Quantum gate between logical qubits in decoherence-free subspace implemented with trapped ions

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We propose an efficient technique for the implementation of a geometric phase gate in a decoherence-free subspace with trapped ions. In this scheme, the quantum information is encoded in the Zeeman sublevels of the ground state and two physical qubits are used to make up one logical qubit with ultra long coherence time. The physical realization of a geometric phase gate between two logic qubits is performed with four ions in a linear crystal simultaneously interacting with single laser beam. We investigate in detail the robustness of the scheme with respect to the right choice of the trap frequency and provide a detailed analysis of error sources, taking into account the experimental conditions. Furthermore, possible applications for the generation of cluster states for larger numbers of ions within the decoherence-free subspace are presented.

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I. INTRODUCTION

Trapped ions are among the most promising physical systems for implementing quantum information due to the long coherence time [1]. The robust quantum memory is a crucial part of the realization of an ion trap based quantum computer [2]. One can clearly distinguish between two possibilities for encoding a qubit in a trapped ion: First, one can use a long lived metastable state and drive coherent transitions on the corresponding optical transition [3], which sets very challenging requirements on the laser source and ultimately limits the coherence time to the lifetime of the metastable state. Second, a qubit can be encoded in sublevels of the electronic ground state. It can be encoded either in hyperfine ground state levels [4] or Zeeman ground states [5] and the coherent manipulations are performed by means of stimulated Raman transitions. For hyperfine levels, so called *clock states* can be used whose energy difference exhibits very small Zeeman shift. The quantum information is therefore retained in the presence of magnetic field fluctuations [6]. An elegant alternative to improve the stability of the qubits is encoding two physical Zeeman qubits in one *logical qubit* and use the decoherence-free subspace (DFS) [7, 8, 9] of odd Bell states as the computational basis: $|0\rangle \equiv |\uparrow\downarrow\rangle$ and $|1\rangle \equiv |\downarrow\uparrow\rangle$. As the basis states have equal components from both spin levels, a magnetic field fluctuation will add no extra phase to any superposition $\alpha|\uparrow\downarrow\rangle+\beta|\downarrow\uparrow\rangle$ of these states. With current available technology, entangled states between two ions with a coherence time of up to 20 s and single qubit operations in DFS have been demonstrated [10, 11]. Universal set of single and two qubit operations between logical qubits has been performed [12]. The necessary overhead to handle two ions is negligible compared to the efforts that would be needed to implement error correction [13]. Segmented micro ion traps [14, 15, 16] pave the way to operations with a large number of ions. The remaining challenge is therefore to find a simple scheme to carry out two-qubit gates between two logical qubits without leaving DFS at any time of the gate operation.

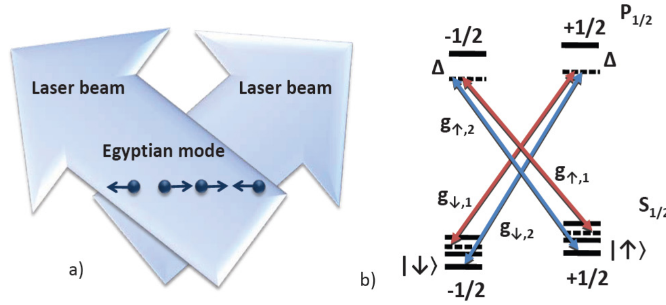

FIG. 1: (Color online) a) The state-dependent force is created by two laser beams with frequency difference $\omega_p+\delta$, and nonzero wavevector difference $\mathbf{\Delta k}$ along the trap axis. Here ω_p is the vibrational frequency with $p = 1\ldots 4$ and δ is detuning. As an example we show the gate implementation mediated by the third vibrational mode (Egyptian mode). b) Level scheme of a $^{40}Ca^{+}$. The qubit is encoded in the Zeeman sublevels $|m_J = -1/2\rangle = |\downarrow\rangle$ and $|m_J = 1/2\rangle = |\uparrow\rangle$ of the $S_{1/2}$ ground state. Here $g_{s_i,a}$ is the single Rabi frequency of the coupling between the ground states $s_i = \downarrow, \uparrow$ and the excited state of the ith ion, associated with each beam ($a = 1, 2$).

