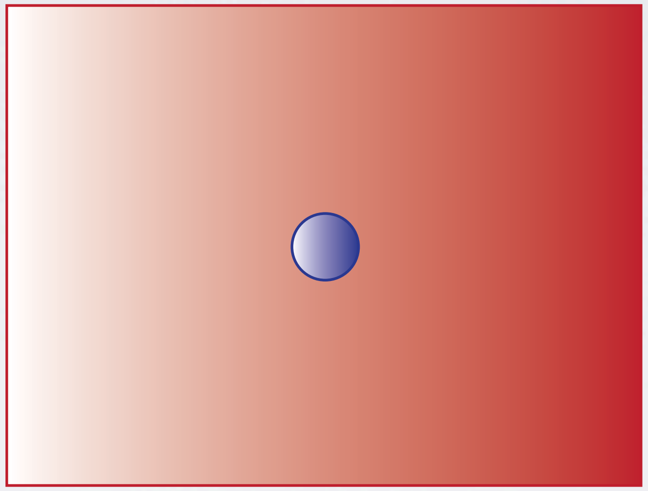
- similar time scales for both

Alternative: solving the whole dynamics

- large number of degrees of freedom involved
- typically a truncation in environment dof required

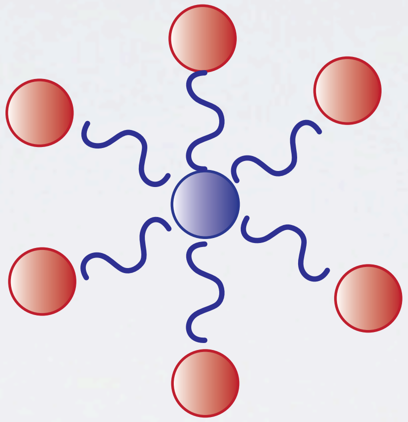
CHAIN MAPPINGS



CHAIN MAPPINGS

discretized environment

star geometry



$$H = H_S + \sum_\lambda \omega_\lambda a_\lambda^\dagger a_\lambda + \sum_\lambda g_\lambda (a_\lambda^\dagger L + L^\dagger a_\lambda)$$

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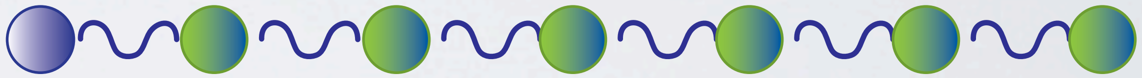
discretized environment

chain geometry



tight-binding model

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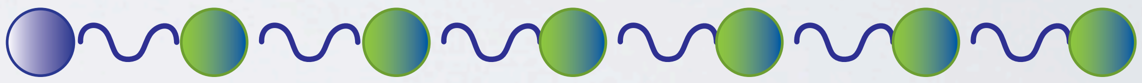

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NRG approach: exponentially decaying couplings

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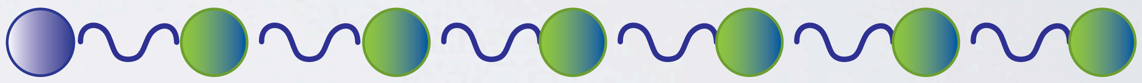

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Continuous environment mapped to semiinfinite chain

Prior et al., PRL 2010
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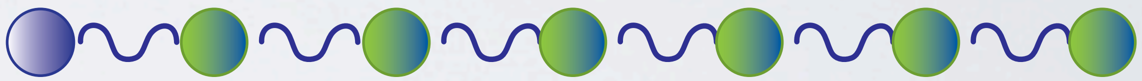

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State of system and bath represented as MPS

$$|\Psi(0)\rangle = |\psi_0\rangle_S \otimes |0_E\rangle$$

T=0 bath corresponds to vacuum

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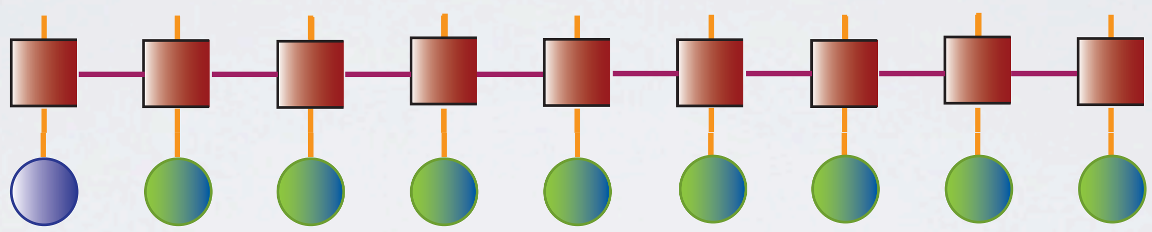

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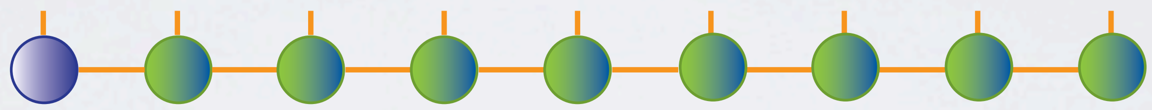

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CHAIN MAPPINGS

Can also be used for $T>0$ environment

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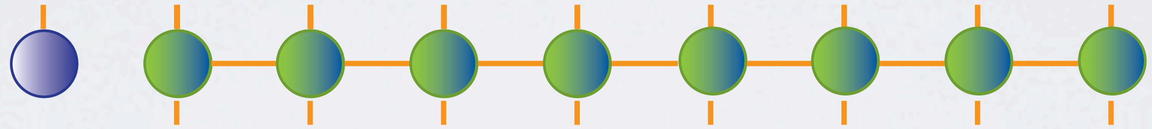

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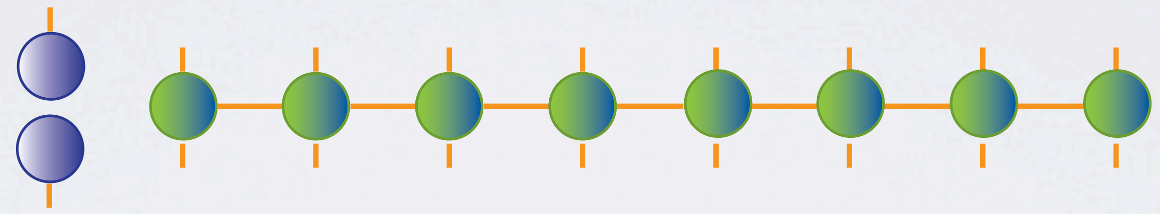

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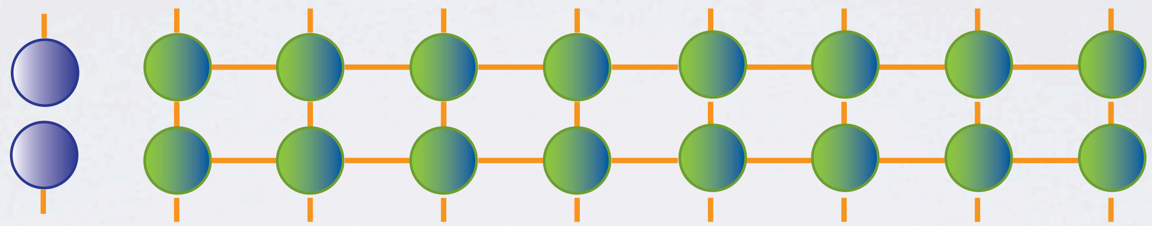

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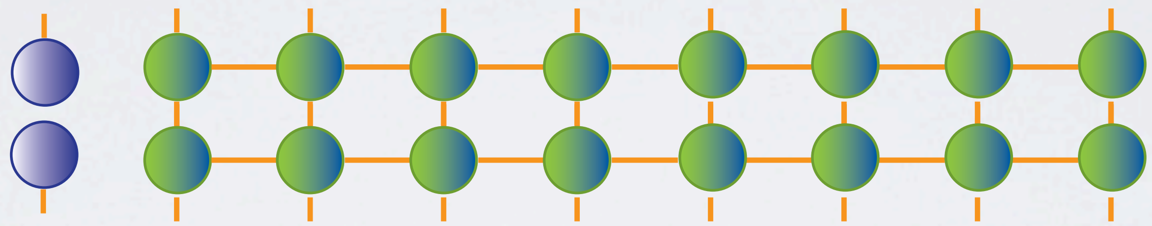

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computational overhead

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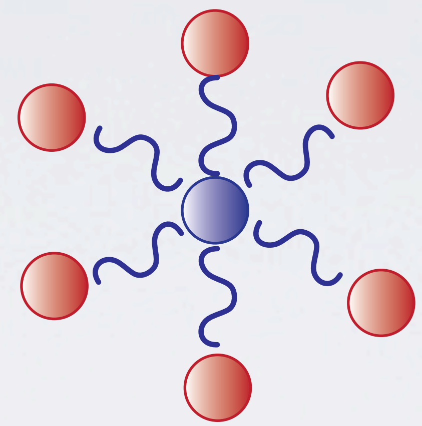
THERMOFIELD APPROACH



An alternative to deal with $T>0$ also with pure MPS

Takahasi 1975

I. de Vega, MCB, PRA 92, 052116 (2015)

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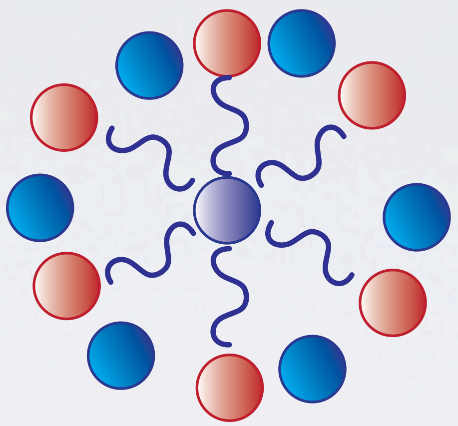

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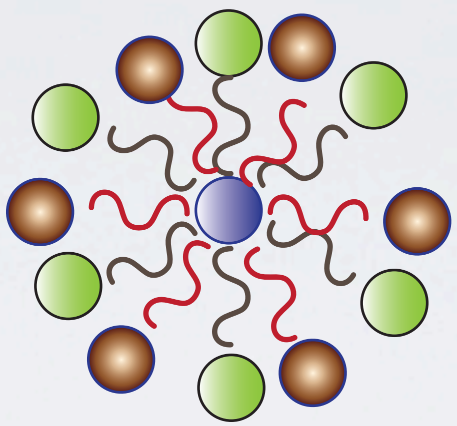

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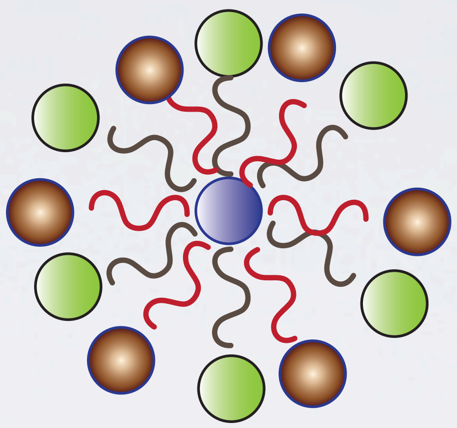

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thermal vacuum

$$|\Omega\rangle \propto e^{-\beta H_B/2} \sum_n |E_n\rangle_b |E_n\rangle_c$$

$$\rho_B(\beta) = \mathrm{tr}_c(|\Omega\rangle\langle\Omega|)$$

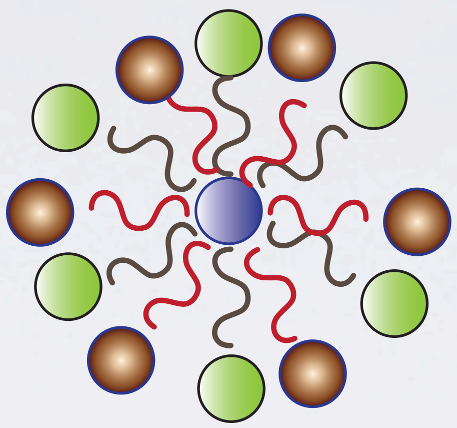
I. de Vega, MCB, PRA 92, 052116 (2015)

THERMOFIELD APPROACH

$$a_{1k}(\theta),\ a_{2k}(\theta)$$

$$\cosh\theta_k = \sqrt{1+n_k}$$

$$n_k = \frac{1}{e^{\beta\omega_k}-1}$$



$$\hat{H} = H - \sum_k \omega_k c_k^\dagger c_k$$

An alternative to deal with $T>0$ also with pure MPS

introduce auxiliary decoupled environment

thermal Bogoliubov transformation

Takahasi 1975

thermal vacuum

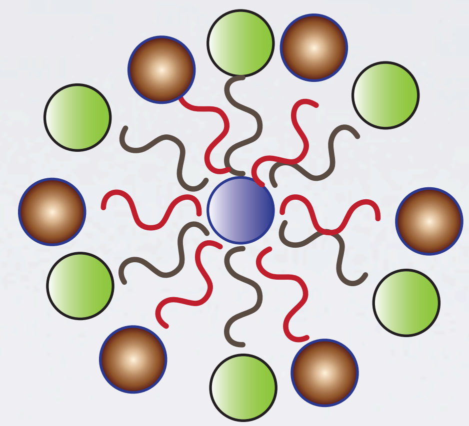
$$|\Omega\rangle \propto e^{-\beta H_B/2}\sum_n |E_n\rangle_b |E_n\rangle_c$$

$$\rho_B(\beta) = \mathrm{tr}_c(|\Omega\rangle\langle\Omega|)$$

$$\langle\Omega|b_k^\dagger b_k|\Omega\rangle = n_k$$

I. de Vega, MCB, PRA 92, 052116 (2015)

THERMOFIELD APPROACH



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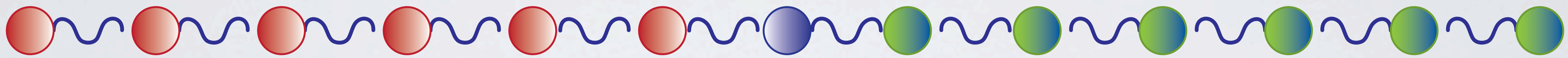
Thermal state mapped to two environments at ***T*=0**

initially no excitations
in the environment

I. de Vega, MCB, PRA 92, 052116 (2015)

THERMOFIELD APPROACH



Thermal state mapped to two environments at $T=0$

Double chain mapping

initially no excitations
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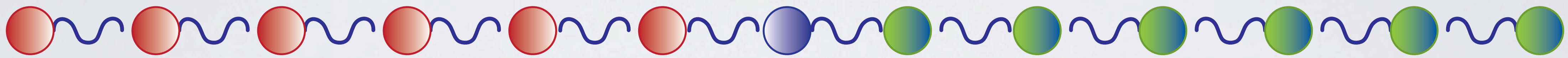
I. de Vega, MCB, PRA 92, 052116 (2015)

THERMOFIELD APPROACH



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Pure MPS methods can be applied

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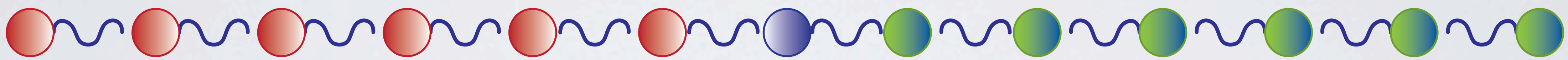

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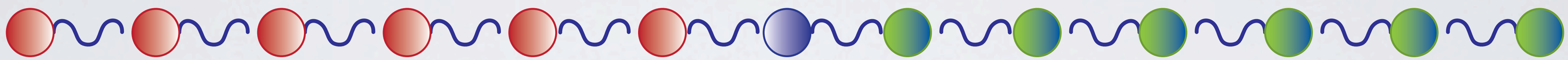

Thermal state mapped to two environments at $T=0$

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Population imbalance between chains gives deviation of bath from thermal distribution

