Theoretical Quantum Optics

Sheet 8 **SS 2017**
Date of issue: 19-06-2017 Discussion: 30-06-2017

Exercise 12 *Ramsey interferometer: to measure ω_{eg}*

Let us consider a two-level system with the resonance frequency ω_{eg} interacting with a series of near-resonant laser pulses, shown in Fig. 1.

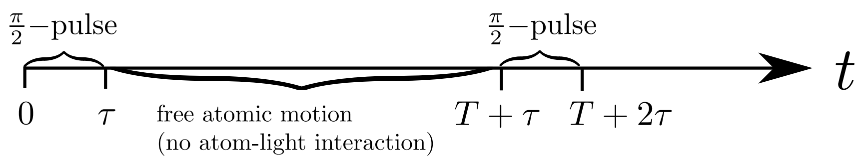

Figure 1: The Ramsey interferometer consisting of a series of two $\frac{\pi}{2}$-pulses, separated by the region of free atomic evolution.

Mathematically, the interaction of an atom with the near-resonant lasers pulses takes the form

$$\Omega_0(t) = \Omega_0 \begin{cases} 0\,, & t \leq 0 \\ 1\,, & 0 < t \leq \tau \\ 0\,, & \tau < t \leq T+\tau \\ 1\,, & T+\tau < t \leq T+2\tau \\ 0\,, & t > T+2\tau\,. \end{cases}$$

By neglecting any relaxations, the internal state of the atom $|\psi(T+2\tau)\rangle$ then reads

$$|\psi(T+2\tau)\rangle = \hat{U}_{\frac{\pi}{2}}(T+2\tau, T+\tau)\hat{U}_{free}(T+\tau,\tau)\hat{U}_{\frac{\pi}{2}}(\tau,0)\,|\psi(0)\rangle\ ,$$

where

- $\hat{U}_{\frac{\pi}{2}}(t_f,t_i)$ is the time evolution operator due to the interaction of an atom with a near-resonant laser pulse (of the frequency ω_L and the phase $\varphi = 0$) during the time interval (t_i,t_f).
- $\hat{U}_{free}(t_f,t_i)$ is the time evolution operator for a free atom (without any field) during the time interval (t_i,t_f).

a) Obtain the time evolution operator $\hat{U}_{\frac{\pi}{2}}(t_f,t_i)$ for a given detuning $\Delta = \omega_{eg} - \omega_L$ and find the condition for the pulse to be a $\frac{\pi}{2}$-pulse (pulse area equal to $\frac{\pi}{2}$).

(3 points)

b) Obtain the time evolution operator $\hat{U}_{free}(t_f,t_i)$.

(1 point)

c) Find the probability $w_e(T+2\tau)$ to find the atom in the excited state directly after the second pulse (at $t = T+2\tau$), if initially the atom is in the ground state, $|\psi(0)\rangle = |g\rangle$.

(3 points)

d) Consider the case of $|\Delta| \ll \Omega_0$ and plot the probability $w_e(T+2\tau)$ as a function of the time interval T.

(1 point)

Exercise 13 *Ramsey interferometer with relaxations*

To take into account relaxation characterized by two relaxation times T_1 and T_2, we represent the optical Bloch equations in the matrix form

$$i\frac{\partial}{\partial t}\hat{\rho} \;=\; \hat{\mathcal{H}}\hat{\rho}$$

for $\hat{\rho} = (\rho_{ee}, \rho_{gg}, \rho_{eg}, \rho^*_{eg})^T$.

a) Find the time evolution operator $\hat{\mathcal{U}}_{free}(t_f, t_i)$, corresponding to the case of $\Omega_0 = 0$, that is

$$\hat{\rho}(t_f) \;=\; \hat{\mathcal{U}}_{free}(t_f, t_i)\hat{\rho}(t_i)\ .$$

(1 point)

b) Find the evolution operator $\hat{\mathcal{U}}(t_f, t_i)$ corresponding to the atom-field interaction during the time interval (t_i, t_f) with $\Omega_0(t) = \Omega_0$ for $t_i \le t \le t_f$ and $|t_f - t_i| \ll T_{1,2}$. Obtain the condition, under which the pulse area is equal to $\frac{\pi}{2}$.

(3 points)

c) Obtain the population $\rho_{ee}(T+2\tau)$ of the excited state, see Fig. 1, and plot its dependence on the time interval T.

(3 points)