In this paper we propose a physical implementation of a geometric phase gate between two logical qubits with trapped ions. The geometric phase gate is based on state-dependent light force and has successfully been used to entangle hyperfine [17, 18], optical [19, 20] and spin [21] qubits. In our proposal, we do not need to individually address the ion crystal with laser beams at specific positions which would be technically demanding but illuminate the ions *uniformly* with laser frequency difference close to one of the vibrational modes. This ensures a high fidelity of our scheme as compared to other methods [12]. The laser-ion interaction gives rise to a spin-dependent force, displacing the motional state of the ion string depending of the internal qubit states of the ions. The spin-dependent force appears due to the differential Stark shift of the spin states. By proper adjustment of the trap frequency the force cancels for two of the logical qubit states and acts only on the remaining two logical states. For one cycle in phase space the motional state returns to the origin while the spin states acquire a non-zero geometric phase. The effect of the spin-dependent force therefore is a state-dependent geometric phase gate between two logical qubits. The geometric phase gate and single-qubit rotations on the DFS then generate a universal set of quantum gates. The technique is also suitable for creation of highly entangled cluster states, which are the major resource for the one-way quantum computer [22].

We shall specify the derivation for the gate implementation mediated by the third (*Egyptian*, in brief E-mode) vibrational mode and compare the gate fidelity to other vibrational modes. For the E-mode, the laser-ion couplings are equal in magnitude for all four ions, but different in phase for the middle two ions, Fig. 1a. The DFS gate mediated by the E-mode has a advantage due to a reduction of the off-resonant transitions.

The present paper is organized as follows: Sec. II describes the implementation of the decoherence-free subspace gate between two logical qubits using a spin-dependent force. In Sec. III we show that the gate is suitable for the creation of the four linear cluster state without leaving the DFS. In Sec. IV we analyze several error sources relevant to an experimental implementation of our method. Finally, in Sec. V we give a summary of the results.

II. STATE-DEPENDENT FORCE

We consider a linear crystal of four ions confined in a linear Paul trap. The qubit is encoded in the Zeeman ground states levels $|\uparrow\rangle = |m_J = 1/2\rangle$ and $|\downarrow\rangle = |m_J = -1/2\rangle$ of the $S_{1/2}$ ground state of the ion [5]. The linear ion crystal simultaneously interacts with two non-copropagating laser beams with frequency difference $\omega_p + \delta$, where $\omega_p = \sqrt{\gamma_p}\omega_z$ [23] is the frequency of vibration of the ion crystal, with ω_z being the axial trap frequency and δ is the detuning from the vibrational frequency ($\omega_p \gg |\delta|$). In contrast to the center-of-mass mode, the higher energy vibrational modes are less sensitive to the heating due to fluctuating ambient electric field because it requires short-wavelength components of the field to heat it [24]. The laser is detuned from the $S_{1/2} \rightarrow P_{1/2}$ transition with large detuning Δ and couples only the vibrational levels for each of the spin states according to Fig. 1b. The interaction Hamiltonian for a string of four ions simultaneously interacting with a single laser pulse in the Lamb-Dicke limit and rotating wave

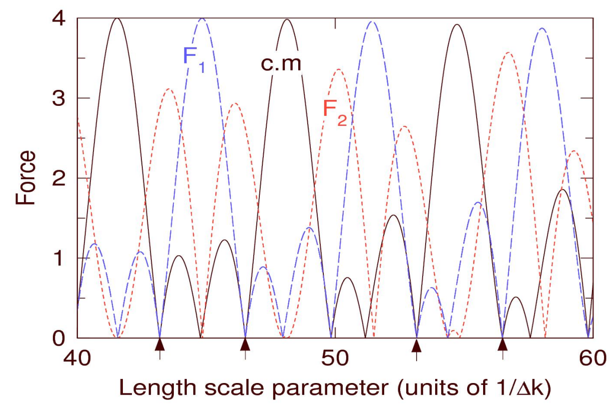

FIG. 2: (Color online). Absolute values of the spin-dependent forces $F_1 = F^{(3)}_{\uparrow\downarrow\uparrow\downarrow} = F^{(3)}_{\downarrow\uparrow\downarrow\uparrow}$ (dashed) and $F_2 = F^{(3)}_{\downarrow\uparrow\uparrow\downarrow} = F^{(3)}_{\uparrow\downarrow\downarrow\uparrow}$ (dotted), Eq. (2) as a function of the length scale parameter l for a gate mediated by the E-mode. Here we assume that $\Omega_\uparrow = -\Omega_\downarrow$. The forces F_1 vanishes for $\Delta k l = 2\pi n/(u_4 - u_2)$, with n integer. At the same points the forces $F^{(1)}_{\uparrow\downarrow\downarrow\uparrow}$ and $F^{(1)}_{\downarrow\uparrow\uparrow\downarrow}$ (solid) for the center of mass mode (in brief c.m) are zero, as indicated by the arrows.