Applies to bosonic and fermionic systems

I. de Vega, MCB, PRA 92, 052116 (2015)

THERMOFIELD APPROACH

EXAMPLE

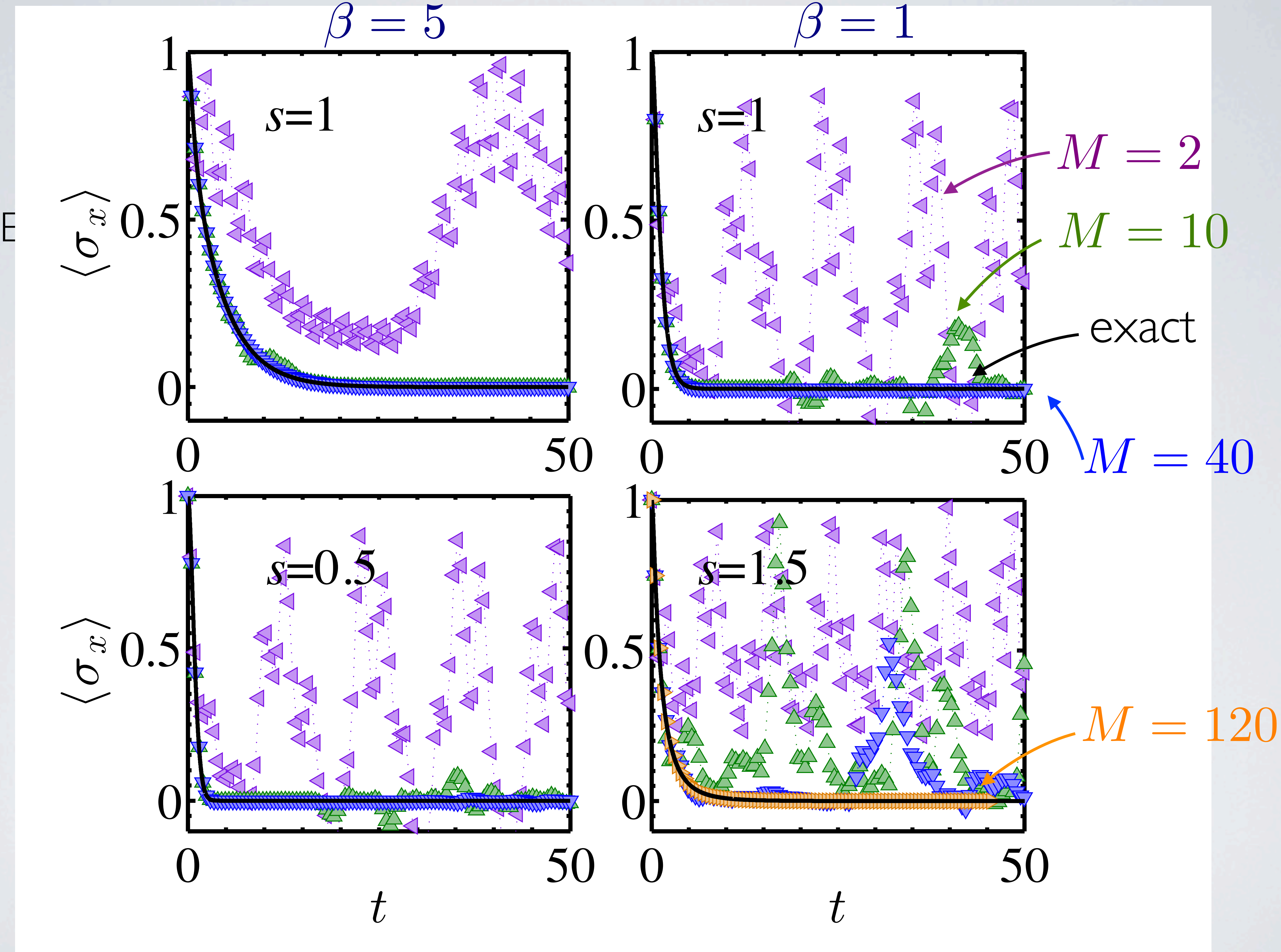
spin in a bosonic bath

$$J(\omega) = \eta \omega^s e^{-\omega/\omega_c}$$ Caldeira-Legget model

exact solution for $H_S \propto \sigma_z$

$$L \propto \sigma_z$$

I. de Vega, MCB, PRA 92, 052116 (2015)



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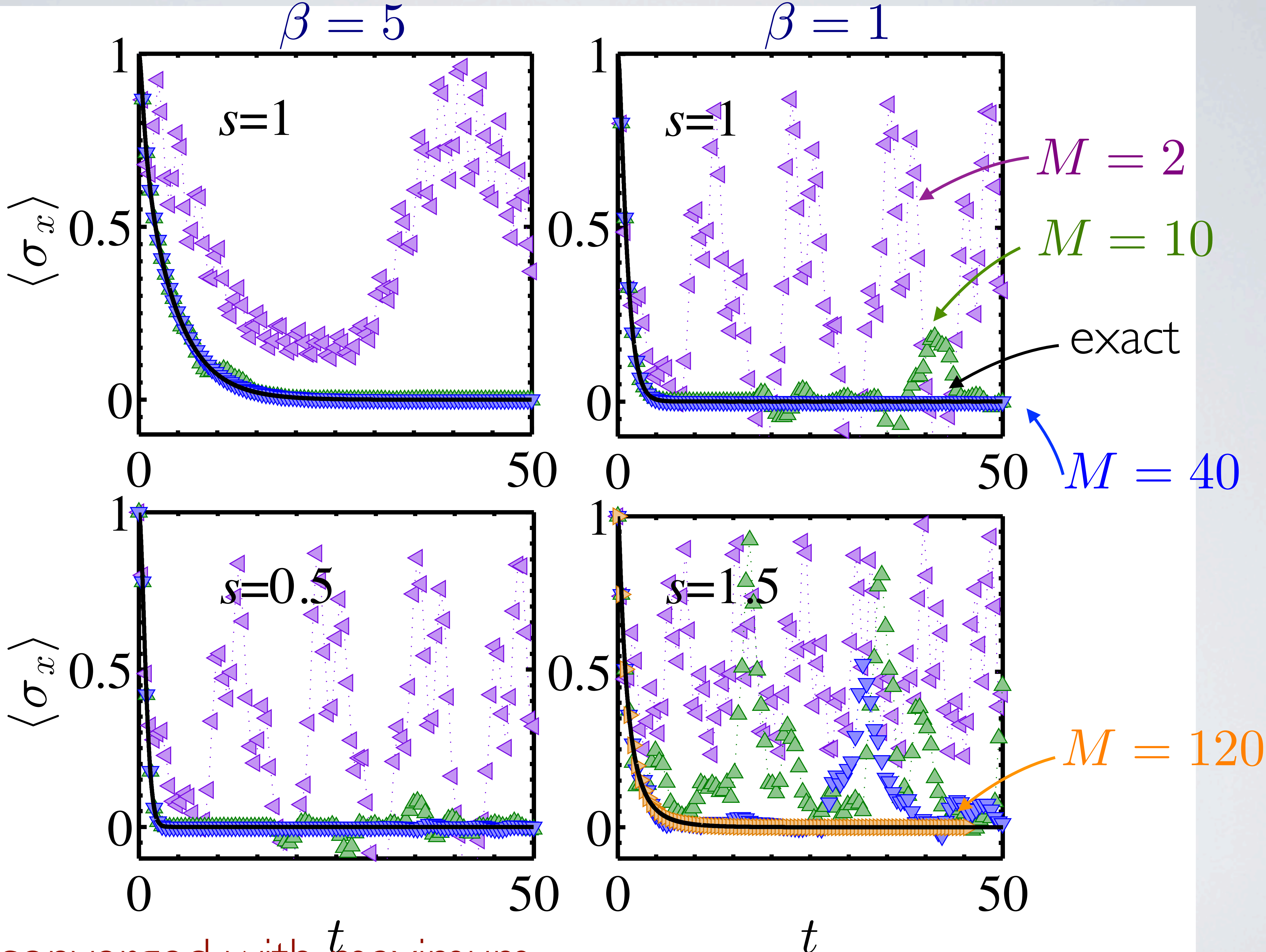

converged with maximum occupation n=3

I. de Vega, MCB, PRA 92, 052116 (2015)

THERMOFIELD APPROACH

EXAMPLE

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$J(\omega) = \eta\omega^s e^{-\omega/\omega_c}$ Caldeira-Legget model

not exactly solvable for $H_S \propto \sigma_z$

$L \propto \sigma_x$

can only compare to master equation

I. de Vega, MCB, PRA 92, 052116 (2015)

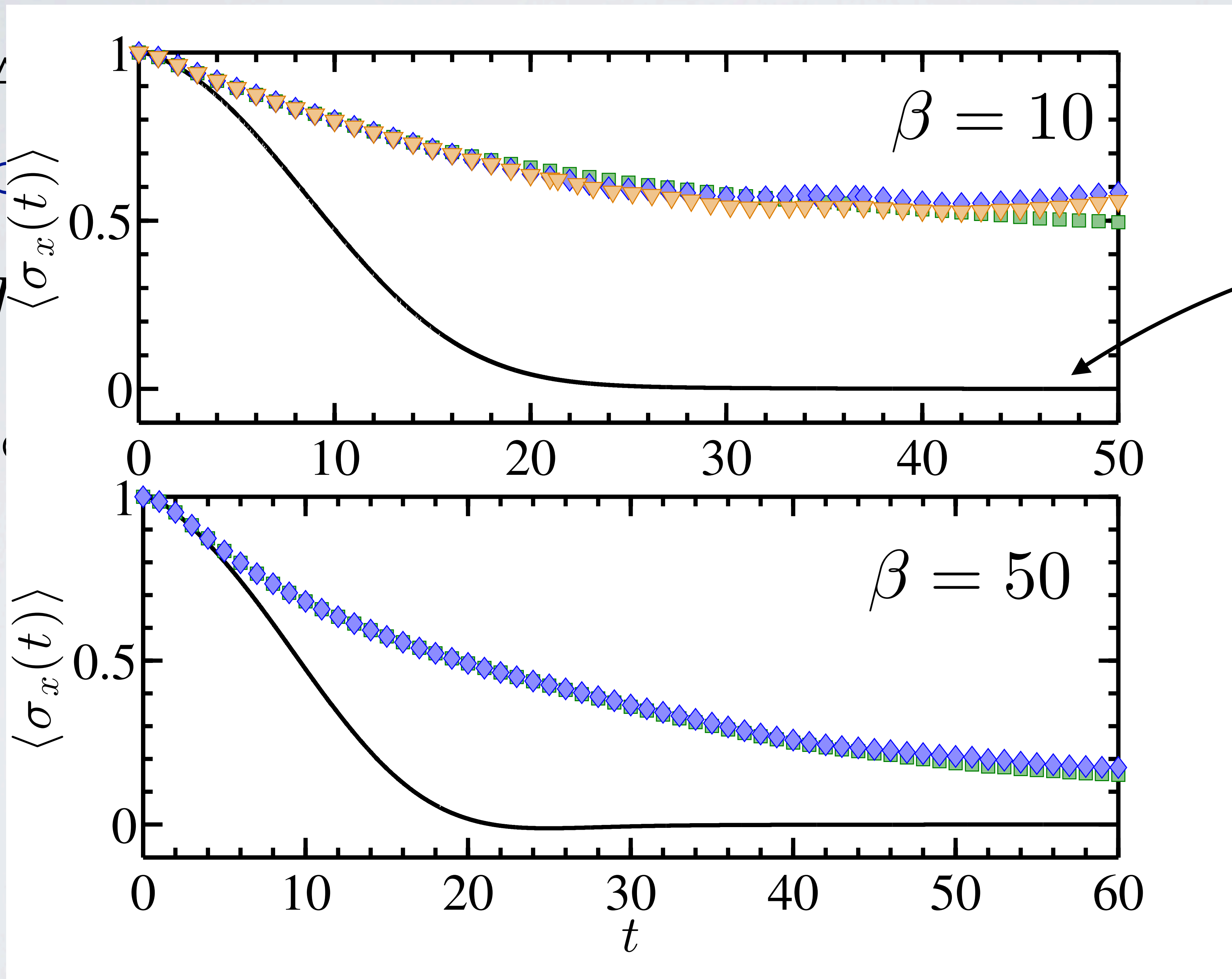
THERMOFIELD APPROACH

EXA

sp

J

n



master equation

aster equation

$$\eta = 0.1$$

I. de Vega, MCB, PRA 92, 052116 (2015)

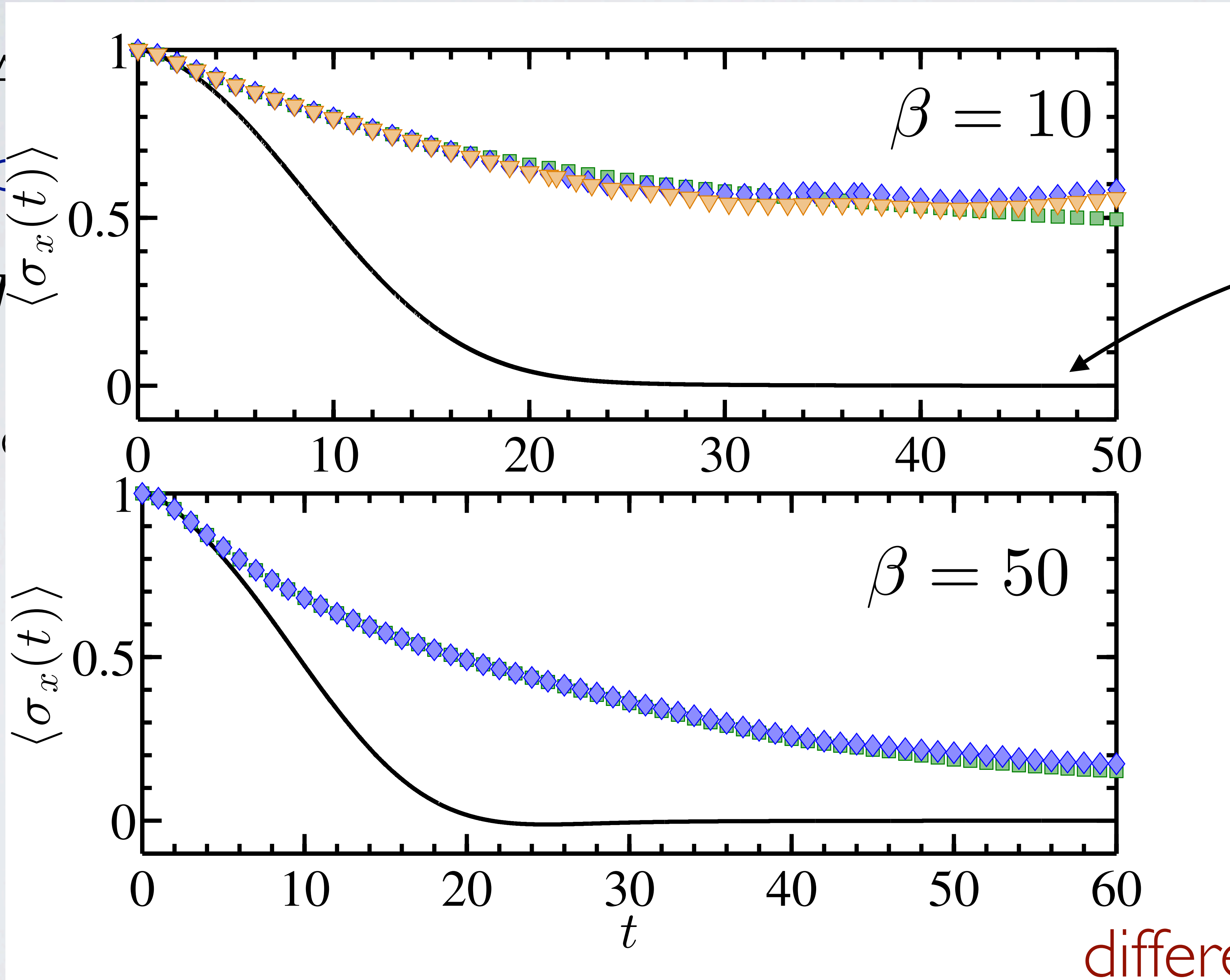
THERMOFIELD APPROACH

EXA

sp

J

n



master equation

aster equation

different truncations in occupation

$\eta = 0.1$

I. de Vega, MCB, PRA 92, 052116 (2015)

TO CONCLUDE

Generally speaking, out of equilibrium dynamics is hard for TNS/MPS, but...

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modelling system-bath interactions beyond master equation

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Applications of MPS/MPO to non-equilibrium

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THANKS!

MPQ
Max Planck Institut
of Quantum Optics
(Garching)

DICKE MODEL

Dicke, 1954
Hepp, Lieb, 1973
Carmichael, 1980

DICKE MODEL

N 2-level atoms coupled to same EM mode

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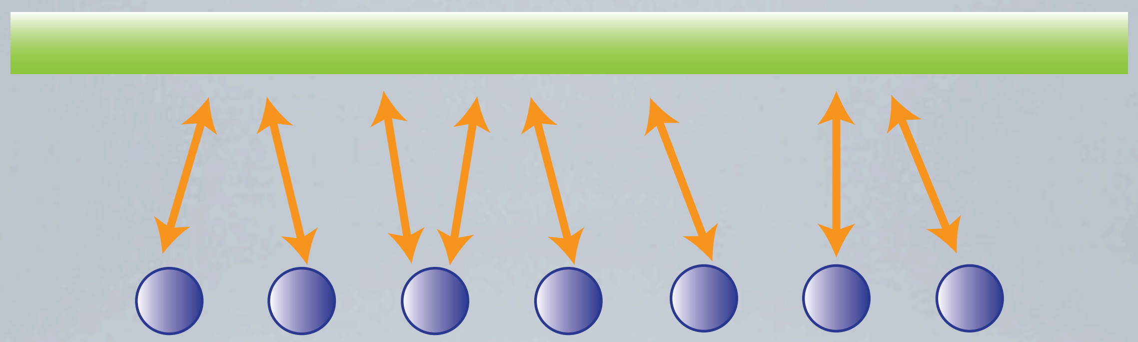

