Exercise 1
Proof the following estimation for $x \to +\infty$:
\[
\log \left( e^{2x \cos(x)} + e^x \right) = \mathcal{O}(x)
\]
(1 Point)

Exercise 2
Derive the following estimation for $x \to +\infty$:
\[
\left[ x + 1 + \mathcal{O}\left( \frac{1}{x} \right) \right]^x = e^{x^x} + \mathcal{O}(x^{x-1}).
\]
(1 Point)

Exercise 3
Let the function $f(x)$ have an asymptotic expansion for $x \to +\infty$
\[
f(x) \sim f_0 + \frac{f_1}{x} + \frac{f_2}{x^2} + ...
\]
For $f_0 \neq 0$ we can define the asymptotic expansion for the function
\[
\frac{1}{f(x)} \sim a_0 + \frac{a_1}{x} + \frac{a_2}{x^2} + ...
\]
Find the first three coefficients $a_0, a_1, a_2$ in terms of $f_n$ with $n = 0, 1, 2, \ldots$ (2 Points)

Exercise 4
For $x \to +\infty$, the function $f(x) \sim x^\nu$ with $\text{Re}(\nu) = -1$ and $\text{Im}(\nu) \neq 0$. By considering an example of such a function, $f(x) = x^\nu + 1/(x \log^\mu(x))$ with $\mu > 0$, find the condition on the parameter $\mu$, under which the following estimation of the integral
\[
\int_a^x f(t) dt = \mathcal{O}(1)
\]
for $x \to +\infty$ and any finite $a > 0$ is correct. (2 Points)

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