approximation is given by [25]

$$\hat{H}_I(t) = \sum_{\mathbf{s}_i} \left(F^{(p)}_{\mathbf{s}_i} z_p e^{-i\delta t}\hat{a}^\dagger + F^{(p)*}_{\mathbf{s}_i} z_p e^{i\delta t}\hat{a} \right) |\mathbf{s}_i\rangle \langle \mathbf{s}_i| . \tag{1}$$

Here $\hat{a}$ and $\hat{a}^\dagger$ are the creation and annihilation operators of phonons in the pth vibrational mode, $z_p = \sqrt{\hbar/2M\omega_p}$ is the spread of the ground state wavepacket for the respective vibrational mode, M is the ion mass and $\mathbf{s}_i = \{s_1, s_2, s_3, s_4\}$ runs over all spin configurations of the four ions. For the E-mode ($\omega_3 = \sqrt{5.81}\omega_z$) the first and fourth ions oscillate out of phase and with equal amplitudes with the second and third ions, Fig. 1a. The magnitude of the laser-ion coupling is the same for all four ions, but opposite in sign with respect to the middle two ions. The absolute static AC-Stark shift of the energies of the qubit states is gives by $|g_{s_i,a}|^2/\Delta$, where $g_{s_i,a}$ ($a = 1, 2$) is the single Rabi frequency, Fig. 1b. This shift is generally different for the qubit states $|\uparrow\rangle$ and $|\downarrow\rangle$, however it can be equalized by proper choice of the laser polarizations. Then we have only a spatiotemporally varying differential shift $\Omega_{s_i} = g^*_{s_i,1} g_{s_i,2}/\Delta$ which gives rise to the spin dependent force. The force on the collective spin states due to differential Stark shift for the E-mode is given by

$$F^{(3)}_{s_1,s_2,s_3,s_4} = \frac{\hbar\Delta k}{2}\left(\Omega_{s_1}e^{i\zeta_1} - \Omega_{s_2}e^{i\zeta_2} - \Omega_{s_3}e^{i\zeta_3} + \Omega_{s_4}e^{i\zeta_4}\right), \tag{2}$$

where Δk is the laser wave vector difference along the trap axis. The position-dependent phase is equal to $\zeta_i = \Delta k z_i^0 - \Delta\phi$, where $z_i^0 = l u_i$ with u_i is the dimensionless equilibrium position of the ith ion and $l^3 = Z^2 e^2/4\pi\epsilon_0 M\omega_z^2$ is the length scale parameter. $\Delta\phi$ is the phase difference between the driving fields. The unitary

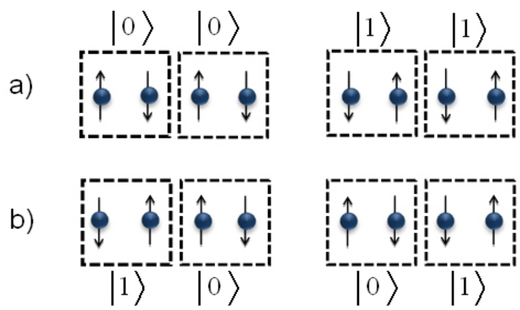

FIG. 3: (Color online). a) When the spins of middle two ions are aligned in opposite direction the force cancels. b) If and only if the middle two spins are aligned in the same direction up (down) the force pushes the ions in different direction. After one cycle in the phase space the spin states acquire non-zero geometric phase. The dashed box shows one logical qubit state which is invariant under the effect of collective dephasing. The effect of the spin-dependent displacement is the geometric phase gate between two-logical qubits.