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phase transition to superradiant phase $\frac{\Omega}{\Gamma} = \frac{N}{2}$

analytic solution conserved total spin

Dicke, 1954
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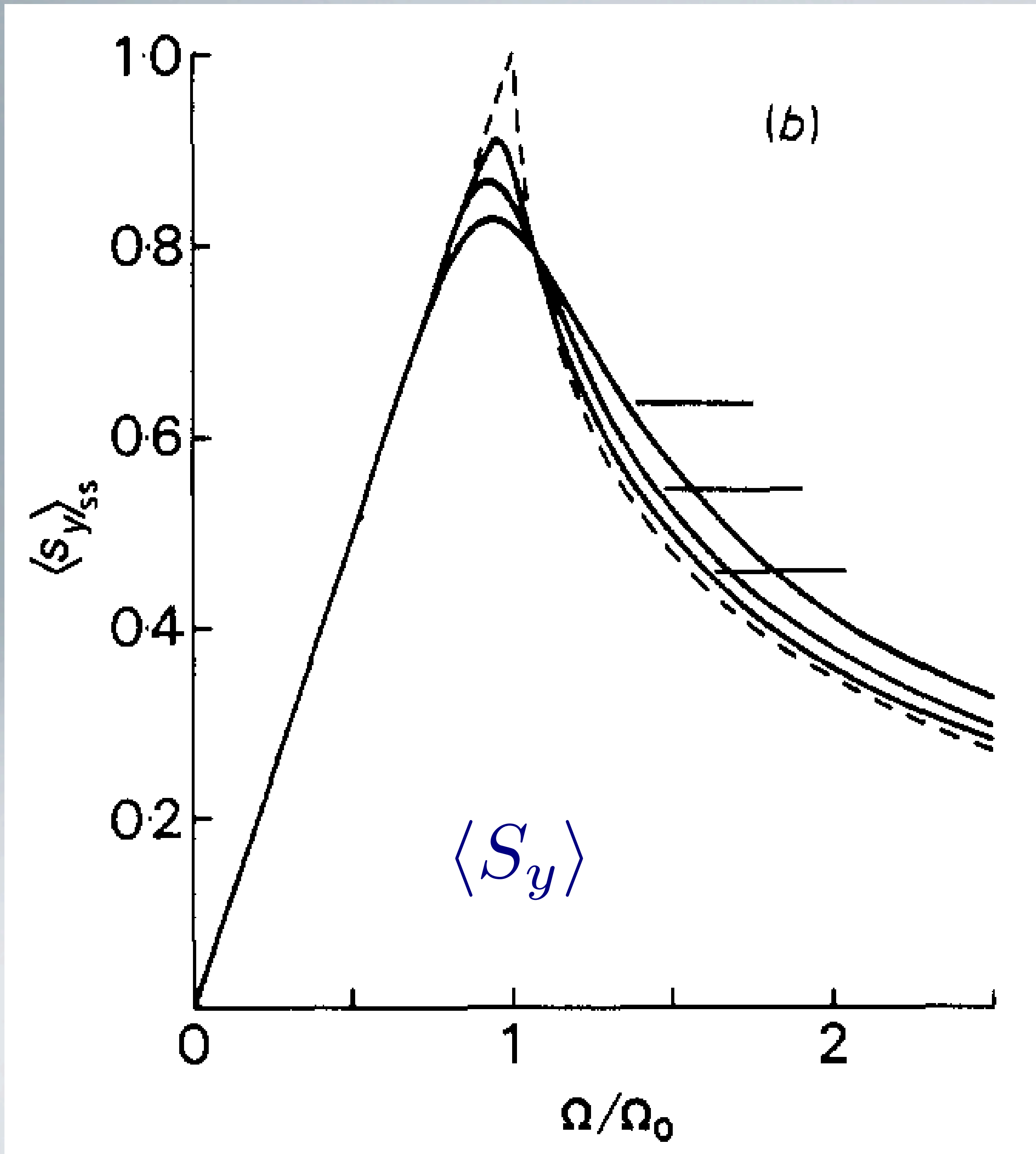

from Carmichael, J Phys B 1980

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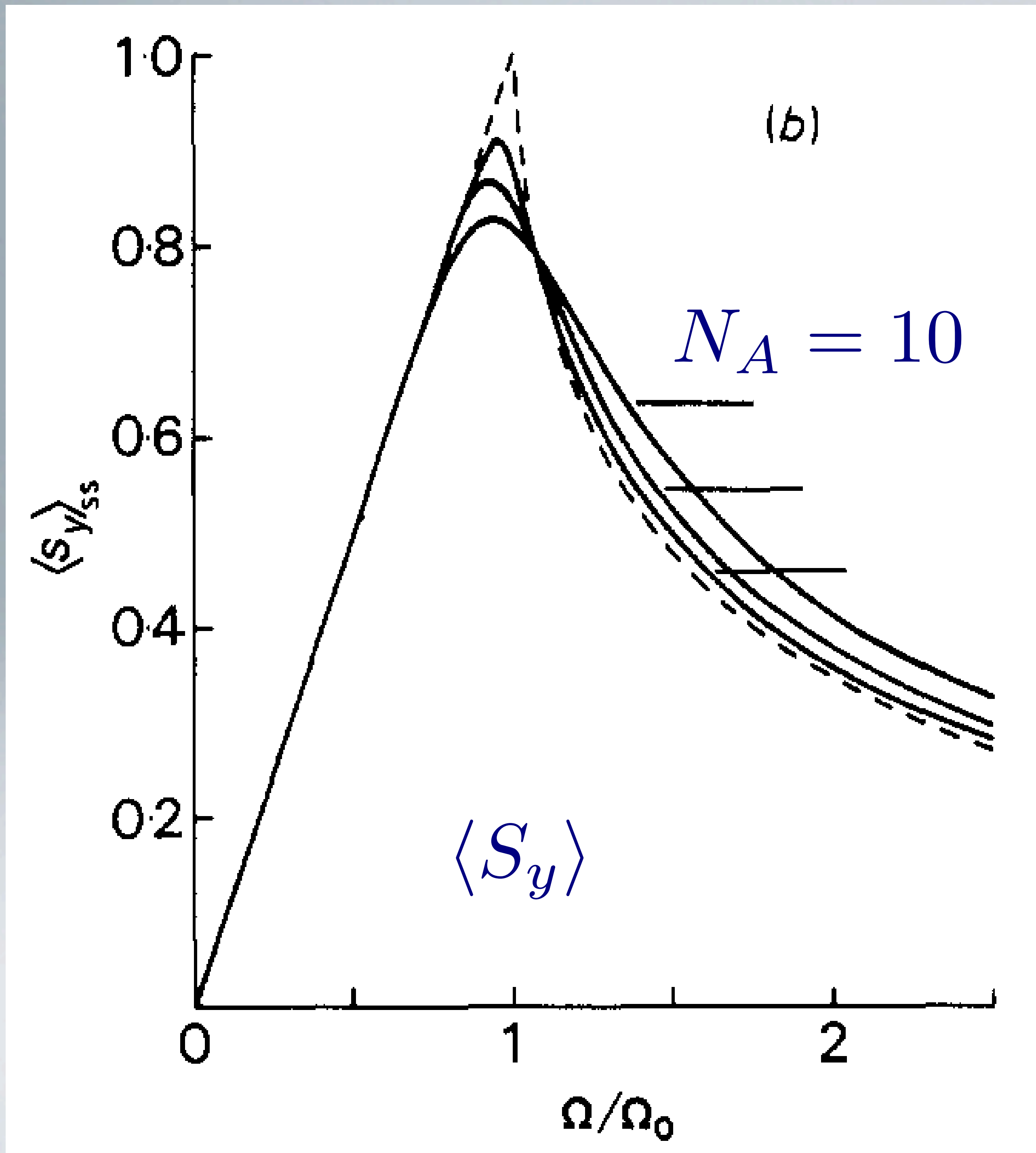

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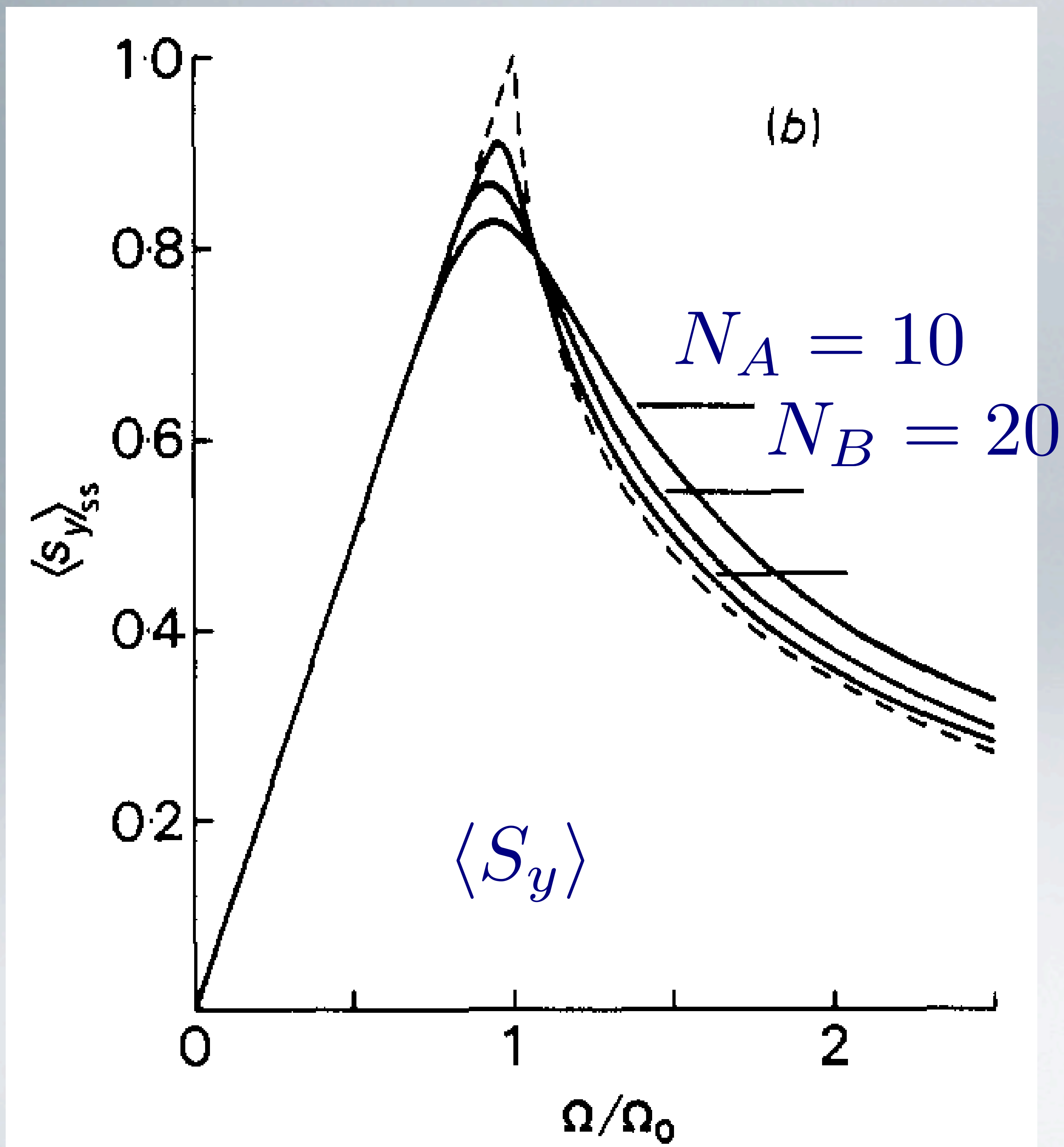

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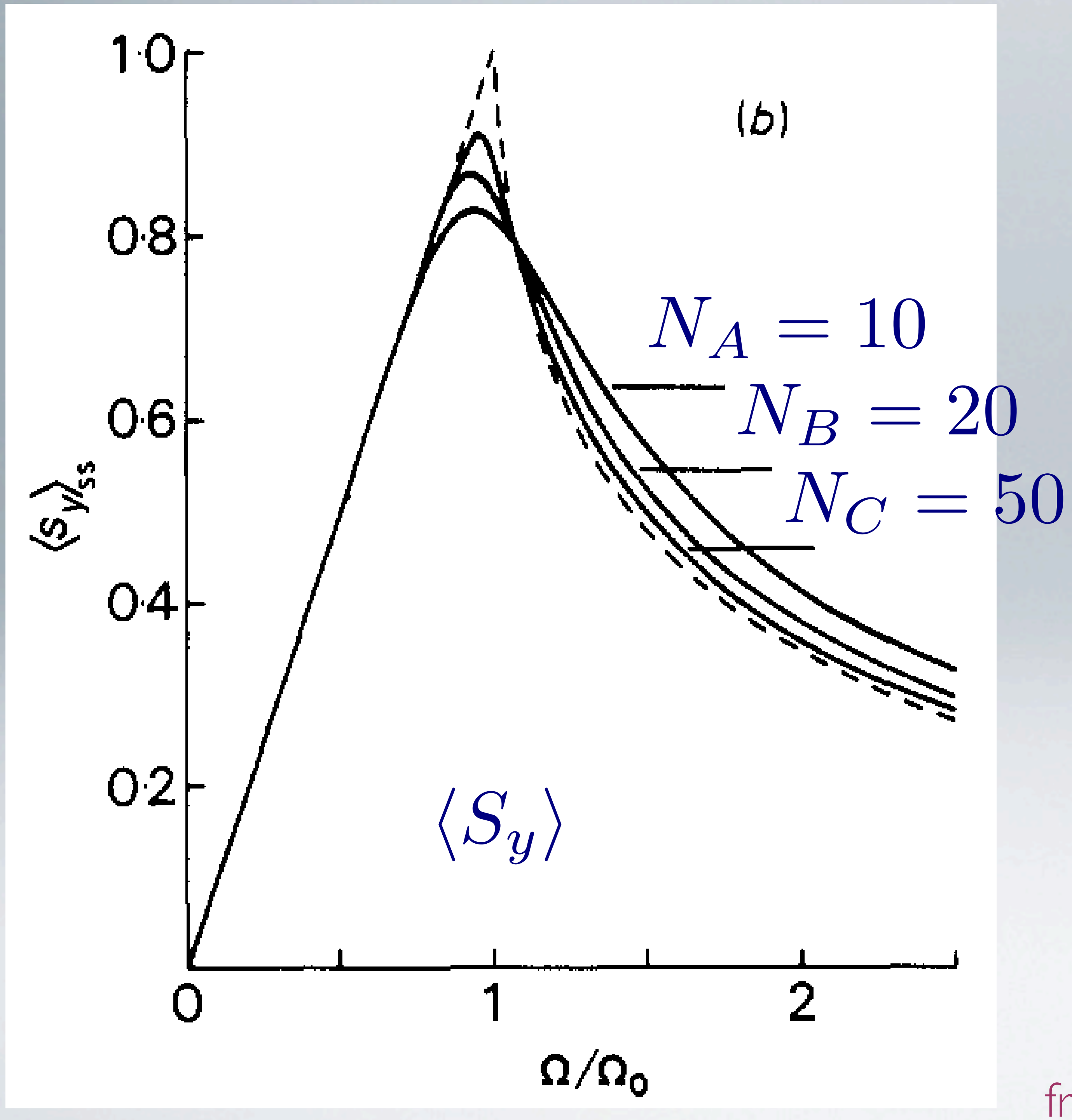

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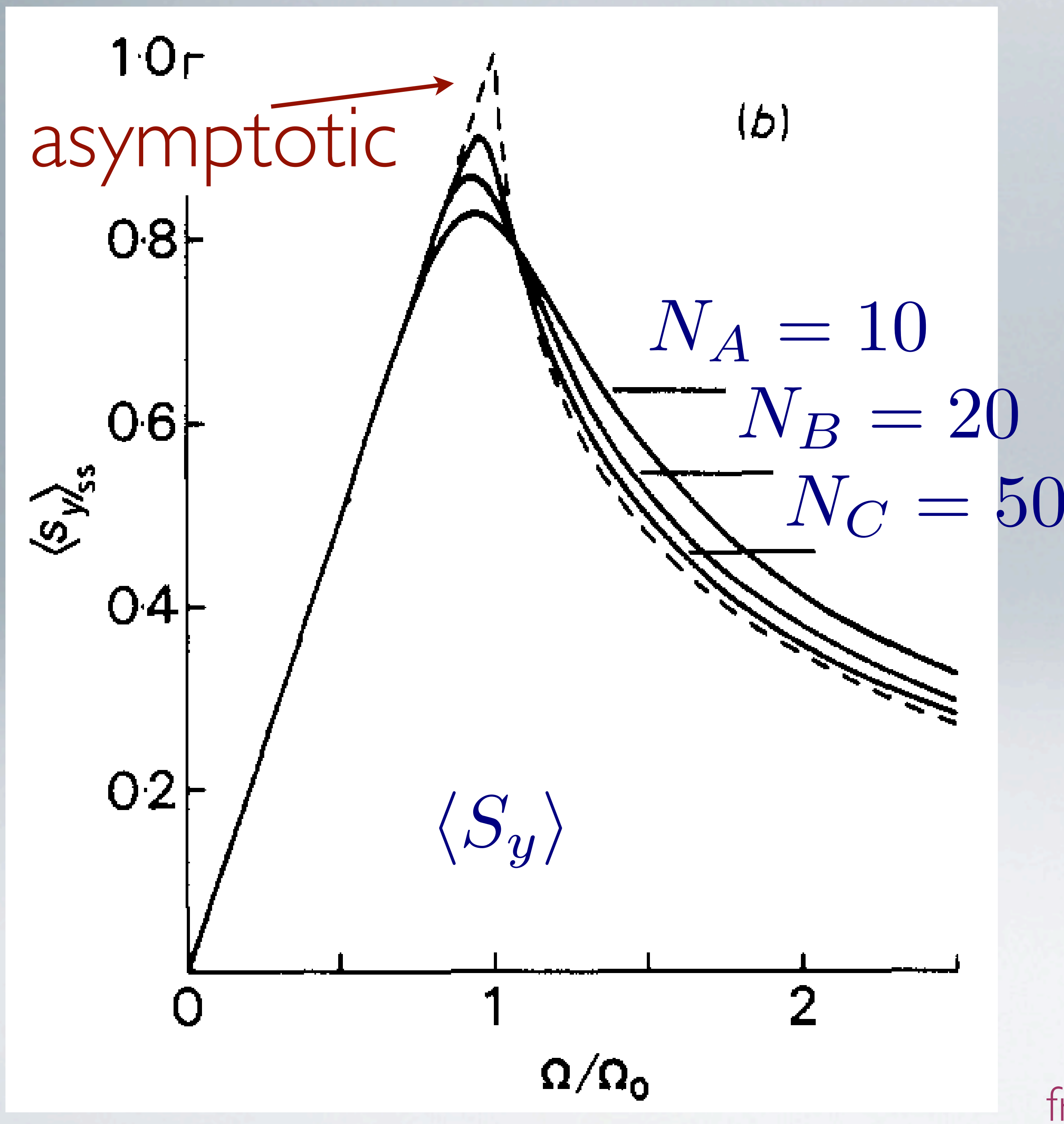

