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Transport dynamics of single ions in segmented microstructured Paul trap arrays

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Introduction (content available from arXiv **quant-ph/0606237**)

Small groups of ions as qubit carriers in miniaturized, two-dimensional electrode arrays might be a scalable approach for large-scale quantum computation[1,2].

By this method processing of quantum information is achieved by shuttling ions to and from separate memory and qubit manipulation zones enabling quantum computation via principles of quantum communication.

Transport of ion groups in this scheme plays a major role and requires precise experimental control and fast shuttling times.

We discuss theoretically the transport performance and limitations associated with shuttling ions in typical miniaturized Paul trap arrays by modeling the process by a dragged, parametrically driven harmonic oscillator.

Here, we present a theoretical framework to describe and optimize these transport processes and discuss implications on trap technology.

1 D.J. Wineland et al., J. Res. Natl. Inst. Stand. Technol. **103**, 259 (1998)

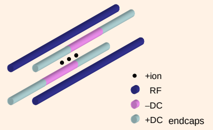
2 D. Kielpinski et al., Nature **417**, 709 (2002)

traditional ion traps

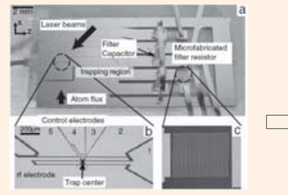
principle:

in the 4rod geometry below, ions are trapped radially by RF fields applied to the blue rods, and axially by two DC endcap electrodes.

trapping ion strings along the center axis requires that the (degenerate) radial trap frequencies are larger than the axial frequency.



current technology



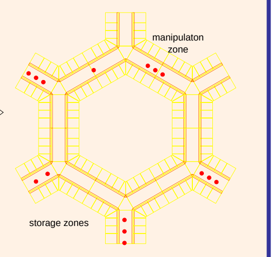
2,5: endcap electrodes; 1,3,4: control electrodes; rf electrode;

Surface-electrode trap at NIST:

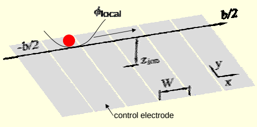
S. Seidelin et al. PRL 96, 253003 (2006)

next years

multiplexed traps & vision



1 Transport equations



assume equispaced configuration for the control electrodes to transport in x direction.

apply time varying potentials to the electrodes in order to translate the trapping well of the ion.

Newton's equation (to describe a transport in lab frame along q=x axis)

$$\ddot{q}(t)+\frac{Q}{m}\frac{\partial\phi(q,t)}{\partial q}=0 \quad \text{+initial conditions}$$

design potential so that

$$\phi(q,t)=\phi_{local}(q-q_0(t))\approx m\omega_0^2(q-q_0(t))^2/2Q$$

$q_0(t)$ *transport function (moves minimum of the well)*

define $\phi_{res}=\phi(q,t)-m\omega_0^2(q-q_0(t))^2/2Q \qquad a_{res}(q)=\frac{Q}{m}\frac{\partial\phi_{res}(q)}{\partial q}$

resulting equation in moving frame with coordinate $u=q-q_0(t)$

$$\ddot{u}+\omega_0^2 u=-\ddot{q}_0(t)+a_{res}(u+q_0(t))$$

expand a_{res} only to 2nd order, neglect higher terms, and use

$$\omega^2(t)=\omega_0^2\left(1-a'_{res}[q_0(t)]/\omega_0^2\right) \qquad f(t)=-\ddot{q}_0(t)+a_{res}[q_0(t)]$$

result is a parametrically driven, forced harmonic oscillator

$$\ddot{u}+\omega^2(t)=f(t) \quad \text{or} \quad \mathcal{H}(t)=\frac{p^2}{2m}+\frac{m\omega^2(t)}{2}u^2-mf(t)u \qquad (1)$$

2 Classical transport

An Ermakov ansatz to the homogeneous equation (f=0) of Eq.(1)

$$u_1=\rho(t)e^{i\mu(t)} \qquad u_2=\rho(t)e^{-i\mu(t)}$$

phase function

amplitude function

gives two equations instead of one

$$\ddot{\rho}+\omega^2(t)\rho=1/\rho^3 \qquad \mu(t)=\int_{-t_0}^{t}dt'\rho(t')^{-2} \qquad (2)$$