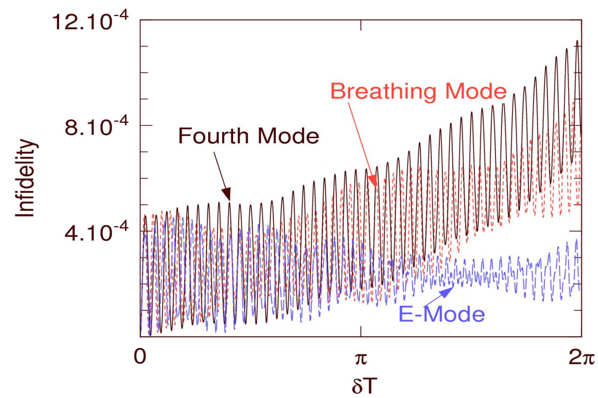

FIG. 4: (Color online). The part of the infidelity $1-F$ due to the off-resonant transitions as a function of δt. The string of four ions is simultaneously addressed with laser fields with frequency close respectively to the breathing mode (dotted), E-mode (dashed), and fourth mode (solid). The Rabi frequencies and the axial trap frequencies are listed in Table I.

operator for the Hamiltonian (1) is given by

$$\hat{\mathbf{U}}_0\left(t\right)=\prod_{\mathbf{s}_i}\hat{D}\left(\alpha_{\mathbf{s}_i}\right)e^{\mathrm{i}\Phi_{\mathbf{s}_i}}, \tag{3}$$

where

$$\hat{D}\left(\alpha_{\mathbf{s}_i}\right)=\exp\left[\left(\alpha_{\mathbf{s}_i}\hat{a}^{\dagger}-\alpha^{*}_{\mathbf{s}_i}\hat{a}\right)\right]\left|\mathbf{s}_i\right\rangle\left\langle\mathbf{s}_i\right| \tag{4}$$

is the state-dependent displacement operator with

$$\alpha_{\mathbf{s}_i}=\left(e^{-\mathrm{i}\delta t}-1\right)F^{(3)}_{\mathbf{s}_i}z_3/\hbar\delta. \tag{5}$$

The state-dependent geometric phase

$$\Phi_{\mathbf{s}_i}=\left(\delta t-\sin\delta t\right)\left|F^{(3)}_{\mathbf{s}_i}z_3\right|^2/\left|\hbar\delta\right|^2 \tag{6}$$

appears due to non-commutativity of the interaction Hamiltonian at different times. We project the unitary operator (3) into the DFS under consideration: $\{|\uparrow\downarrow\uparrow\downarrow\rangle$, $|\downarrow\uparrow\uparrow\downarrow\rangle$, $|\uparrow\downarrow\downarrow\uparrow\rangle$, $|\downarrow\uparrow\downarrow\uparrow\rangle\}$. These states are immune to collective dephasing caused by magnetic field fluctuations. We adjust the trap potential such that

$$\begin{aligned}\Delta k\left(z_4^0-z_1^0\right) &= \Delta k\left(z_2^0-z_1^0\right)+2n\pi,\\ \Delta k\left(z_4^0-z_2^0\right) &= \Delta k\left(z_3^0-z_1^0\right)=2n\pi,\end{aligned} \tag{7}$$

where n is integer. This optimizes the motional coupling to the E-mode as can be seen from Eq. (2). In Fig. 2 we plot the spin-dependent forces for the E-mode as a function of the length scale parameter l. The spin-dependent forces $F^{(3)}_{\uparrow\downarrow\uparrow\downarrow}$ and $F^{(3)}_{\downarrow\uparrow\downarrow\uparrow}$ oscillate as a function of Δkl and vanishes for $\Delta kl=2\pi n/(u_4-u_2)$, while the forces $F^{(3)}_{\uparrow\downarrow\downarrow\uparrow}$ and $F^{(3)}_{\downarrow\uparrow\uparrow\downarrow}$ displace the motional state in opposite directions. At the same point two of the spin-dependent forces for the center-of-mass mode are zero, hence the off-resonant excitations of this mode cancels. During the time evolution the motional state moves along a circular path in phase space and returns to the origin after time $T_g=2\pi/\delta$, while the spin states acquire geometric phases $\Phi_{\mathbf{s}_i}=2\pi\left|F^{(3)}_{\mathbf{s}_i}z_3\right|^2/\left|\hbar\delta\right|^2$. In this case the force displaces the ions if the spins of the middle two ions are aligned in the same direction and cancel if the spins are opposite, Fig. 3. After time T_g the states for which the force is not cancel acquire geometric phases $\Phi_{\downarrow\uparrow\uparrow\downarrow}=\Phi_{\uparrow\downarrow\downarrow\uparrow}$. By proper choice of the Rabi frequencies $\Omega_{\uparrow}$ and $\Omega_{\downarrow}$ one can adjust the geometric phases equal to $\pi/2$ and the action of the gate onto the DFS states is given by