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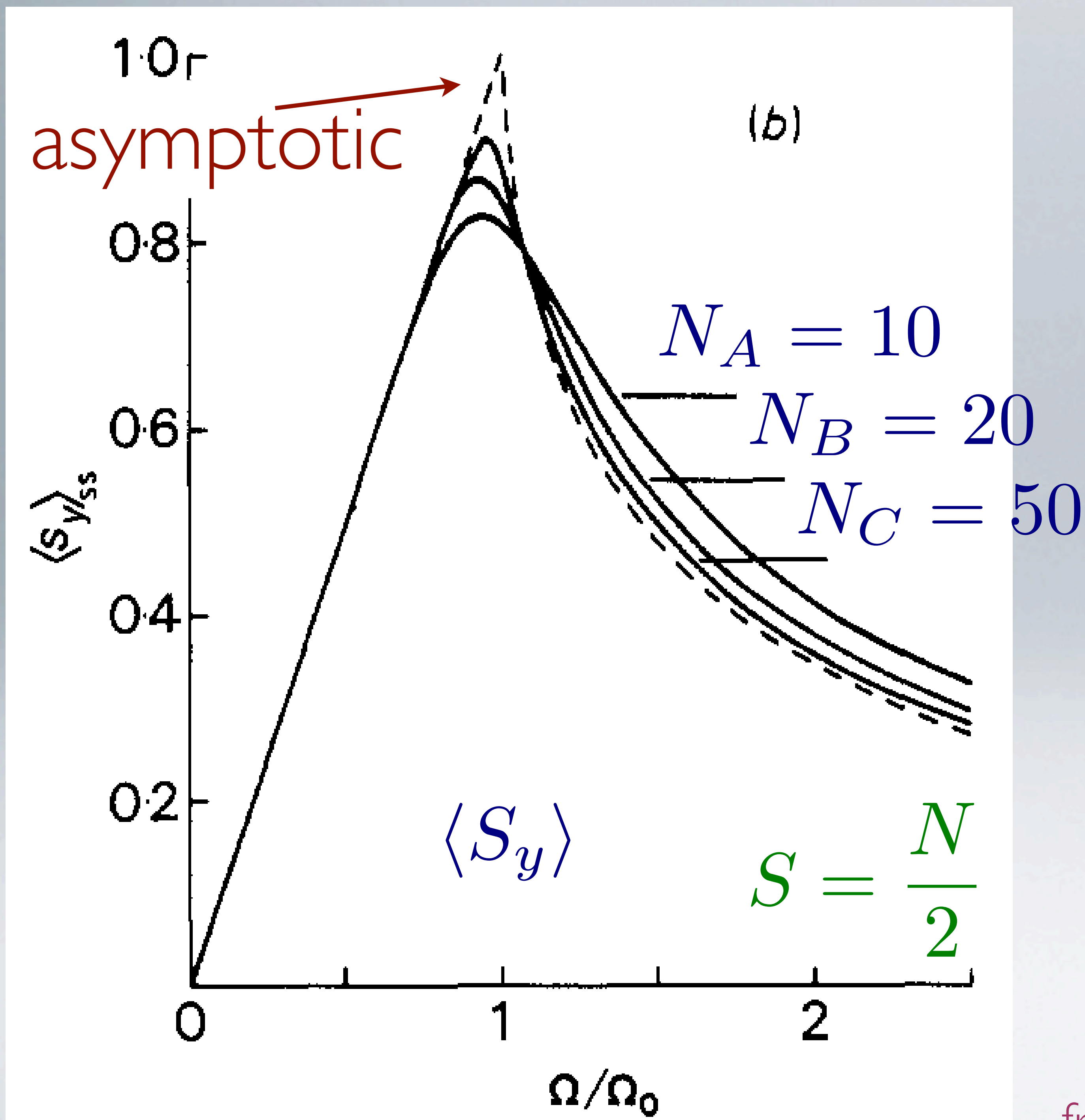

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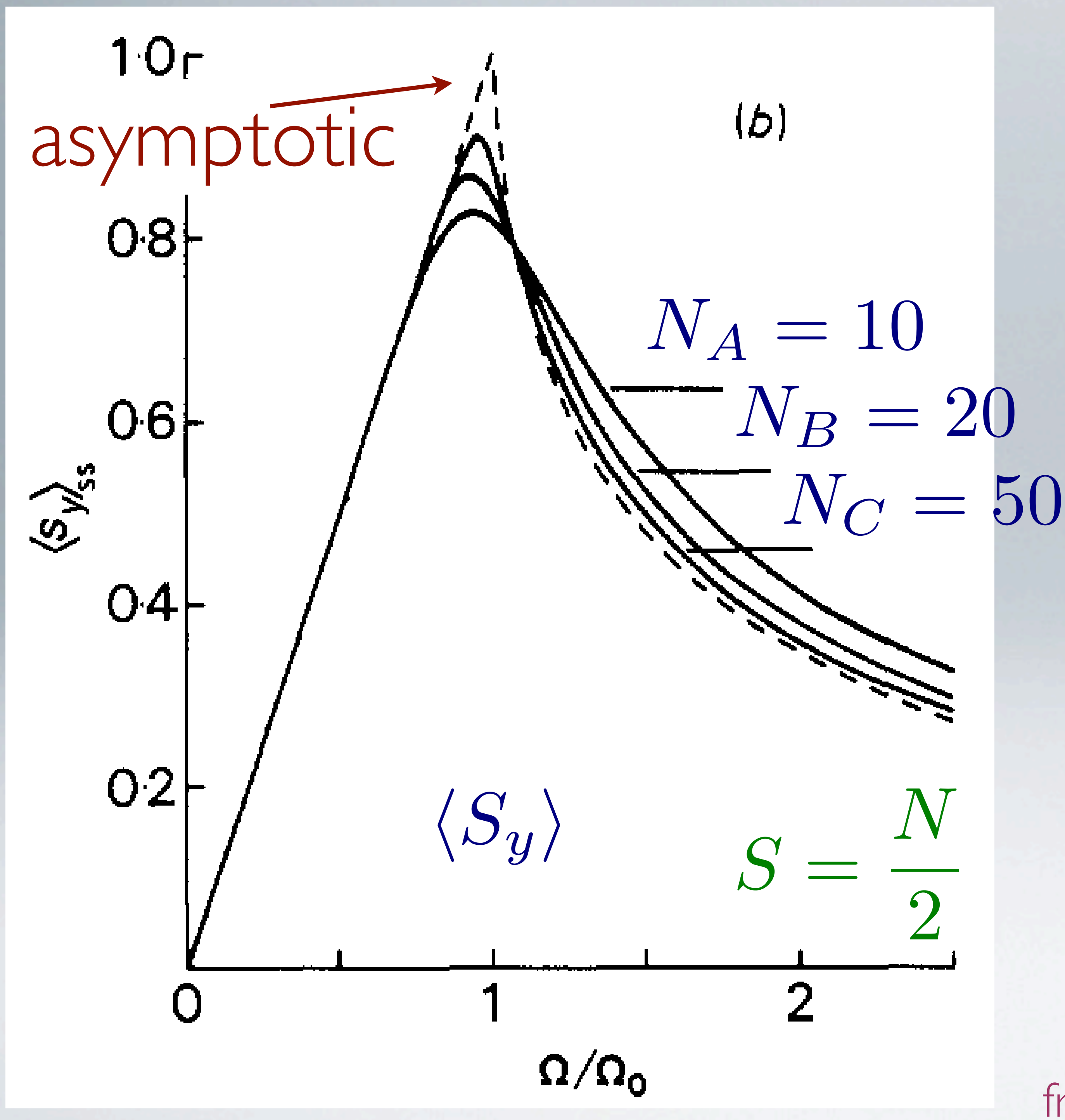

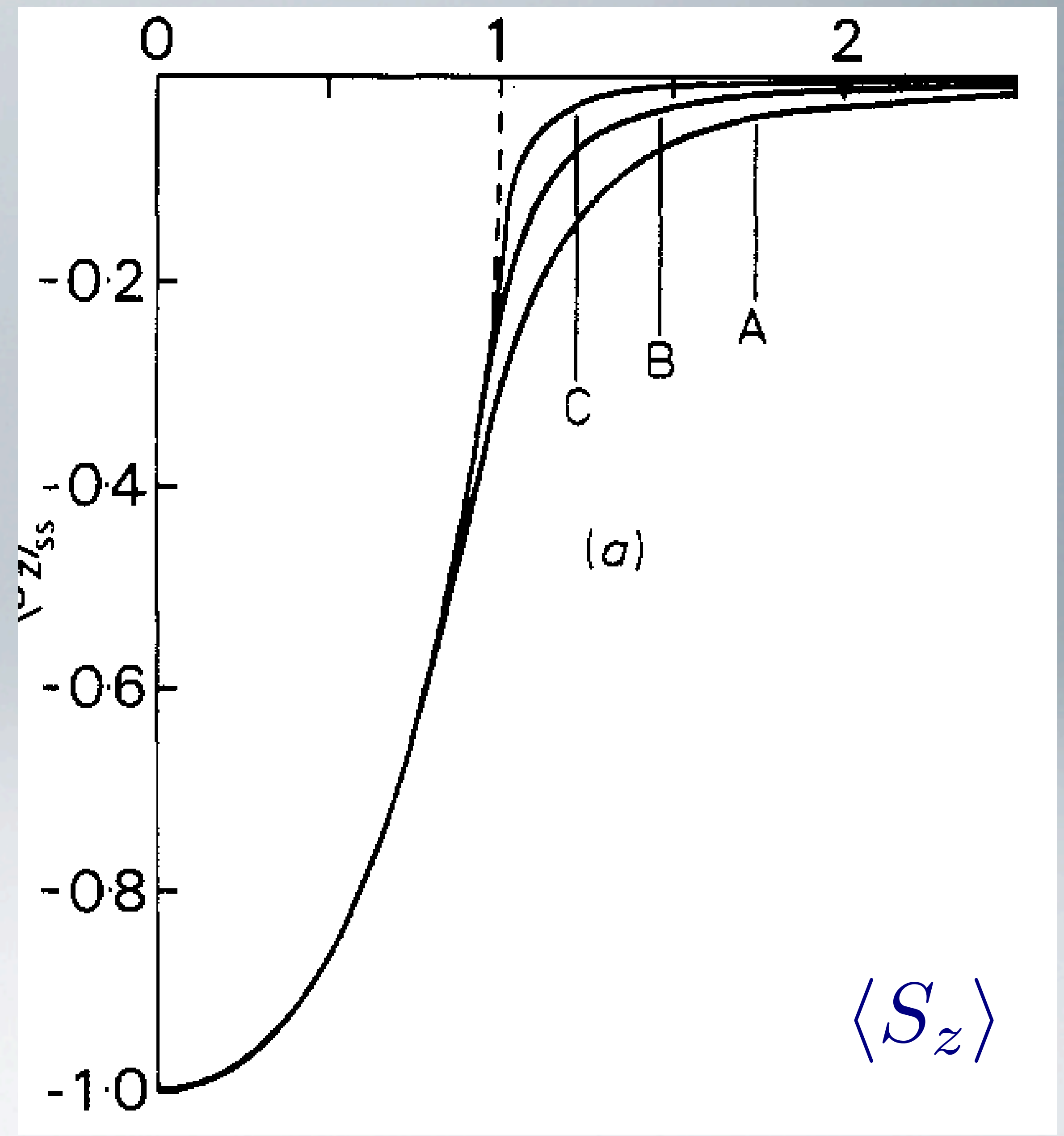

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but experimentally difficult

Baumann et al., 2010
Hamner et al., 2014
Baden et al., 2014

Do *simpler* models show similar phenomena?

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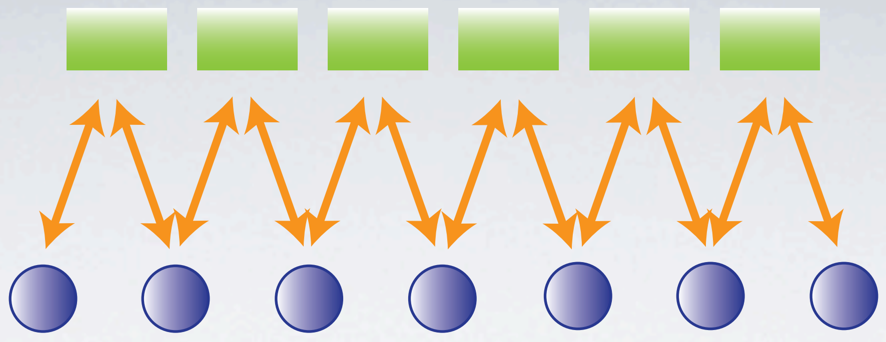
more local



A SIMPLER MODEL

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Lower dimensional version of Dicke model

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N 2-level systems with dissipation coupling NN

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$$\frac{d\rho}{dt} = -i\Omega[S_x, \rho] + \Gamma \sum_n \left(S^-_{n\ n+1} \rho S^+_{n\ n+1} - \frac{1}{2} \rho S^+_{n\ n+1} S^-_{n\ n+1} - \frac{1}{2} S^+_{n\ n+1} S^-_{n\ n+1} \rho \right)$$

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Lower dimensional version of Dicke model

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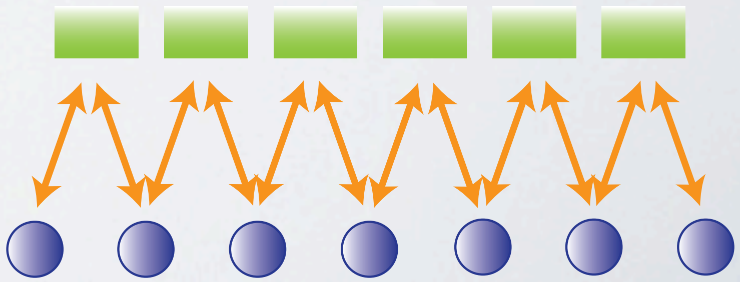
$$S^+_{n\ n+1} = \sigma^+_{n+1} \otimes I + I \otimes \sigma^+_{n+1}$$



LOW DIM DICKE MODEL

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No special symmetry

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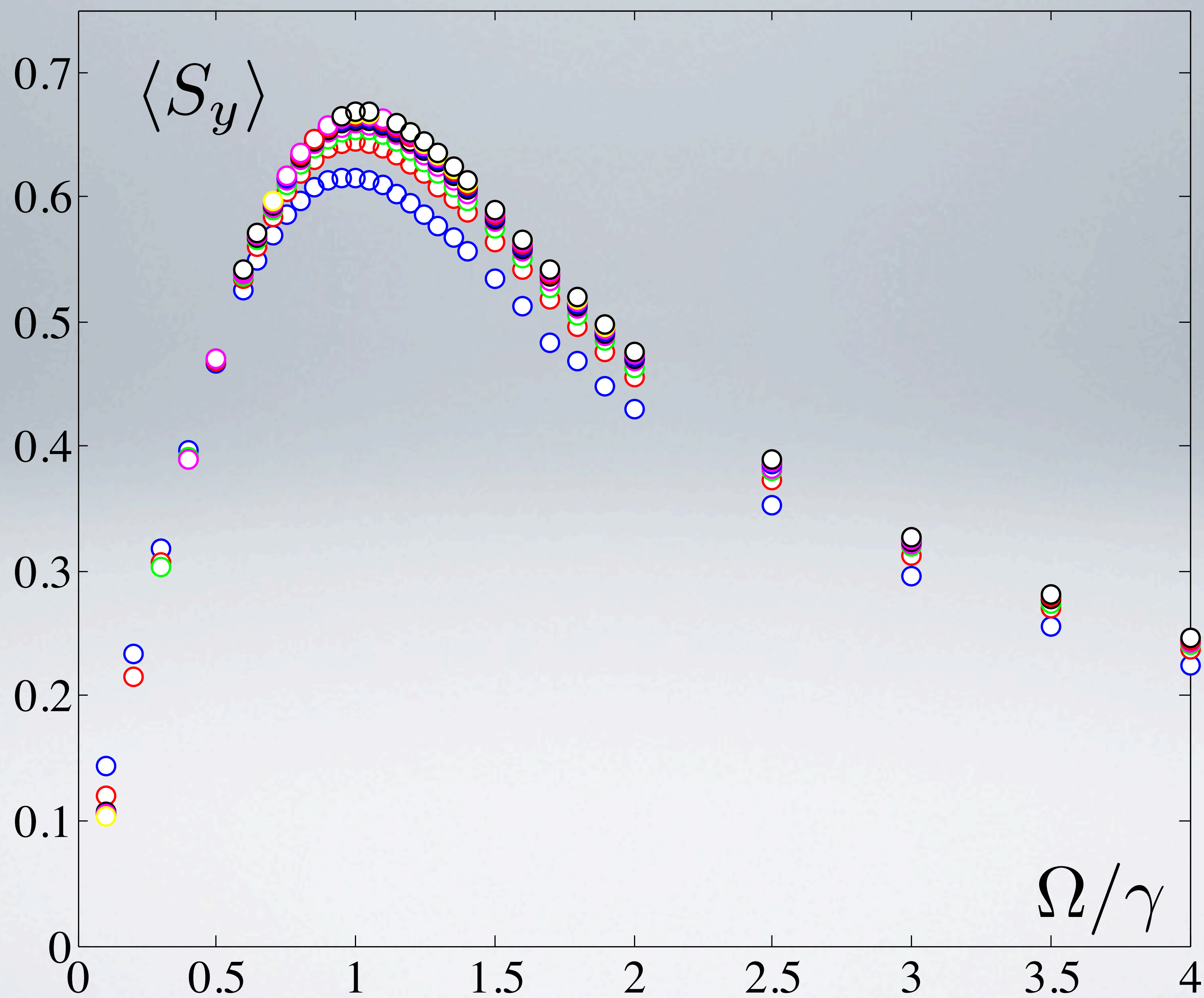