If ω is constant, lhs of equation (2) has the general solution

$$\rho(t)=\pm\omega_0^{-1/2}\sqrt{\cosh\delta+\sinh\delta\sin(2\omega_0 t+\theta)} \qquad (3)$$

with the use of a Green's function we can find a particular solution and thus write the general solution of Eq. (1)

$$u_c(t)=u_h(t)+u_p(t)\equiv\rho(t)/2\left\{a_c e^{i(\mu(t)+\varphi)}+\zeta(t)\right\}+c.c.$$

where $\zeta(t)=ie^{-i\mu(t)}\int dt' e^{i\mu(t')}\rho(t')f(t')$ $\qquad a_c,\varphi$ class. amplitude & phase, resp.

by averaging over the initial classical phase we find the classical mean energy and energy dispersion (ω assumed constant)

$$\langle E\rangle_{E_0}=E_0+W \qquad \langle(\Delta E)^2\rangle_{E_0}=2E_0W$$

Equi-valence

3 Quantum solution

Utilizing a generalized invariant theory (Kim et al. PRA **53**, 3767 (1996)) we can define the annihiliation and creation operators by the class. terms ρ,μ,ζ

$$B(t)=\sqrt{\frac{m}{2}}\left\{\left(\rho^{-1}-i\dot{\rho}\right)\hat{q}(t)-\zeta\right\}+i\frac{\rho}{\sqrt{2m}}\hat{p}(t) \quad \text{and } B^\dagger(t) \text{ similar}$$

We can use their properties to define the coordinate and momentum operator, and Hamiltonian in a Heisenberg picture as a function of time

$$\hat{q}(t)=\rho(t)\left\{\hat{q}(-t_0)\sqrt{\omega_0}\cos\mu(t)+\frac{\hat{p}(-t_0)}{m\sqrt{\omega_0}}\sin\mu(t)\right\}+u_p(t) \text{ similar for } \hat{p}(t)$$

$$\mathcal{H}(t)=\hbar\omega_0\left(B^\dagger(t)B(t)+\frac{1}{2}\right)$$

If we use the standard operators at initial time $-t_0$

$$\hat{q}(-t_0)=\frac{1}{\sqrt{2m\omega_0}}\{B+B^\dagger\} \qquad \hat{p}(-t_0)=-i\sqrt{\frac{m\omega_0}{2}}\{B-B^\dagger\}$$

we can determine the mean values in a coherent state $|\alpha\rangle$ with $\alpha=|\alpha|e^{i\varphi}$ to find the equivalency to the classical trajectory

$$\langle\alpha|\hat{q}(t)|\alpha\rangle=\sqrt{\frac{2}{m}}\rho(t)|\alpha|\cos[\mu(t)+\varphi]+u_p(t)$$

$$\langle\alpha|\hat{p}(t)|\alpha\rangle=\ldots\text{similar}\ldots\equiv m\frac{d}{dt}\langle\alpha|\hat{q}(t)|\alpha\rangle$$

this provides the interlink between the classical and quantal solution. For constant frequency we can also obtain the first two moments of the energy distribution

$$\langle E_m\rangle_n\equiv\hbar\omega_0\left(\sum_m m\,P_{mn}+1/2\right)=\epsilon_n+\hbar\omega_0\gamma$$

$$\langle(\Delta E_m)^2\rangle_n\equiv(\hbar\omega_0)^2\left\langle(m-\langle m\rangle)^2\right\rangle=2\epsilon_n\,\hbar\omega_0\gamma$$

where $\gamma(t)=$ classical energy transfer$/\hbar\omega_0$

We observe squeezing by relating the dispersions to classical amplitude changes

$$\langle\alpha|(\Delta q(t))^2|\alpha\rangle=\rho^2/2m \qquad \langle\alpha|(\Delta p(t))^2|\alpha\rangle=(\rho^{-2}+\dot{\rho}^2)m/2$$

In the constant frequency regime we can employ Eq.(3) to see that $\langle\Delta q^2\rangle,\langle\Delta p^2\rangle$ are proportional to

$(\cosh\delta\pm\sinh\delta\sin(2\omega_0 t+\theta))$, respectively,

and that they exhibit an oscillatory behaviour with opposite phase.

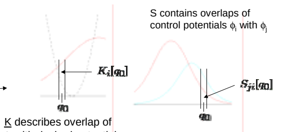
4 Results for the simple ion trap model

How to find optimum waveforms of potentials for safely transporting an ion?

