

Vorkurs Lösungen

Mathematik f. Chemiker

BLOCK I

1. Faktorisieren

- (1) $(a-b)(a+b)(a^2+b^2)$ (6) $(3c-4)(3c-8)$
(2) $(x-3)(x+3)(x^2+9)$ (7) $(x-y)(x^2+xy+y^2)$
(3) $(\frac{c}{3}-\frac{x}{2})(\frac{c}{3}+\frac{x}{2})(\frac{c^2}{9}+\frac{x^2}{4})$ (8) $(2b+5)(4b^2-10b+25)$
(9) $(3x+5)(x^2+1)$
(4) $(n+8)(n-2)$ (10) $(x+1)(x^3+2) =$
(5) $(x-3)(x+2)$ $(x+1)(x+\sqrt[3]{2})(x^2-\sqrt[3]{2}x+\sqrt[3]{4})$

2. Polynomdivision

- (11) $2a-3b$ (12) $3x+8z$
(13) $4a^2-2b^2$
(14) $(a+b)(a^2+b^2) = a^3+a^2b+ab^2+b^3$
(15) $27x^6-18x^4z+12x^2z^2-8z^3$ (16) $1+x+2y$
(17) $6-7a$ (18) $-x^3+2x^2-3x+1$
(19) $\frac{1}{x^{16}(x-y)^2}$ (20) $x^{16}(x-y)(x+y)^2$

3. Gleichungen

- (21) $x = -5$ (22) $x = 1$
(23) $x = 11$ (24) $x = a + n$
(25) $x = 3$

BLOCK III

Logarithmieren, Potenzieren

3.1

- | | |
|---|--|
| <p>1. 1000</p> <p>3. $\frac{1}{1000}$</p> <p>5. 1</p> <p>7. e^4</p> <p>9. 2</p> <p>11. 57</p> <p>13. 5</p> <p>15. $x(\ln 3 + \ln 5)$</p> <p>17. $\lg(q^n - 1) - \lg(q - 1) = \lg(q^{n-1} + q^{n-2} + \dots + q + 1)$</p> <p>18. $\frac{1}{5} \ln(a + b) - \frac{1}{2} \ln(a - b)$</p> <p>19. $\lg(a - b) + \lg(a^6 + a^5b + a^4b^2 + \dots + b^6)$</p> <p>20. $\lg(x^{\frac{1}{6}}) + \lg(y^{\frac{3}{4}})$</p> <p>21. $x = \frac{4}{3}$</p> <p>23. $x = \pm \left(\frac{\ln d}{\ln c + \ln d - 1} \right)^{\frac{1}{2}}$</p> <p>24. $2^{x+1} \left(1 + \frac{1}{8} \right) = 3^{x-1} \left(1 - \frac{1}{9} \right) \rightsquigarrow 2^{x+1} \frac{9}{8} = 3^{x-1} \frac{8}{9} \rightsquigarrow x = 5$</p> <p>25. $x_1 = 1 \quad x_2 = -3$</p> <p>27. $\pm(\lg 2)^{1/2}$</p> | <p>2. $\frac{1}{2} \lg a + \frac{1}{2} \lg 3 + \frac{1}{2}$</p> <p>4. $\frac{2}{3} \ln a - \frac{1}{b} \ln c$</p> <p>6. 1</p> <p>8. $\lg 4 + \lg \pi + 3 \lg r - \lg 3$</p> <p>10. 4</p> <p>12. $\lg 9$</p> <p>14. 1</p> <p>16. $5(\lg a - \lg y) + 3(\lg b - \lg x)$</p> <p>22. $x = 19$</p> <p>26. $x = \frac{\ln 13}{\ln 13 - \ln 15} \approx -17.9$</p> <p>28. $x = y$</p> |
|---|--|