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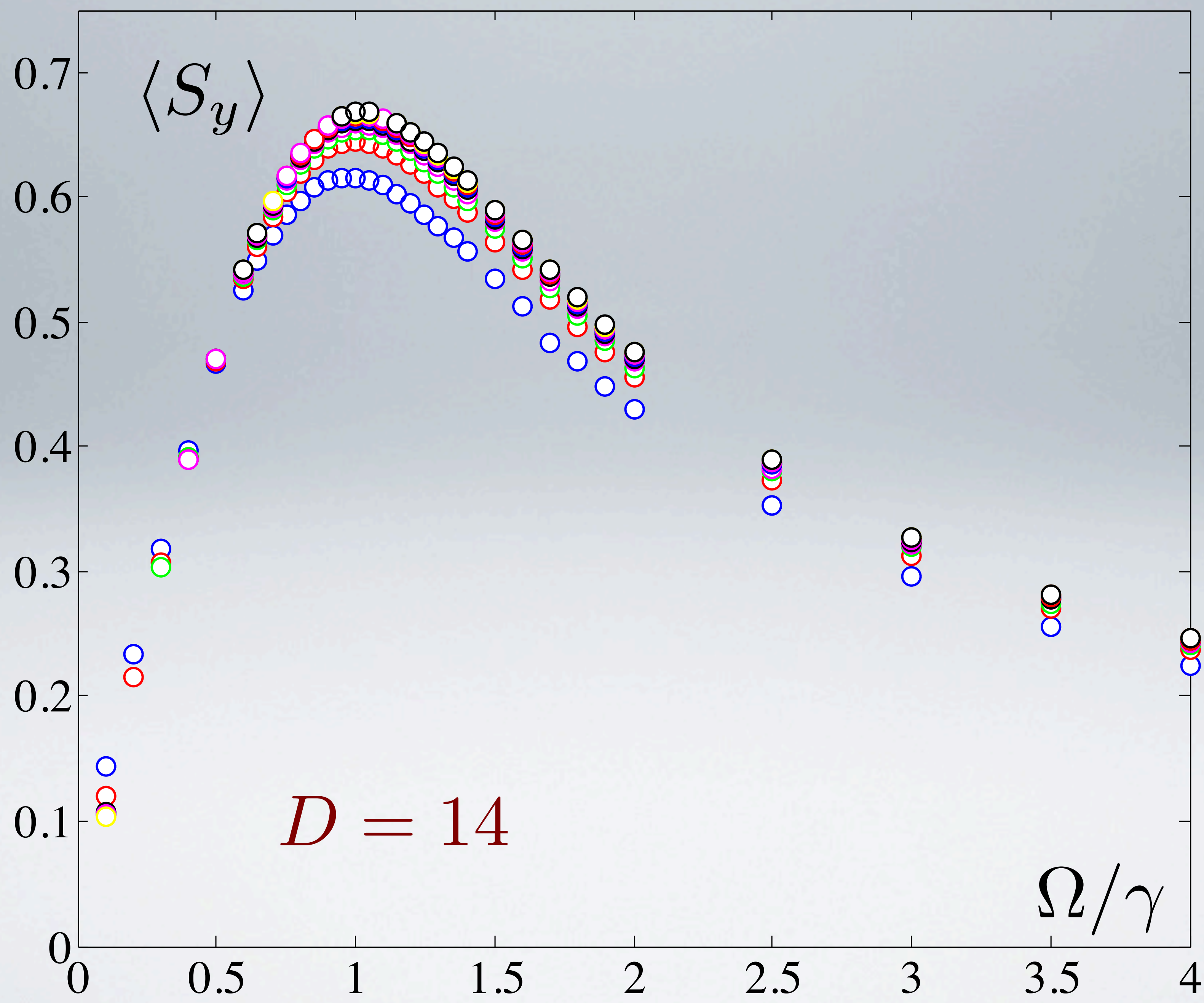

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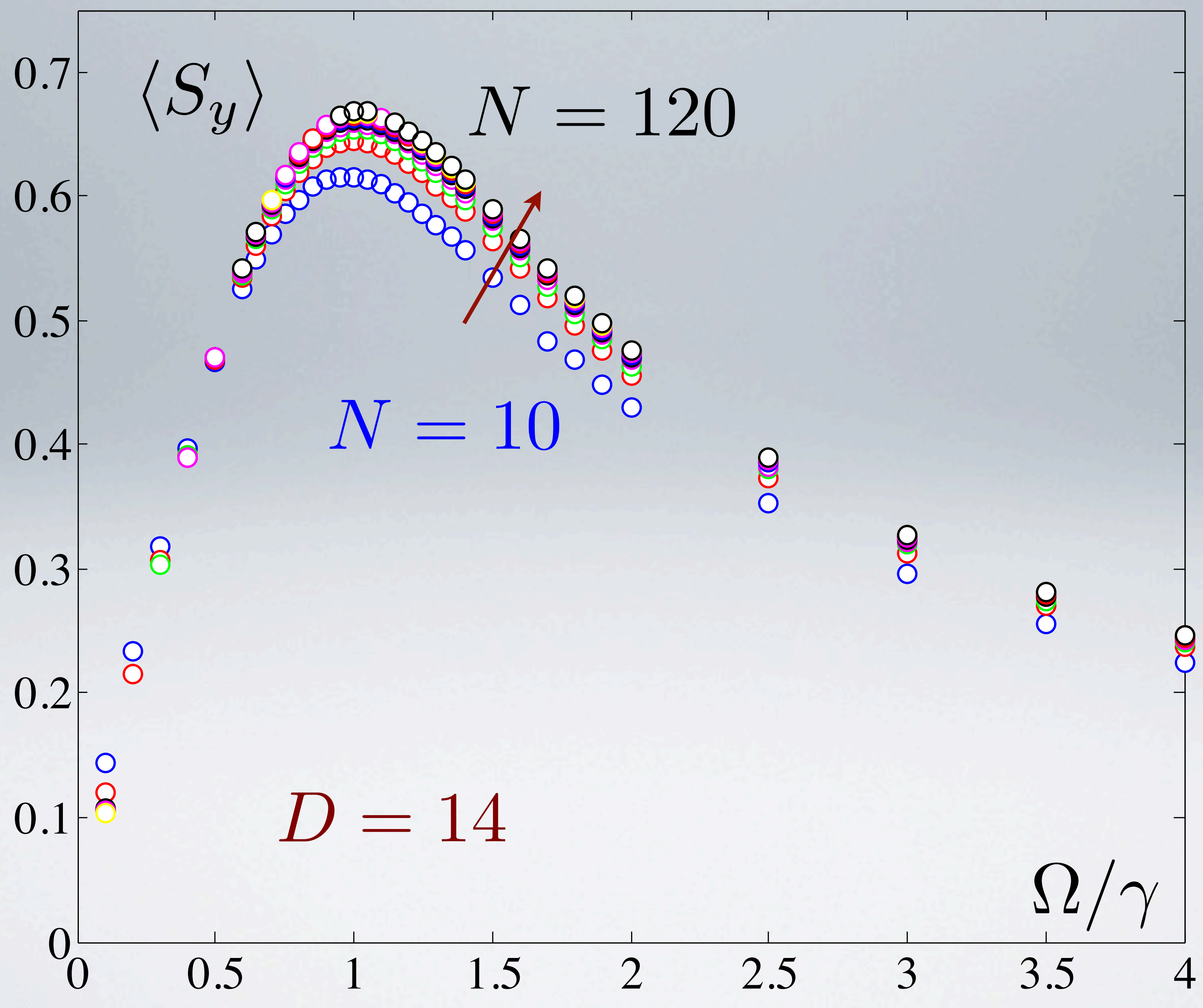

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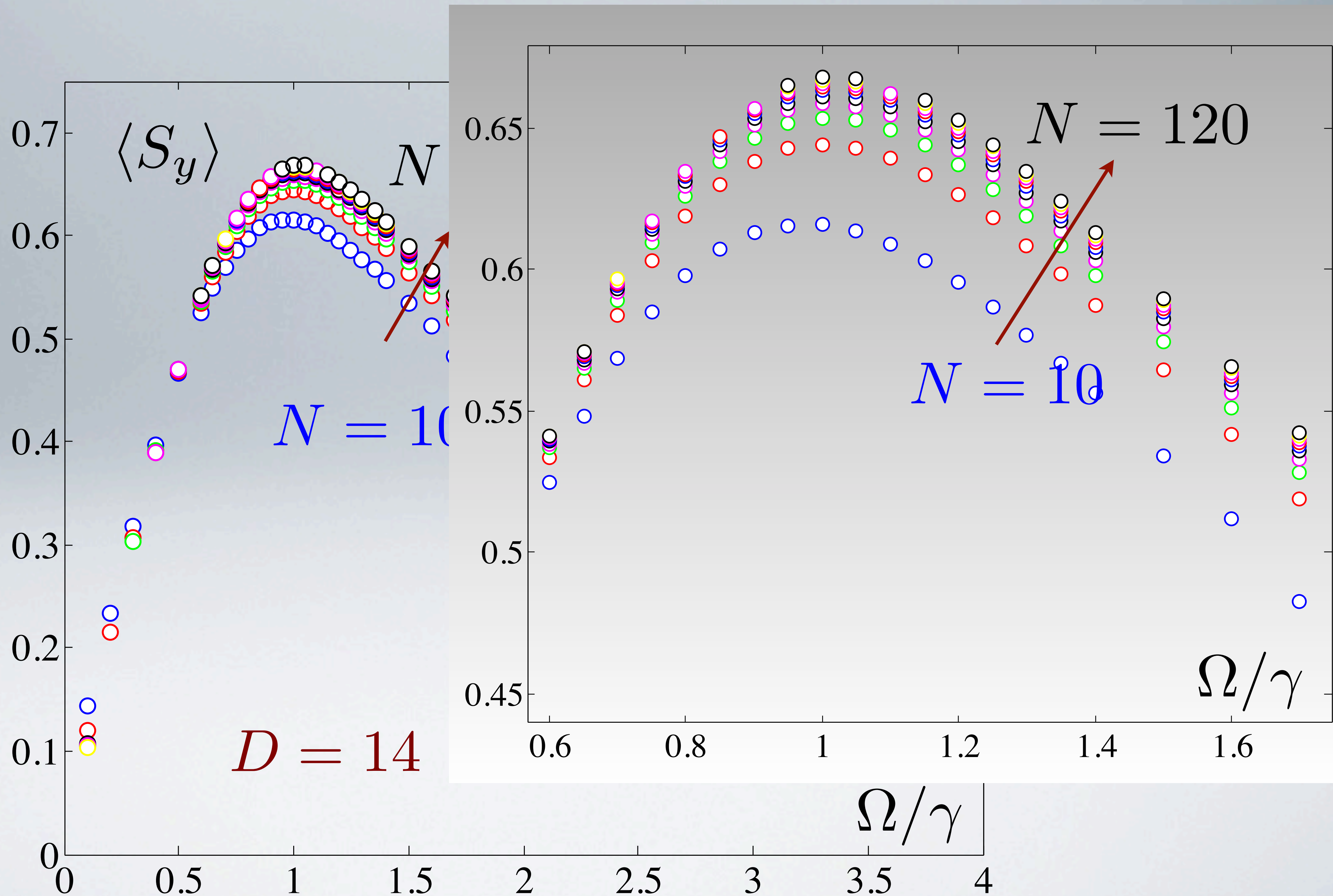

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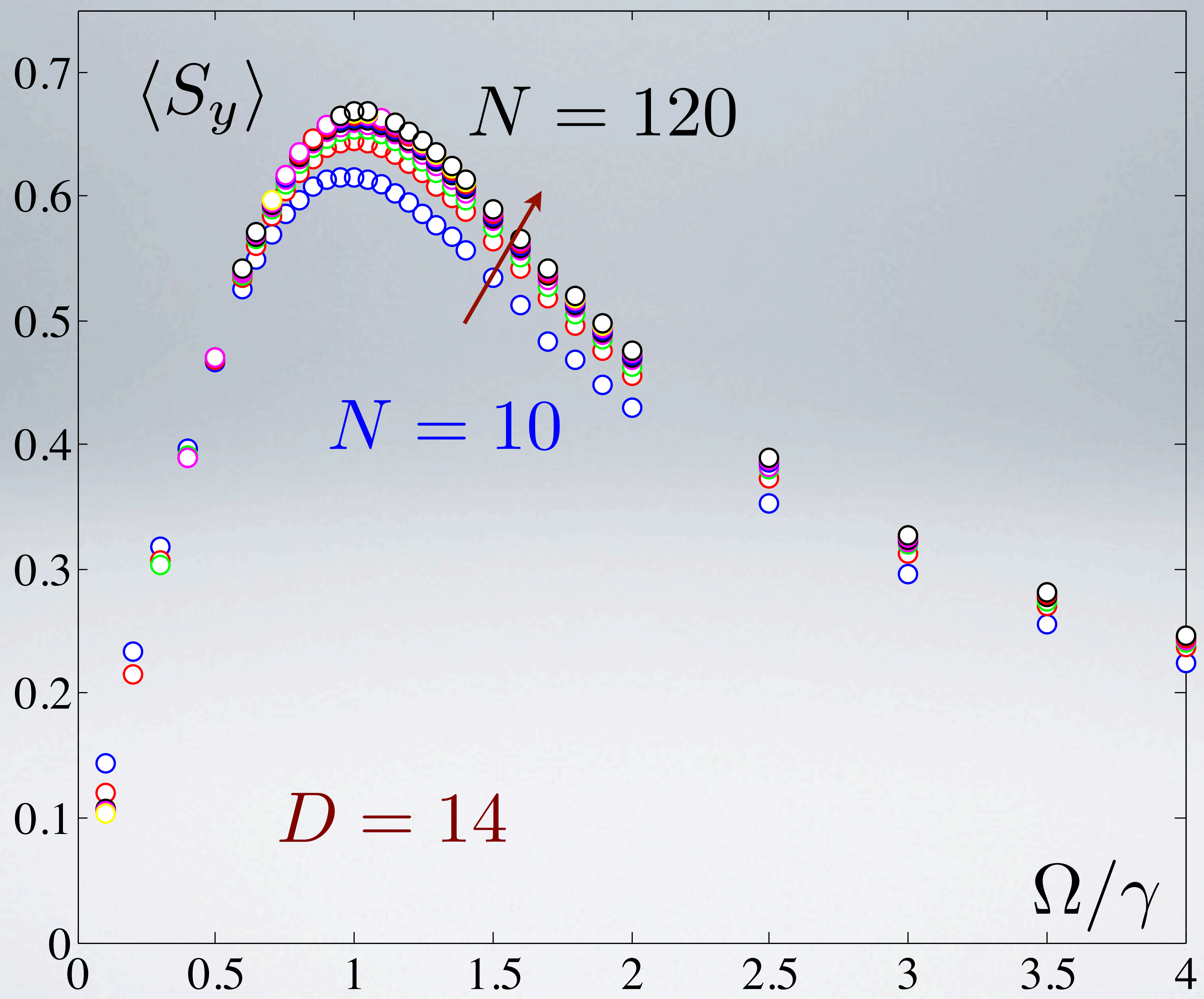

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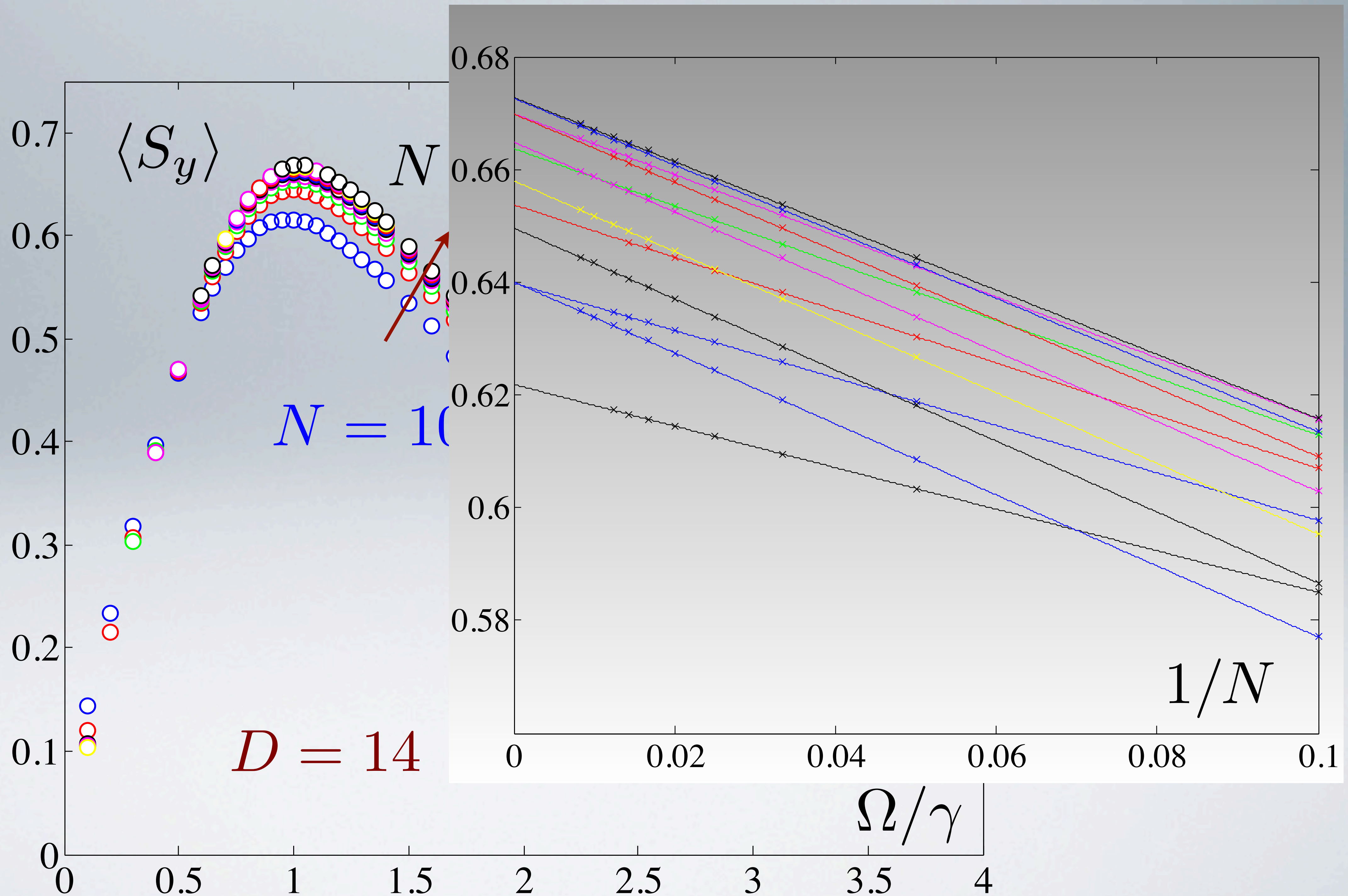