$$\begin{aligned}\left|\uparrow\downarrow\uparrow\downarrow\right\rangle &\to \left|\uparrow\downarrow\uparrow\downarrow\right\rangle,\\ \left|\downarrow\uparrow\uparrow\downarrow\right\rangle &\to \mathrm{i}\left|\downarrow\uparrow\uparrow\downarrow\right\rangle,\\ \left|\uparrow\downarrow\downarrow\uparrow\right\rangle &\to \mathrm{i}\left|\uparrow\downarrow\downarrow\uparrow\right\rangle,\\ \left|\downarrow\uparrow\downarrow\uparrow\right\rangle &\to \left|\downarrow\uparrow\downarrow\uparrow\right\rangle.\end{aligned} \tag{8}$$

The DFS gate (8) is equivalent to the controlled-phase gate between two logical decoherence-free qubits. Hence, the unitary evolution transforms any superposition of the states belonging to the DFS into another superposition of those states.

III. CREATION OF A LINEAR CLUSTER STATE

Cluster states are highly entangled states, which are a major source of the one-way quantum computer [22]. Cluster states have been experimentally demonstrated with atoms in optical lattice [26] and with photons [27]. Multi-qubit cluster states have yet not been created with trapped ions. An ion-trap architecture for high-speed measurement-based quantum computer was pro-

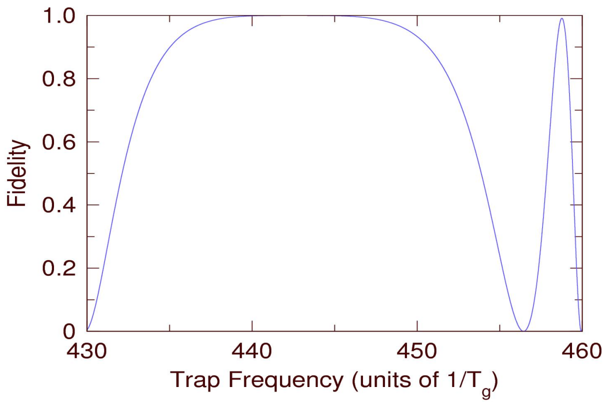

FIG. 5: (Color online). Evolution of the fidelity F Eq. (14) as a function of the axial trap frequency ω_z for a gate Eq. (8) mediated by the E-mode.

posed [28]. Ref. [29] proposed an efficient technique for creation of four, five, and six qubit linear cluster states by collective bichromatic interaction, while in [30] creation of two-dimensional cluster state was suggested by using a spin-spin coupling induced by a magnetic-field gradient. Here, we propose the creation of four linear cluster state, without leaving the DFS. Indeed, if the gate (8) is applied onto the decoherence-free initial state $|\Psi_{in}\rangle = |B_{1,2}\rangle|B_{3,4}\rangle$, which is a product of two Bell states $|B_{i,j}\rangle = (|\uparrow_i\downarrow_j\rangle + |\downarrow_i\uparrow_j\rangle)/\sqrt{2}$, then after T_g the state is transformed into the state

$$|\Psi_c\rangle = (|\uparrow\downarrow\uparrow\downarrow\rangle + \mathrm{i}\,|\uparrow\downarrow\downarrow\uparrow\rangle + \mathrm{i}\,|\downarrow\uparrow\uparrow\downarrow\rangle + |\downarrow\uparrow\downarrow\uparrow\rangle)/2. \quad (9)$$

The final state is highly entangled four qubit cluster state. Since the spin states are not mixed due to the unitary evolution (3), the initial state is transformed into the final cluster state without leaving the DFS. Combining the ion trap architecture large cluster states can be created by fusing several four linear cluster states with the DFS gate.

