Exercise 15
By using Euler-Maclaurin formula, find the asymptotic expansion for the sum
\[
\sum_{n=1}^{\infty} \ln \left( 1 - e^{-nx} \right)
\]
as \( x \to 0 \) and \( x > 0 \), which is valid in the order of \( x \).

**Hint:** To find the correct result, use the definition of the Bernoulli numbers \( B_n \) via the generation function, that is
\[
\frac{x}{e^x - 1} = \sum_{n=0}^{\infty} \frac{B_n}{n!} x^n,
\]
and study the dependence of all terms in the Euler-Maclaurin formula at small values of \( x \).

(4 Points)

Exercise 16
Show that
\[
\sum_{n=1}^{\infty} \frac{x^n}{n!} \ln(n) = e^x \left\{ \ln(x) - \frac{1}{2x} + O \left( \frac{1}{x^2} \right) \right\}
\]
as \( x \to \infty \).

(3 Points)

Exercise 17
Find the asymptotic behavior of the sum
\[
\sum_{k=1}^{\infty} \frac{1}{k(k+n)}
\]
for large positive and integer \( n \).

(3 Points)

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