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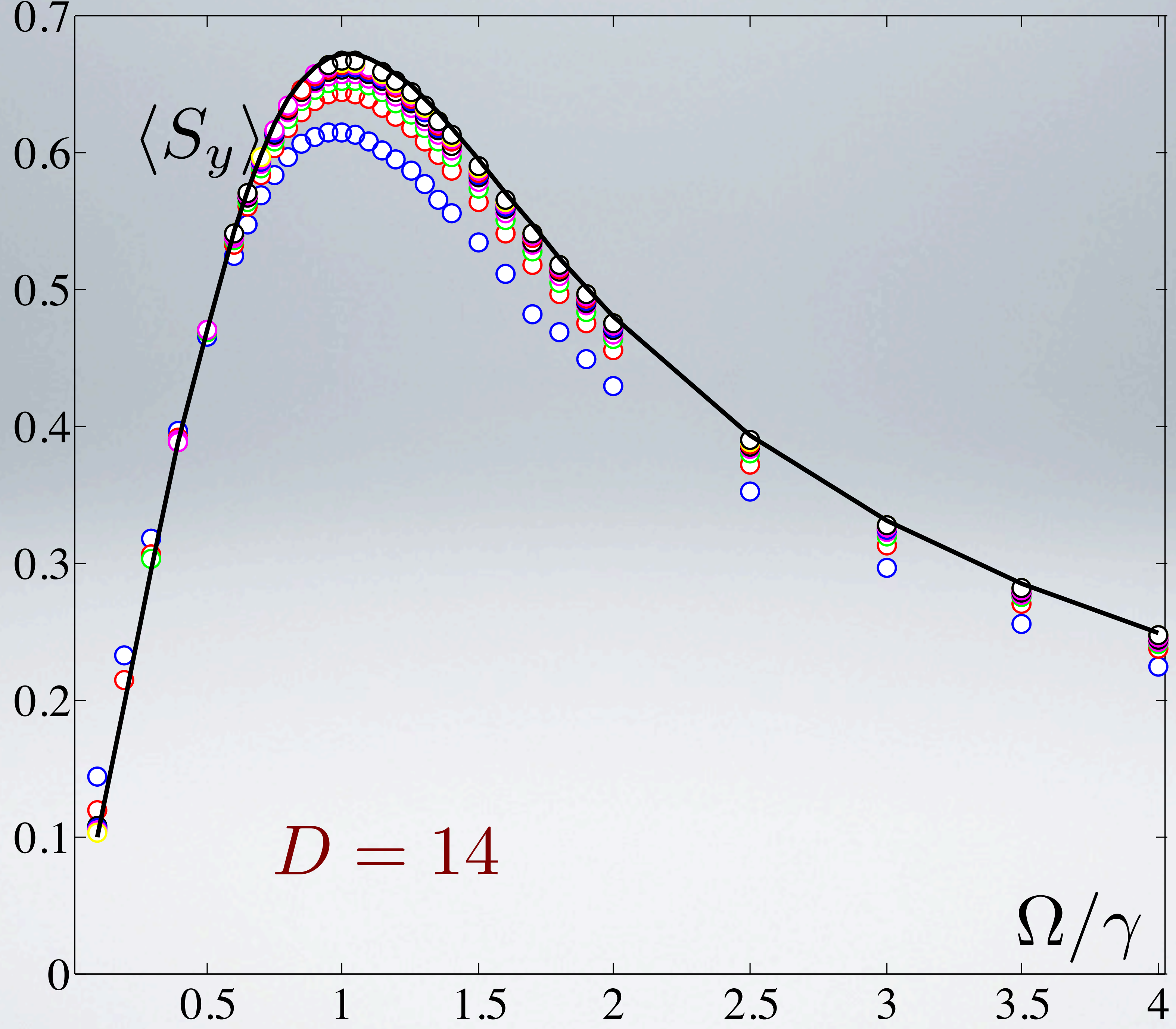

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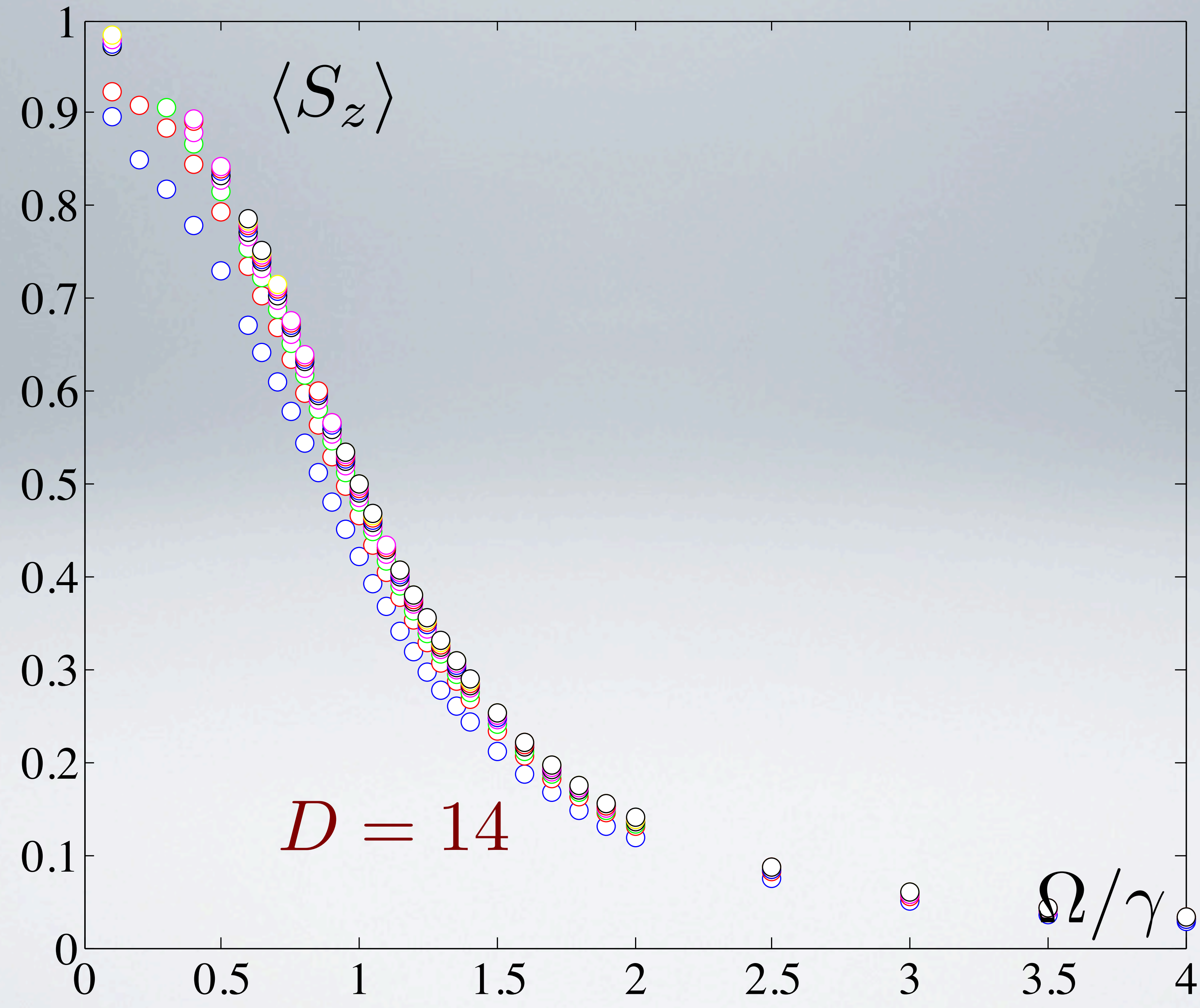

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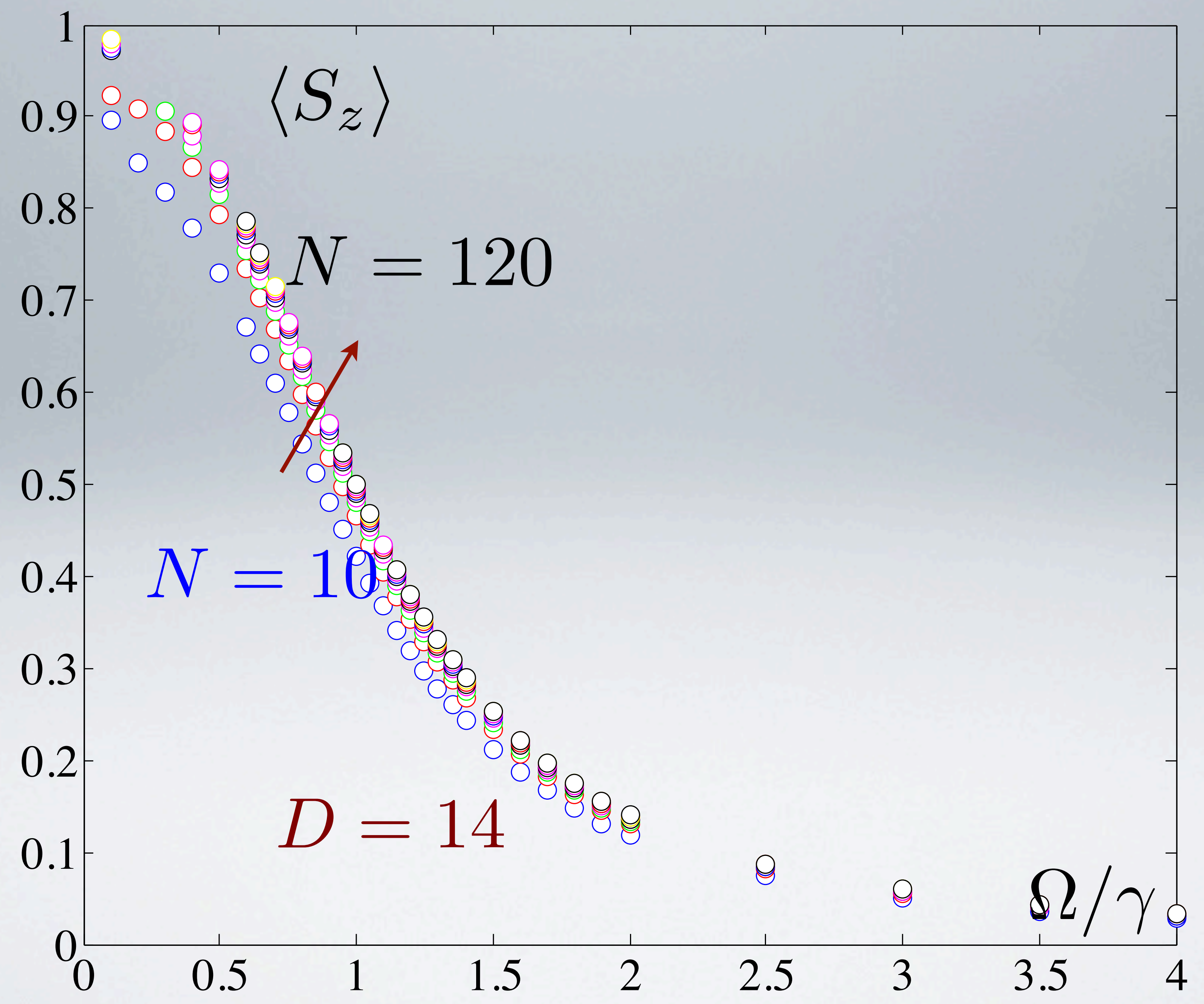

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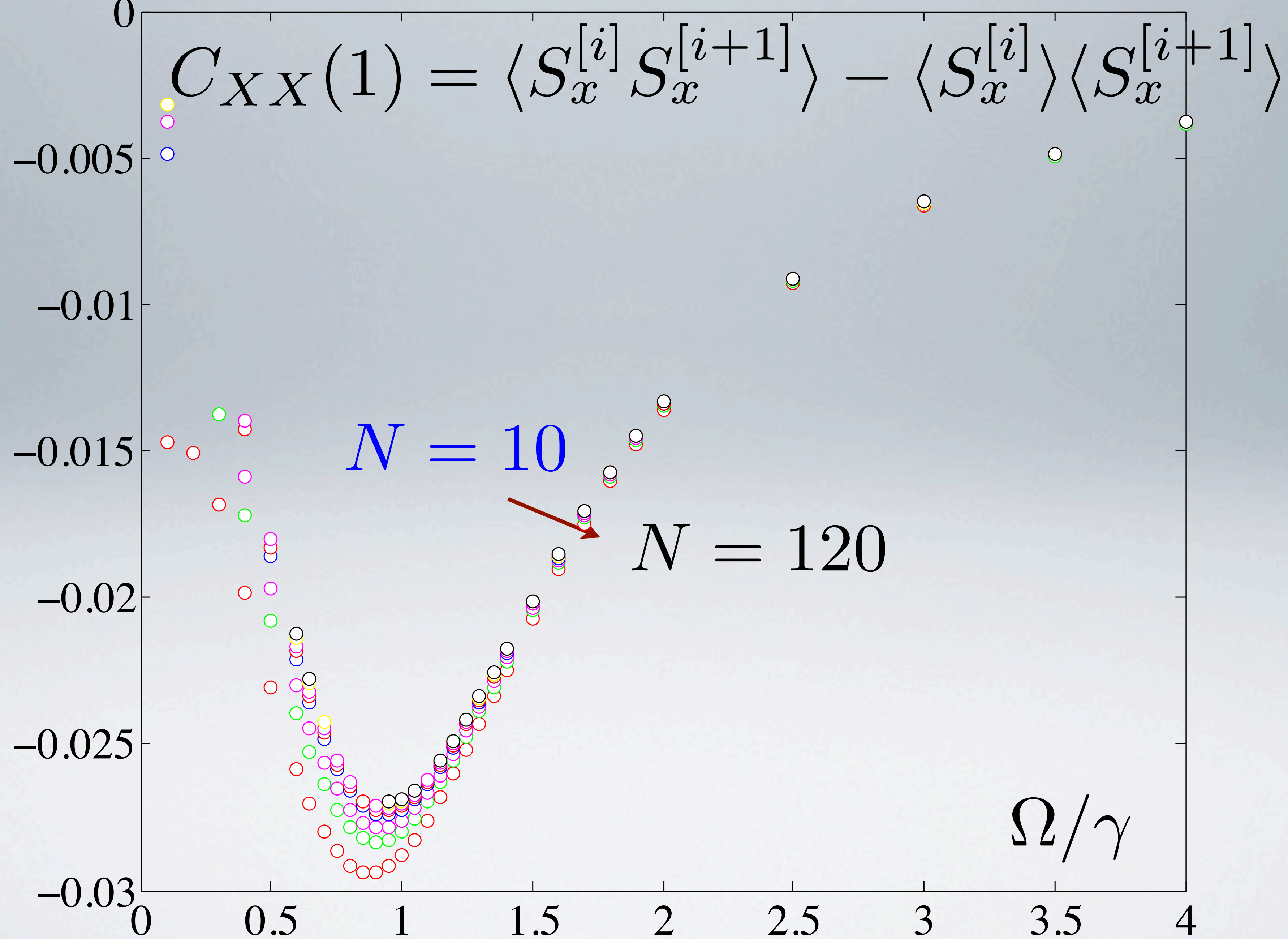