IV. OFF-RESONANT TRANSITIONS AND ROBUSTNESS

During the time evolution the other vibrational modes are off-resonantly excited if there is a non-vanishing coupling strength. For general coupling strengths and $\delta_p = \omega_p - \omega_3 - \delta$, the displacement and geometric phase associated with the pth vibrational mode is given by

$$\begin{aligned} \alpha_{\mathbf{s}_i}^{(p)}(t) &= \frac{F_{\mathbf{s}_i}^{(p)} z_p}{\hbar\delta_p}\left(e^{-\mathrm{i}\delta_p t} - 1\right), \\ \Phi_{\mathbf{s}_i}^{(p)}(t) &= \left|\frac{F_{\mathbf{s}_i}^{(p)} z_p}{\hbar\delta_p}\right|^2 [\delta_p t - \sin(\delta_p t)]. \end{aligned} \quad (10)$$

If now the absolute force magnitude and the final time are fixed such that the geometric phase gate condition is fulfilled, $T_g = 2\pi/\delta_3$, one obtains

$$\begin{aligned} \alpha_{\mathbf{s}_i}^{(p)}(T) &= \frac{F_{\mathbf{s}_i}^{(p)} z_p}{\hbar\delta_p}\left(e^{-\mathrm{i}2\pi\delta_p/\delta_3} - 1\right), \\ \Phi_{\mathbf{s}_i}^{(p)}(T) &= \left|\frac{F_{\mathbf{s}_i}^{(p)} z_p}{\hbar\delta_p}\right|^2 [2\pi\delta_p/\delta_3 - \sin(2\pi\delta_p/\delta_3)]. \end{aligned} \quad (11)$$

An appropriate measure of the gate fidelity is given by [31]:

$$F = \frac{1}{N^2}\sum_{i,j}\langle\tilde{\mathbf{s}}_i|\hat{U}_0^\dagger\hat{U}|\tilde{\mathbf{s}}_i\rangle\langle\tilde{\mathbf{s}}_j|\hat{U}^\dagger\hat{U}_0|\tilde{\mathbf{s}}_j\rangle, \quad (12)$$

where $N = 4$ is the dimension of the DFS and i, j each run over all spin configurations belongs to DFS. $\hat{U}_0$ is the desired unitary transform given by Eq. 8 and $\hat{U}$ is the actual one. Neither $\hat{U}_0$ nor $\hat{U}$ is mixing the spin states. The action of $\hat{U}$ on any basis state is simply given by

$$|\tilde{\mathbf{s}}_i\rangle = |\mathbf{s}_i\rangle|0_1 0_2 0_3 0_4\rangle \rightarrow e^{\mathrm{i}\sum_p \Phi_{\mathbf{s}_i}^{(p)}}|\mathbf{s}_i\rangle\left|\alpha_{\mathbf{s}_i}^{(1)}\ldots\alpha_{\mathbf{s}_i}^{(4)}\right\rangle. \quad (13)$$

Assuming all vibrational modes in the vacuum prior to the gate operation, the fidelity can then be straightforwardly evaluated under consideration of the matrix element $\langle 0|\alpha\rangle = e^{-|\alpha|^2/2}$. Hence, we obtain for the fidelity at time t

$$F = \frac{1}{16}\left|\sum_i\prod_p e^{-\frac{1}{2}|\alpha_{\mathbf{s}_i}^{(p)}|^2 + \mathrm{i}\Phi_{\mathbf{s}_i}^{(p)}}\right|^2. \quad (14)$$

As an example of qubit we consider the $^{40}Ca^+$ ion with qubit states encoded in the Zeeman sublevels of $S_{1/2}$ state. The two-photon Raman transition is driven by laser field with a wave length of $\lambda \approx 397$ nm and wave vector difference along the trap axis with $\Delta k = 2\sqrt{2}\pi/\lambda$. In order to cancel spin forces $F_{\uparrow\downarrow\uparrow\downarrow}$ and $F_{\downarrow\uparrow\downarrow\uparrow}$, Eq. (7) we choose $n = 15$. The Rabi frequencies are $\Omega_\uparrow = -\Omega_\downarrow$ with gate time $T_g = 25$ μs and laser detuning $\delta = 2\pi\times 40$ kHz. Table I lists the values of the axial trap frequency ω_z and the Rabi frequency Ω needed for the synthesis of the DFS gate for the three different vibrational modes. Also

TABLE I: The values of the axial trap frequency ω_z and the Rabi frequency Ω required for implementation of the DFS gate for three different vibrational modes. The infidelity due to the excitation of the off-resonant transitions is also presented. The frequency plateaus, where the minimum gate fidelity is 99% are listed.

