Introduction to Asymptotic Methods

Maxim Efremov
Institut für Quantenphysik, Universität Ulm WS 2017-2018

Sheet 1

Exercise 1
Proof the following estimation for $x \to +\infty$:

$$\log\left(e^{2x\cos(x)} + e^x\right) = \mathcal{O}(x)$$

(1 Point)

Exercise 2
Derive the following estimation for $x \to +\infty$:

$$\left[x + 1 + \mathcal{O}\left(\frac{1}{x}\right)\right]^x = e\,x^x + \mathcal{O}(x^{x-1}).$$

(1 Point)

Exercise 3
Let the function $f(x)$ have an asymptotic expansion for $x \to +\infty$

$$f(x) \sim f_0 + \frac{f_1}{x} + \frac{f_2}{x^2} + ...$$

For $f_0 \neq 0$ we can define the asymptotic expansion for the function

$$\frac{1}{f(x)} \sim a_0 + \frac{a_1}{x} + \frac{a_2}{x^2} + ...$$

Find the first three coefficients a_0, a_1, a_2 in terms of f_n with $n = 0, 1, 2, ...$
(2 Points)

Exercise 4
For $x \to +\infty$, the function $f(x) \sim x^\nu$ with $\mathrm{Re}(\nu) = -1$ and $\mathrm{Im}(\nu) \neq 0$. By considering an example of such a function, $f(x) = x^\nu + 1/(x\log^\mu(x))$ with $\mu > 0$, find the condition on the parameter μ, under which the following estimation of the integral

$$\int_a^x f(t)dt = \mathcal{O}(1)$$

for $x \to +\infty$ and any finite $a > 0$ is correct.

(2 Points)

Date: 18.10.2017
Discussion: 24.10.2017