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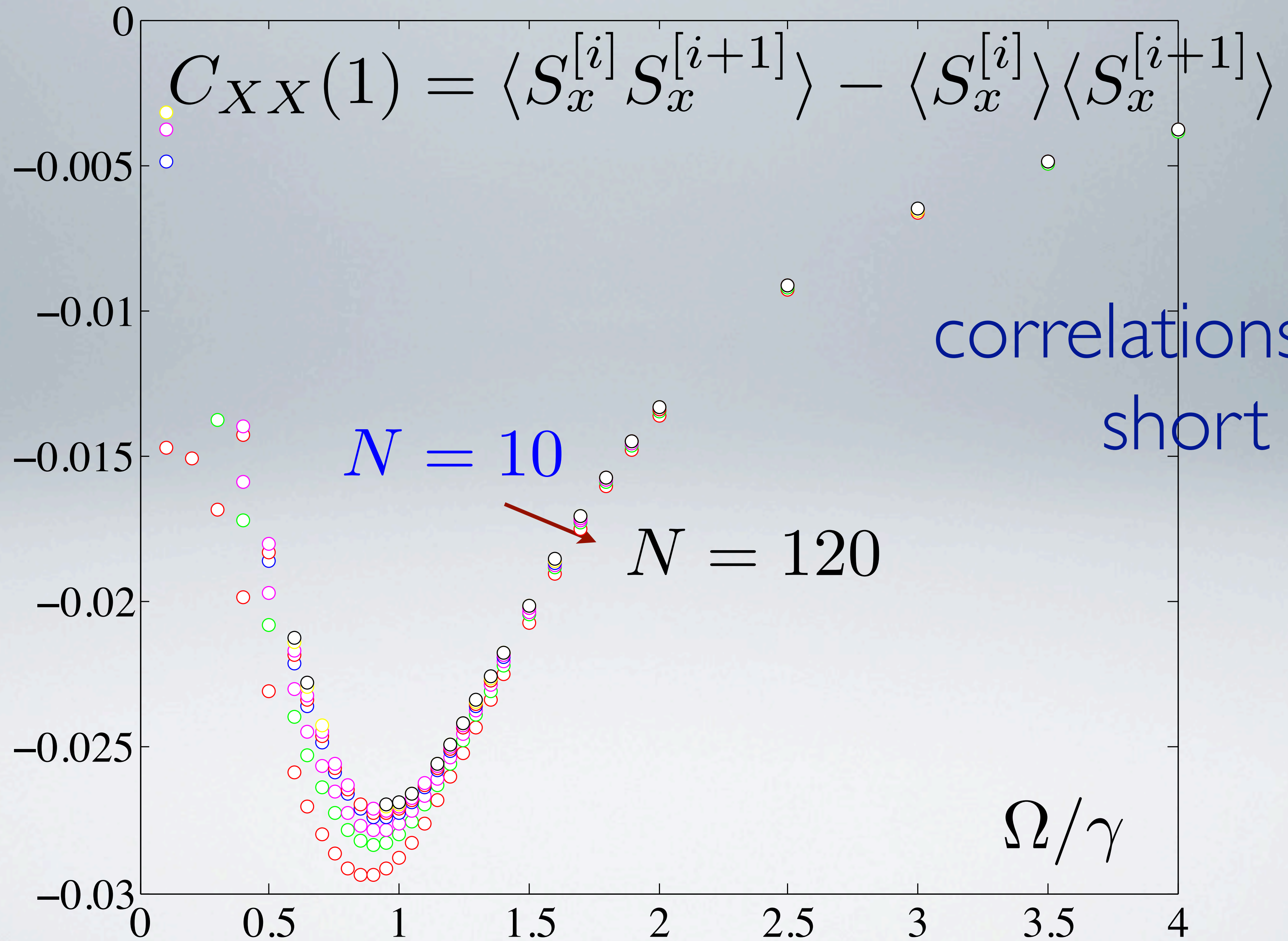

correlations, but short

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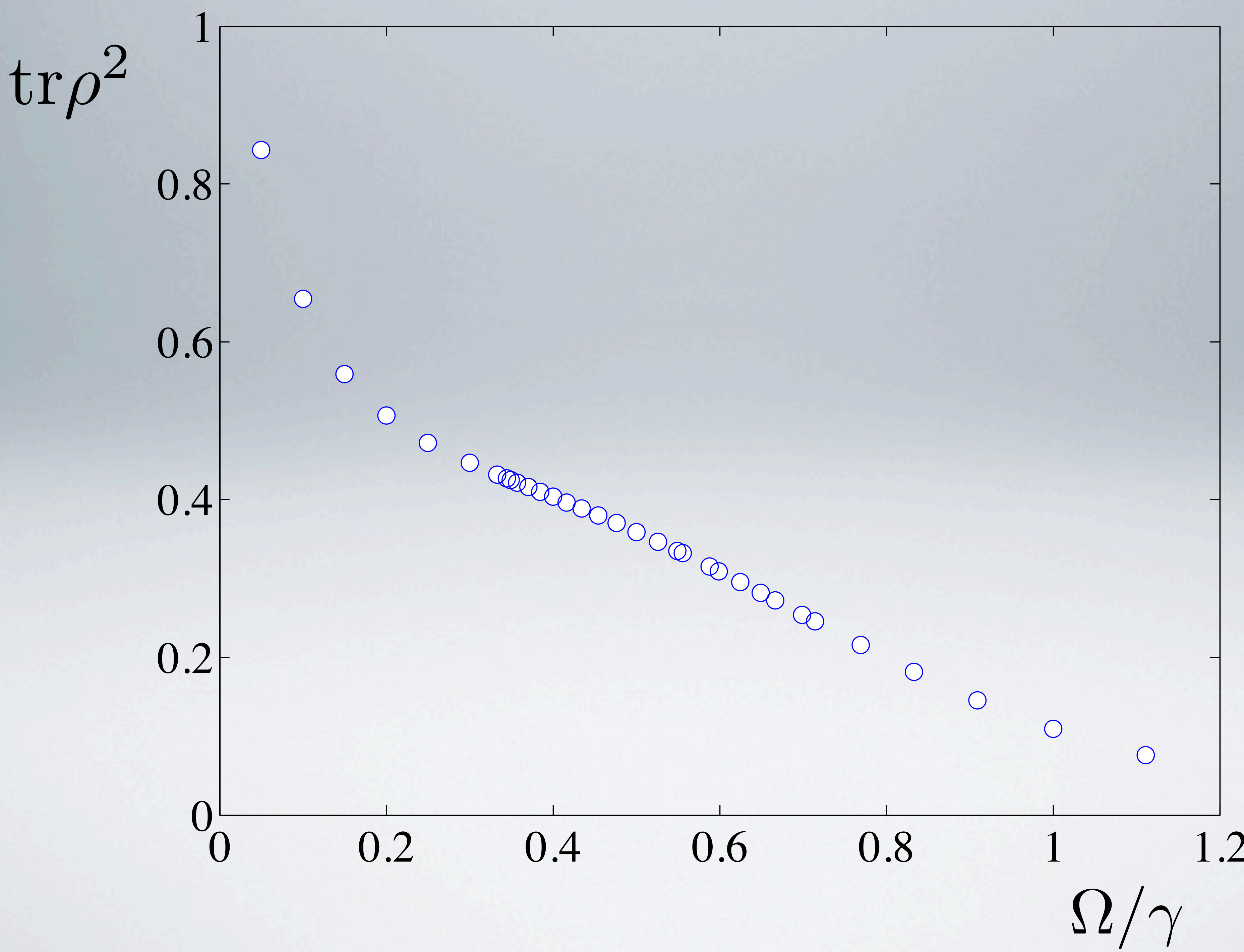

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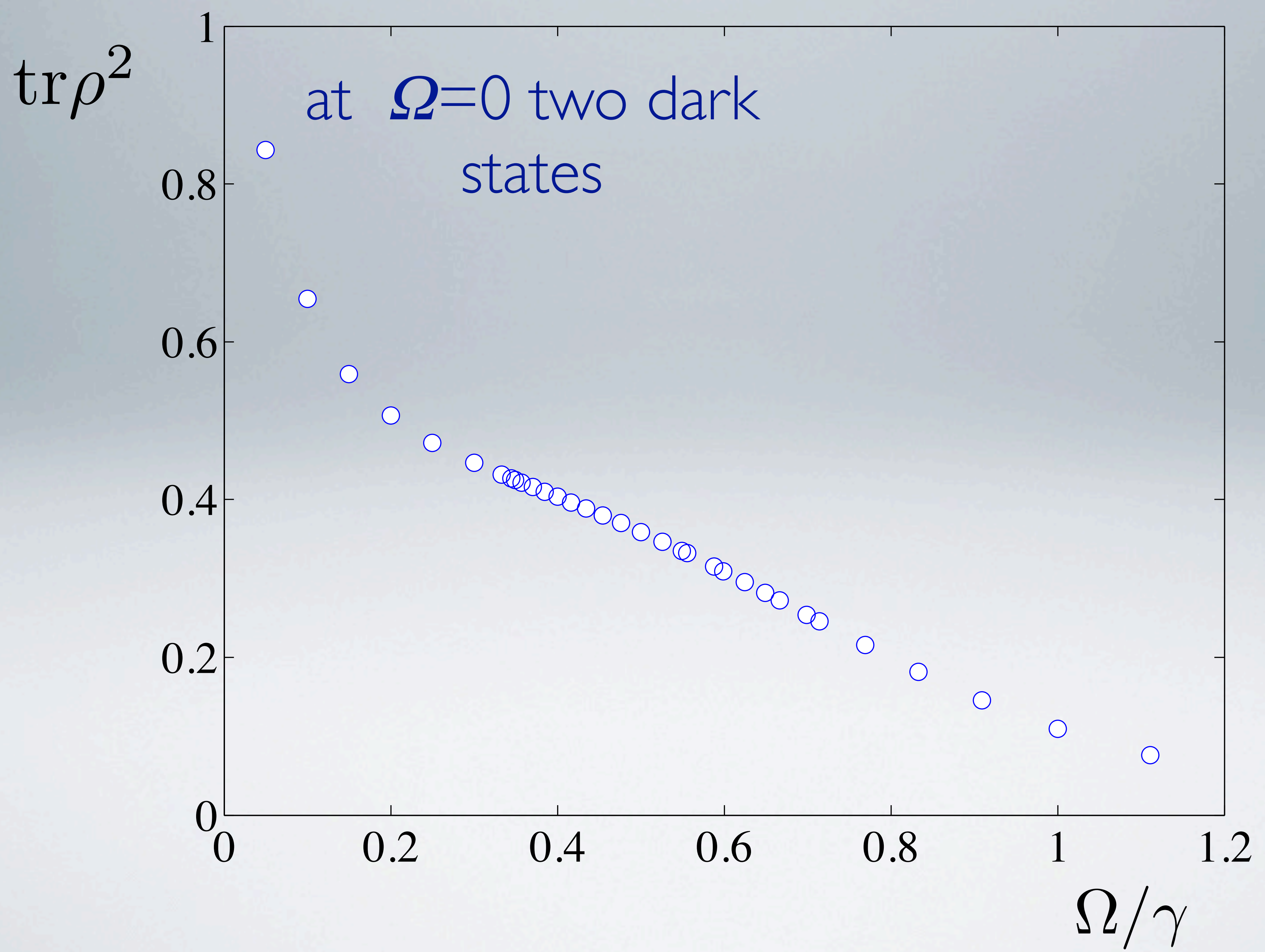

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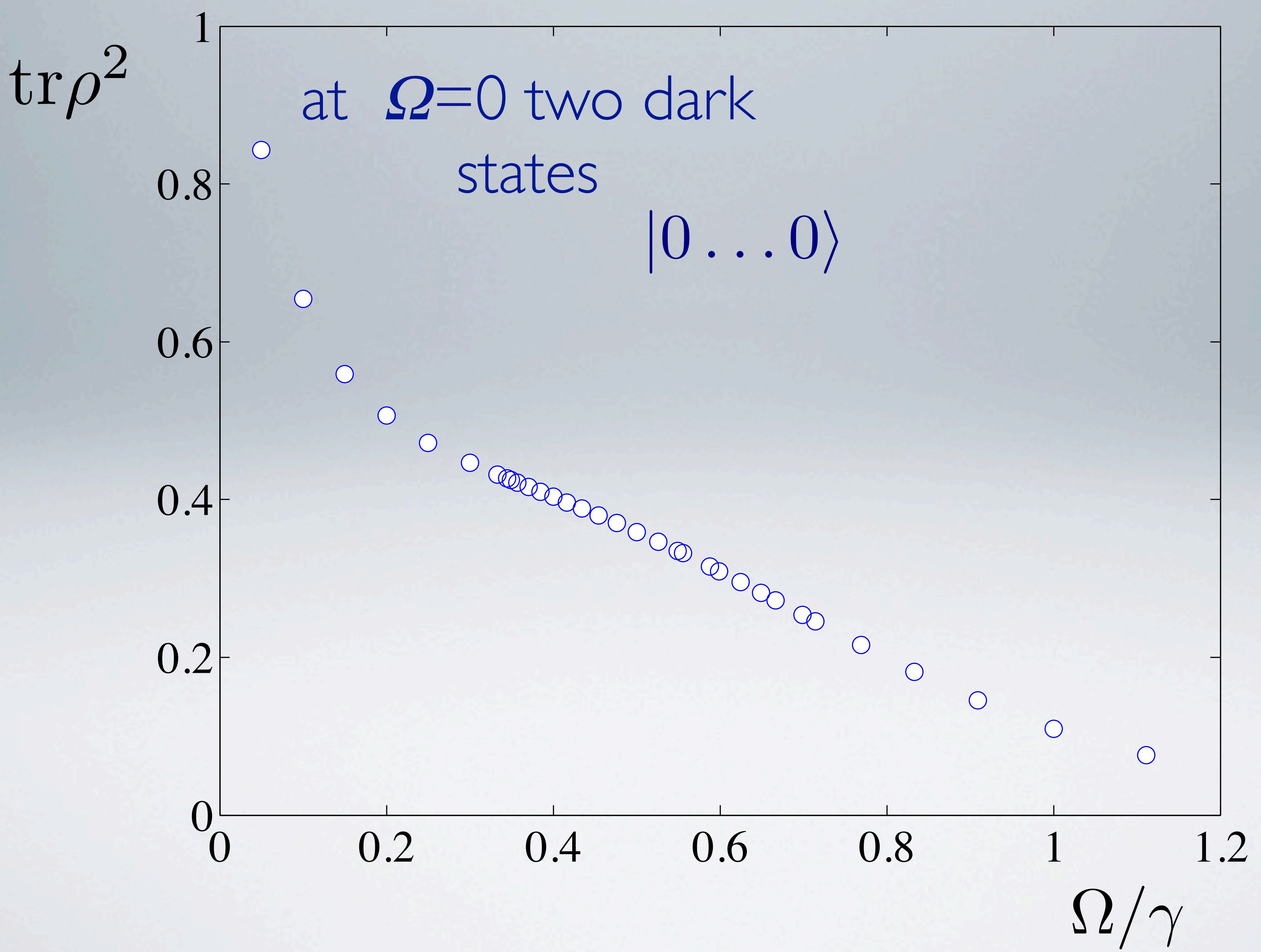

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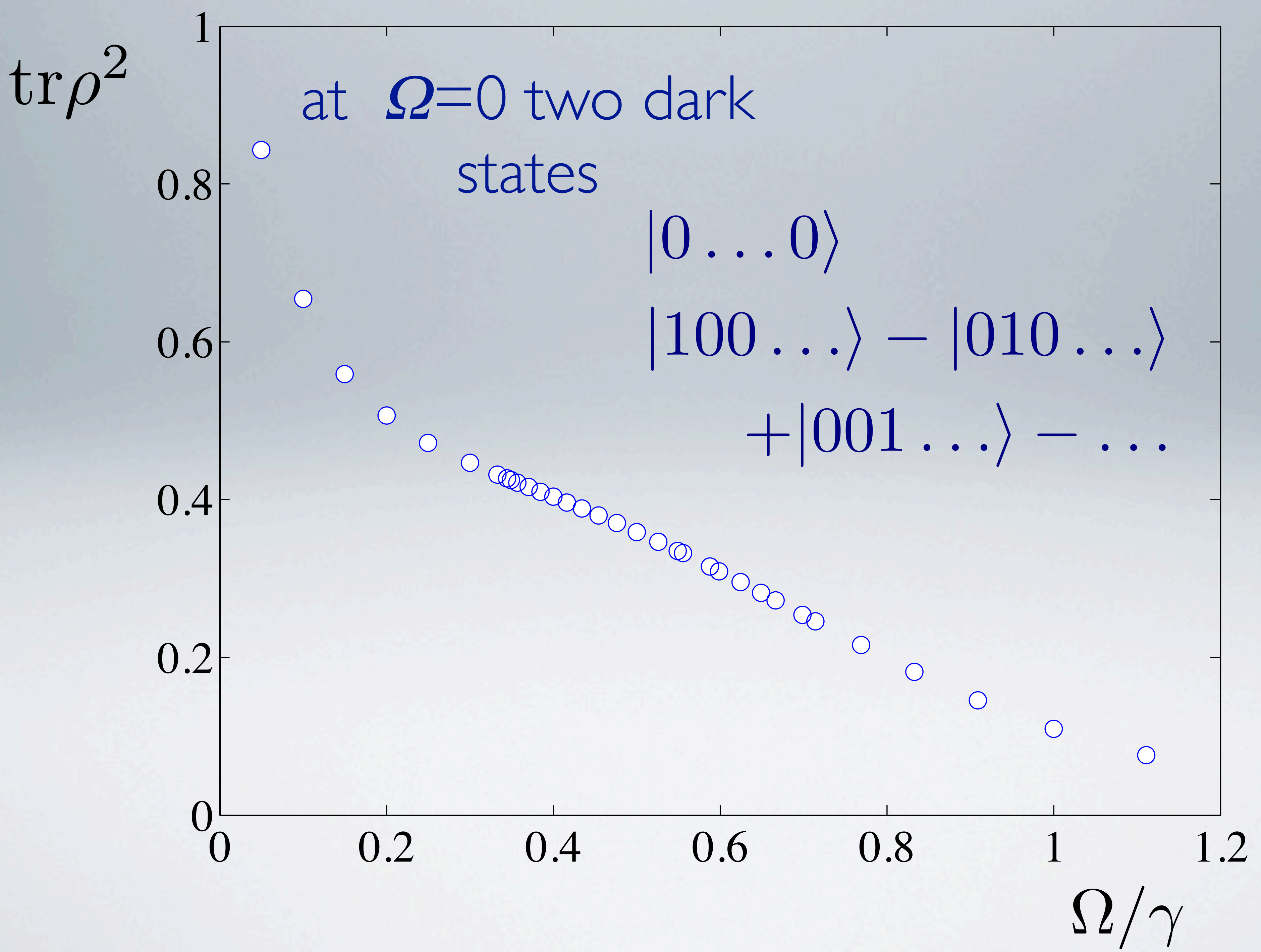