Mode	Breathing	E-mode	Fourth
$\omega_z/2\pi$ (MHz)	2.86	2.82	2.68
$\Omega/2\pi$ (kHz)	119.64	130.62	130.47
Infidelity	8.1×10^{-4}	1.8×10^{-4}	7.7×10^{-4}
$\Delta\omega_z/2\pi$ (kHz)	53	65	92

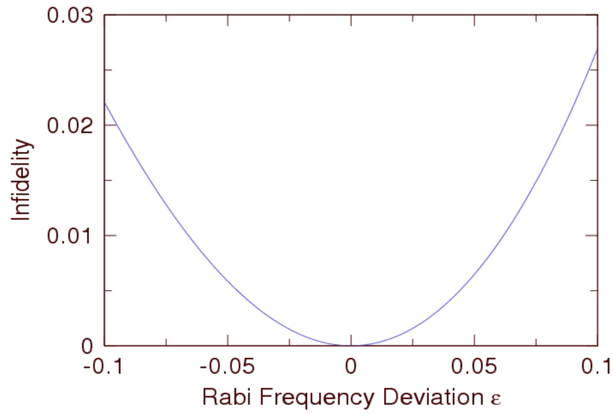

FIG. 6: (Color online). Infidelity as a function of the deviation ϵ of the Rabi frequencies defined as $\Omega_{\{s_i\}}(t) = \Omega_{\{s_i\}}(1+\varepsilon)$ for the implementation of the decoherence-free gate (8) mediated by the E-mode.

we compare the minimum gate infidelity $1-F$ mediated by different vibrational modes due to the off-resonant transitions, (see, Fig. 4). Even for one cycle in phase space $\delta T_g = 2\pi$ the infidelity for the E-mode is smallest since the off-resonant transitions to the center-of mass mode for spin states $\uparrow\downarrow\downarrow\uparrow$ and $\downarrow\uparrow\uparrow\downarrow$ vanishes. In experiments, one may simply ground state cool the E-mode, but leave the other modes with Doppler cooling and a thermal state. The fidelity is obtained by using the overlap integrals of displaced Fock states $\langle n|\alpha, n\rangle = e^{-|\alpha|^2/2}L_n(|\alpha|^2)$ with the Laguerre polynomials $L_n(x)$. Assuming average number of phonons $\overline{n} = 1.3$, the infidelity due to the off-resonant transition for a gate mediated by the E-mode reaches 6.1×10^{-4}.

In Fig. 5 we show the fidelity (14) as a function of the axial trap frequency ω_z for fixed gate time T_g. The Rabi frequency is chosen such that the condition $\Phi_{\downarrow\uparrow\uparrow\downarrow} = \Phi_{\uparrow\downarrow\downarrow\uparrow} = \pi/2$ is fulfilled. The frequency plateaus, where the minimum fidelity is better than 99% for the gate implementation mediated by the different vibrational modes are listed in Table I.

Additionally, the proposed gate scheme shows a remarkable robustness against laser intensity fluctuations, As both laser beams are derived from the same laser source, we assume that the intensity fluctuations are common. The corresponding Rabi frequencies fluctuate therefore simultaneously for all ions. As a result, we observe that the fidelity only decrease quadratically with the fluctuations, see Fig. 6.

The motional decoherence of the vibrational motion of the trapped ions is the most serious limiting factor in ion trap quantum information processing. In order to achieve high fidelity the gate time T_g required for the implementation of DFS gate (8) must be much shorter than the heating time τ. The measured heating time of the center of mass mode for $^{40}Ca^{+}$ ion in a segmented micro-ion traps [5] is 3.3 ms. The heating time for higher energy modes is expected to be even longer. Hence the gate time is more than two order of magnitude faster than the heating time such that the fidelity of the gate is not affected.

V. CONCLUSION

In conclusion we proposed a simple and robust technique for the construction of a decoherence-free controlled phase gate between two logical qubits. We studied in detail the fidelity of the gate implementation taking into account various error sources such as off-resonant transitions, laser fluctuations, and the deviation of the right choice of the axial trap frequency existing in an experiment. We have compared the error sources for a gate mediated by three different modes, and we have shown that the gate mediated by the Egyptian mode optimizes the off-resonant transitions. Our scheme includes the creation of a linear cluster state within a decoherence free subspace manifold of four qubits - a starting point for decoherence-free ion trap one way quantum computing.

VI. ACKNOWLEDGMENTS

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