Recipe: solve linear near-singular system for waveform vector $\underline{a}$ at any position q_0:

$$\underline{a}_\nu=\arg\min_{\underline{a}}\left\{\|\mathbf{S}\cdot\underline{a}-\eta\underline{K}\|^2\right\} \quad \text{with}$$

S contains overlaps of control potentials ϕ_i with ϕ_j



$\underline{K}$ describes overlap of ϕ_i with desired potential (dashed curve)

ϕ_i is the potential of control electrode i

We employ a Tikhonov regularization to find a[q0]

$$\underline{a}_\nu=\arg\min_{\underline{a}}\left\{\|\mathbf{S}\cdot\underline{a}-\eta\underline{K}\|^2+\nu^2\|\mathbf{L}(\underline{a}-\underline{a}^*)\|^2\right\}$$

The common minimization of these two terms stabilizes the solution because it feeds back information to the system.

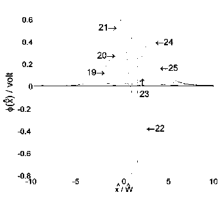
Results for an examplary configuration for a set of 40 equispaced electrodes as illustrated in section 1.

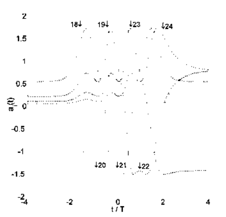
Time-dependent waveforms to move the parabolic well from electrode 19 to electrode 23, i.e. over 4 electrode widths (a_i=2 means 2V applied).

Typical superposition of nearby electrodes to form a local parabolic potential

$$\phi(q,t)=\sum_i a_i(t)\phi_i(q)$$

dashed curve is desired parabolic potential, vertical lines define the optimization range; participating electrodes are numbered from left to right.





a well controlled transport is where $\omega[q_0(t)]/\omega_0\approx 1$

and $|a_{res}(q_0(t))|\ll|\ddot{q}_0(t)|$

blue dotted curve shows the comparison to the transporting force.

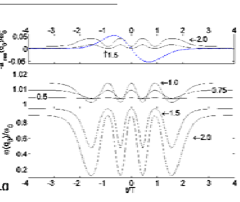


figure on the right shows residual frequency and force modulation for different aspect ratios of $\tilde{W}=W/z_{ion}$ (see fig in 1) during a transport for typical trap parameters.

Adding up more electrodes for smaller $\tilde{W}$ improves the performance but also increases resources. Our example configuration suggests that a value of $\tilde{W}=0.5-1.0$ to be optimal.

5 Minimal energy transfer and phase-insensitive transport

In the adiabatic limit $\dot{\omega}/\omega^2\ll 1$ we can find an approximate solution to Eq.(2)

$$\rho(t)=\frac{1}{\sqrt{\omega(t)}}+\frac{1}{8}\frac{\ddot{\omega}(t)}{\omega(t)^{7/2}}+\cdots \quad \text{and} \quad \dot{\mu}(t)=\omega(t)-\frac{1}{4}\frac{\ddot{\omega}(t)}{\omega(t)^2}+\cdots$$

Then we can define the adiabatic suppression amplitude

$$\Xi(t_1)=\sqrt{\omega(t_1)}\int_{-t_0}^{t_1}dt' f(t')\omega(t')^{-1/2}e^{i\Delta\mu_{1t'}} \quad \text{with} \quad \Delta\mu_{1t'}=\int_{t'}^{t_1}d\tau\,\omega(\tau) \qquad (4)$$

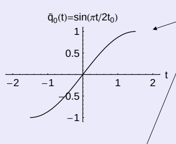
that governs the classical energy transfer to the oscillatory motion by the external time-dependent force according to

$$W(t)=m|\Xi(t)|^2/2$$

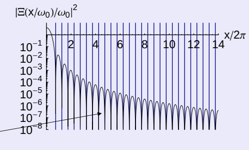
In a classical description the energy transfer is phase-sensitive and depends on the temporal shape of the transport function $q_0(t)$.

What is the optimum choice for $q_0(t)$? (let's assume a constant frequency and $a_{res}=0$)

We give two examples that have quite different results.

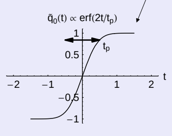


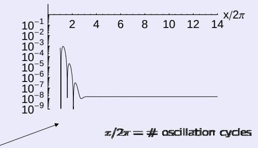
Normalize q_0 so that start location is at -1 and final destination at +1. The transport duration is given by x/π.



For typical experimental parameters we find:

there is a slow decay of the adiabatic amplitude for a sine as transport function. Transport of a single ion over a distance of about 1.2 mm requires ~100 oscillation periods to stay near the ground state *.





$x/2\pi=\#$ oscillation cycles

For an error function transport, the same transport can be done in about 6 oscillation Periods with similar amount of energy transfer.

* consistent with experimental observations of Rowe et al. Quant. Inf. Comp. **4**, 257 (2002)