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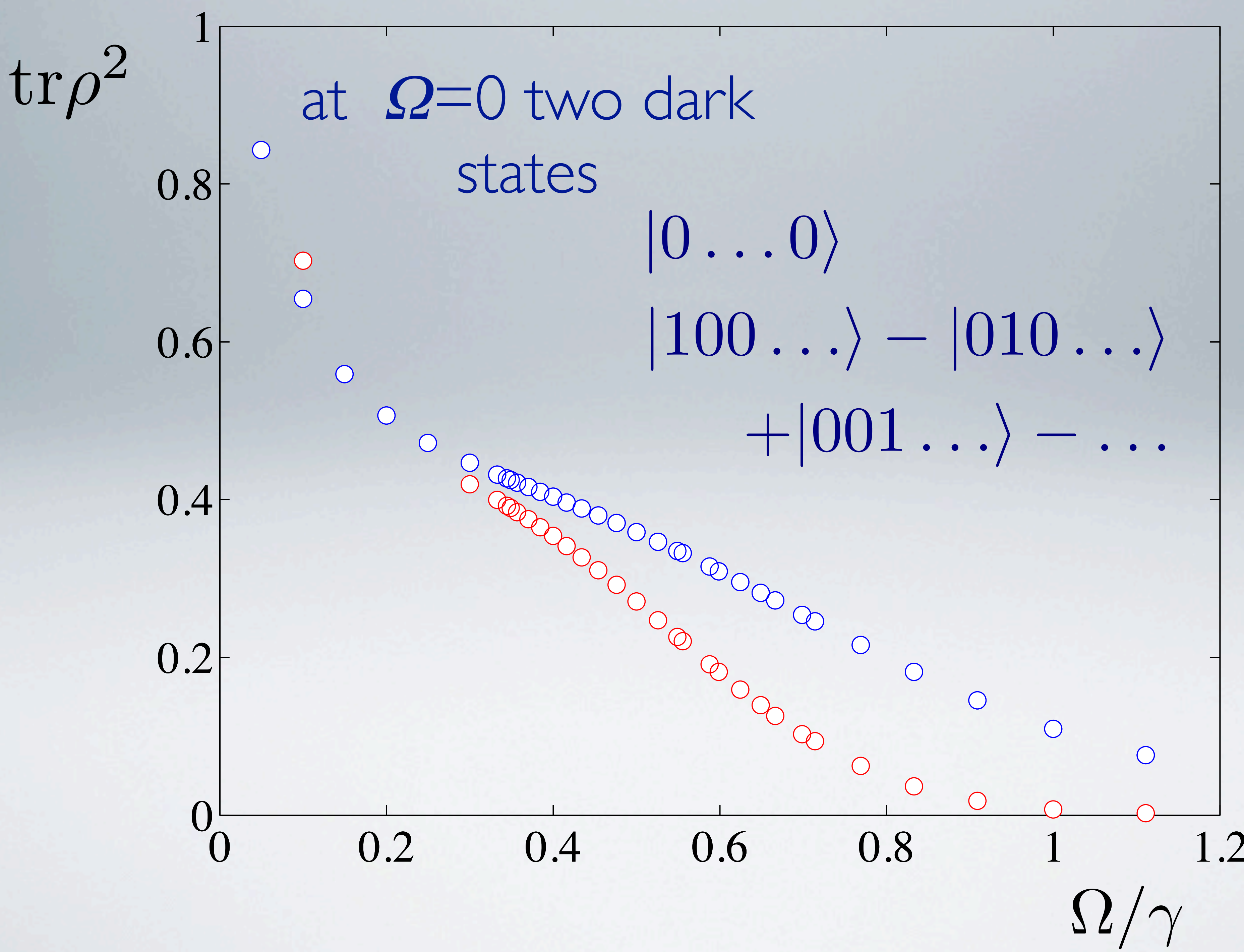

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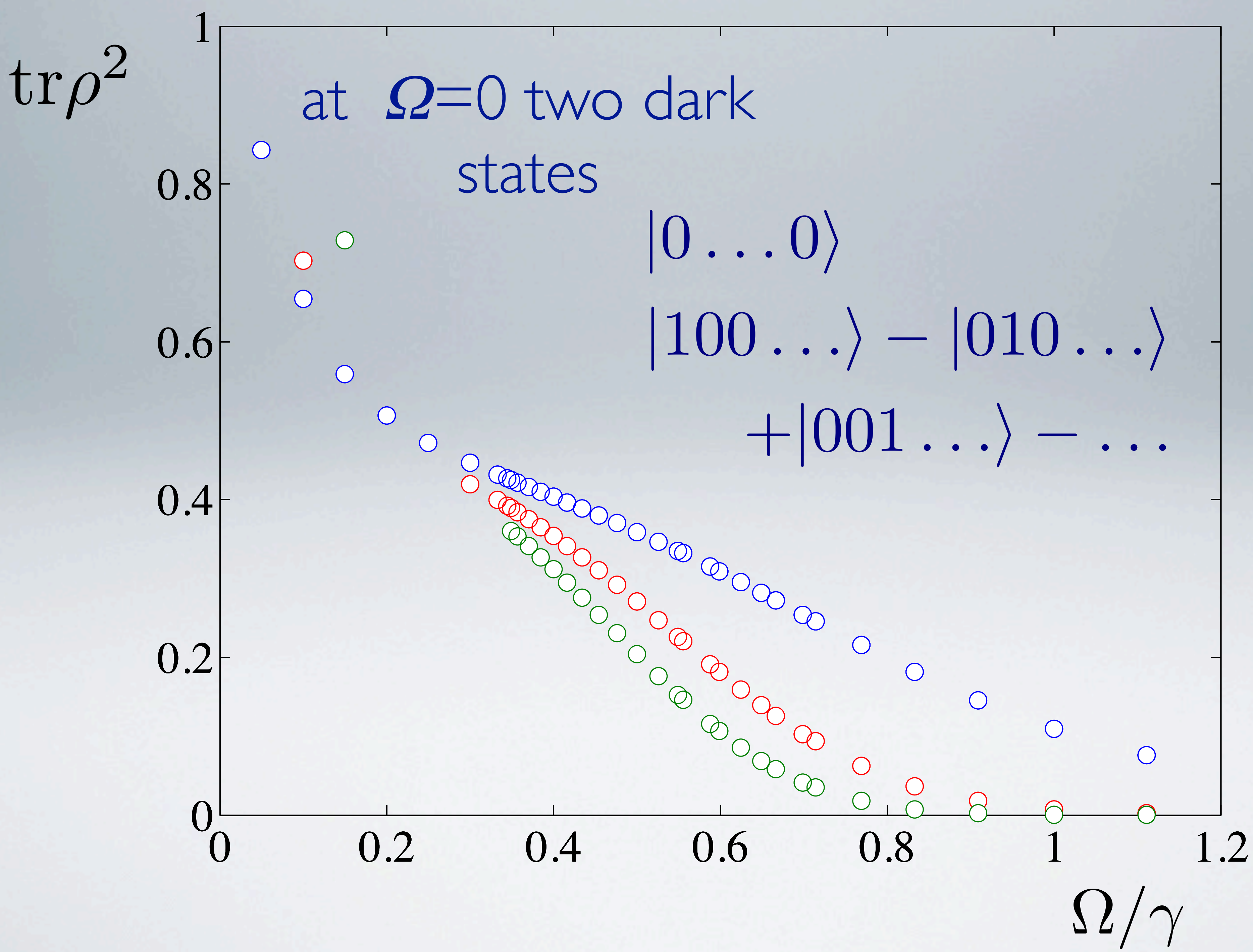

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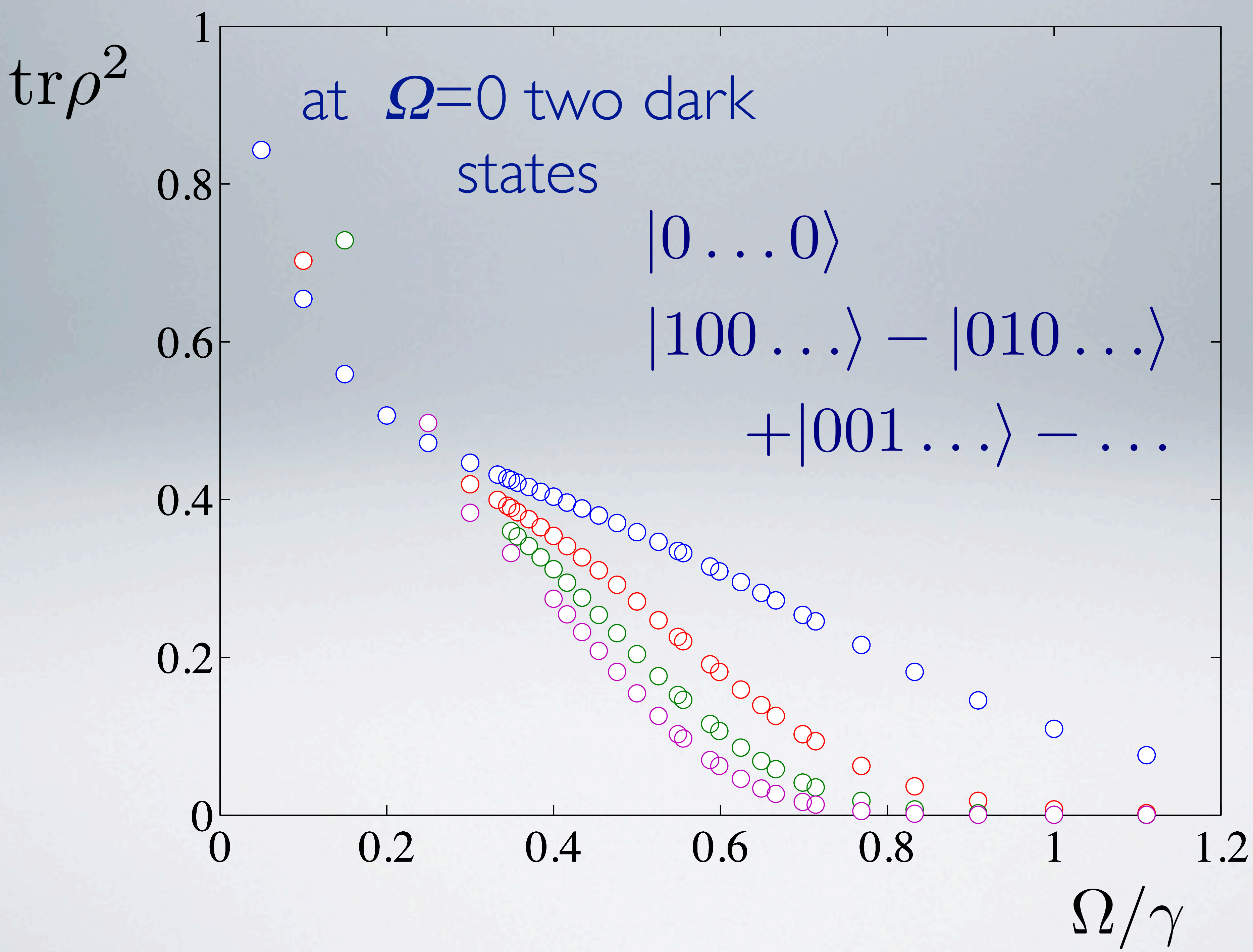

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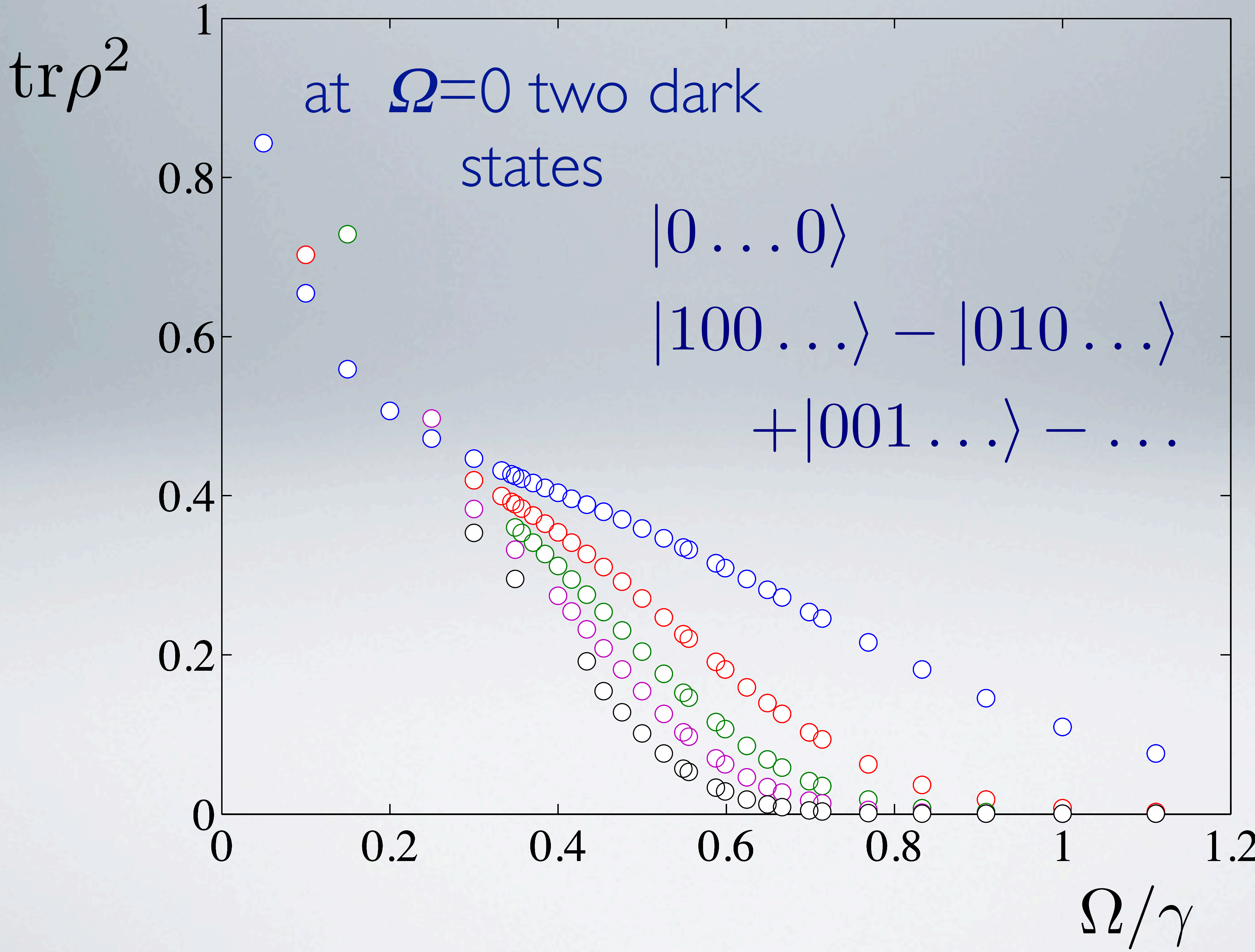

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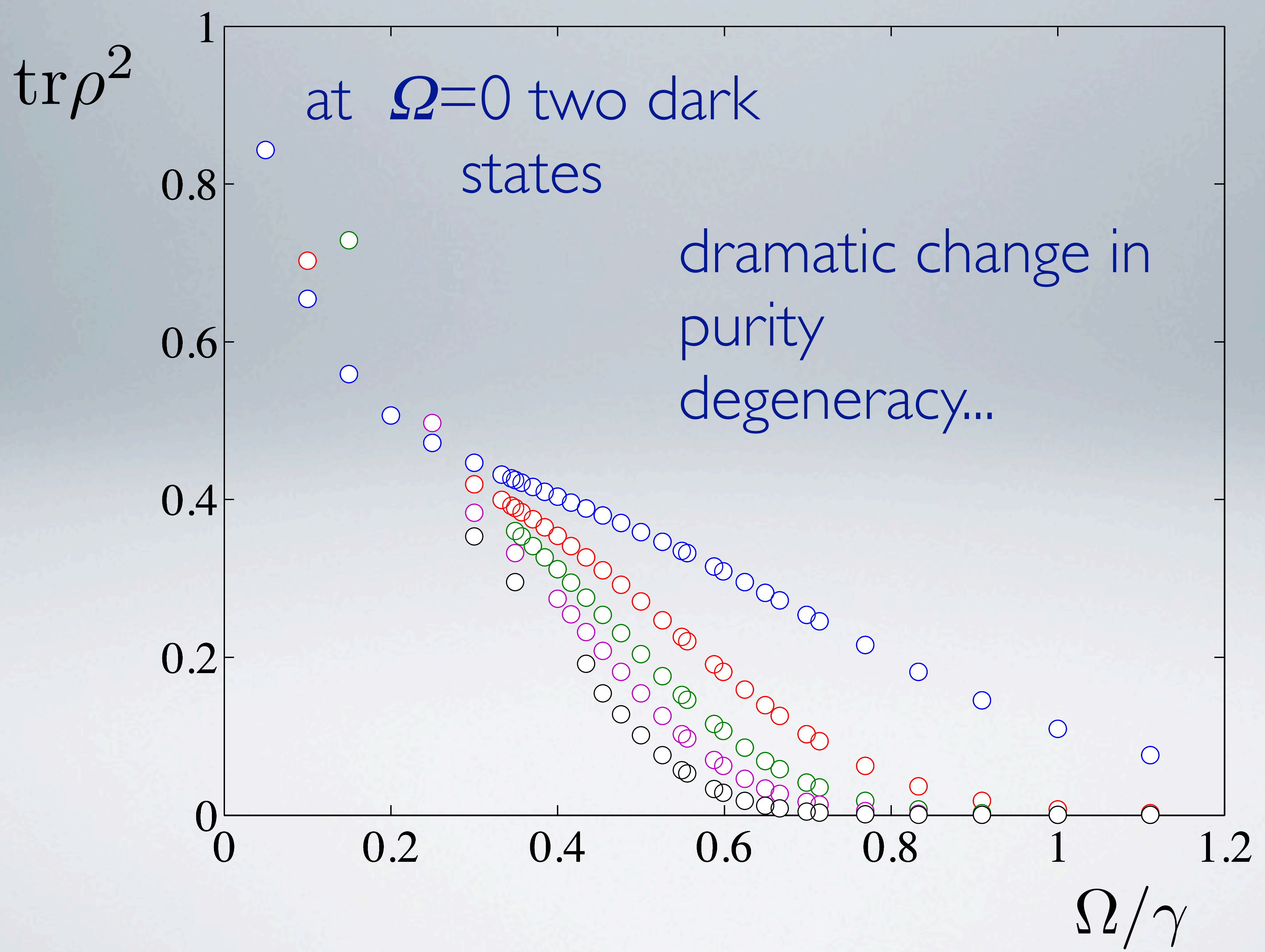

Other interesting models...

Interesting models...

SUMMARY

PRL 114, 220601 (2015)

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NESS can be found variationally

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Future: symmetries, trace one, degeneracies...

much to understand about MPDOs representations

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