29. $e^{\lg x} = 10^{\lg x} \quad | \ln$
 $\ln x = \ln(10^{\lg x}) = \ln 10 \cdot \lg x$
analog $\lg x = \ln x \cdot \lg e$
insbesondere $\ln 10 = \frac{1}{\lg e}$
30. ${}^a \log x = {}^b \log x \cdot {}^a \log b$
31. $\lg x \left(\stackrel{29.}{=} \frac{\ln x}{\ln 10} \right) = \ln y \ln 10 \leadsto x = y^{(\ln 10)^2}$
32. $x = y^{\lg e}$
33. $x_1 = 2 \quad x_2 = 3$
34. $x = 13$
35. $x = -4$
36. $x = 9a$
37. $y = 4 \quad x = 5$
38. $x = 1 \quad y = 2$
39. $x = 2 \quad y = 1$
40. 3
41. $a^{\frac{1}{ax}}$
42. 1
43. $4b^{\frac{n+5}{5}}$
44. b^{2a-1}
45. $|a + b + c|$
46. $4\sqrt{x}$
47. x^{-2b}
48. $16a^2 \sqrt[4]{b^9}$
49. $2\sqrt[3]{c+d}$
50. $x = 5$

3.2 Vermischte Aufgaben

1. $x = 5$
2. $x = 1/4$
3. $x = 5/2$
4. $x = \pm 4$
5. $x = \pm 3, \quad x = \pm 2$
6. $x = \pm 15$
7. $x = \pm(a - b)$
8. $x = \pm 13$
9. $y = \pm 9 \quad x = \pm 6$
10. $x_1 = \pm 1/2, \quad y_1 = \pm \sqrt{17/3}$
 $x_2 = \pm \frac{1}{2} \sqrt{17}, \quad y_2 = \pm \sqrt{1/3}$
11. $x_1 = 2, \quad y_1 = 3 \quad x_2 = -7/5, \quad y_2 = -7/9$

BLOCK IV

Differentiation

4.1 Algebraische Ausdrücke

$$1. \quad y' = 2 - 6x - 15x^2 - 23x^3 + 45x^4$$

$$2. \quad y' = -\frac{1}{x^2} - \frac{6}{x^3} - \frac{6}{x^4}$$

$$3. \quad y' = \frac{1}{\sqrt{x}} + \frac{2}{\sqrt[3]{x^2}} - 3\sqrt{x}$$

$$4. \quad y' = \frac{2x}{\sqrt[3]{9x^4}} = \frac{2}{3}\sqrt[3]{\frac{3}{x}}$$

$$5. \quad \frac{ds}{dt} = 8t(t^2 - 3)^3$$

$$6. \quad f'(x) = \frac{x+3}{\sqrt{x^2+6x+3}}$$

$$7. \quad y' = \frac{2x^3 - 13x}{\sqrt{(4-x^2)^3}}$$

$$8. \quad y' = -\frac{1}{x^3} - \frac{30x(1-5x)^5 + \frac{2}{3}(1-5x)^6}{\sqrt[3]{x^5}}$$

$$9. \quad y = 2x(x^2 + 3)^3 (11x^3 + 9x - 20)$$

$$10. \quad y' = \frac{1}{\sqrt{x-1}} - \frac{x}{2\sqrt{(x-1)^3}}$$

$$y'' = -\frac{1}{\sqrt{(x-1)^3}} + \frac{3x}{4\sqrt{(x-1)^5}}$$

$$11. \quad y' = -\frac{x}{\sqrt{(x^2 - 9)^3}}$$

$$y'' = -\frac{1}{\sqrt{(x^2 - 9)^3}} + \frac{3x^2}{\sqrt{(x^2 - 9)^5}}$$

$$12. \quad y' = \frac{-2(x^5 + 6x^4 + 8x^3 + 4x + 8)}{(x^4 - 4)^2}$$

$$13. \quad y' = -\frac{1}{2\sqrt{1-x}} \text{ da für } x \neq -1 \text{ gilt } y = \sqrt{1-x}$$

$$14. \quad y = \left(\frac{x+1}{x-1}\right)^2 \quad \forall x \neq 1 \quad y' = \frac{-4x-4}{(x-1)^3}$$

$$15. \quad \dot{x} = 2 - \frac{1}{t^2} \quad \ddot{x} = \frac{2}{t^3}$$

$$16. \quad y' = \frac{1}{\sqrt{1-x}} + \frac{1+x}{2\sqrt{(1-x)^3}}$$

$$17. \quad y' = \frac{2}{\sqrt{1-x^2}(\sqrt{1+x} + \sqrt{1-x})^2}$$

$$18. \quad y' = \frac{-1}{2\sqrt{a^2-x^2}} - \frac{\sqrt{a-x}}{2\sqrt{(a+x)^3}}$$

$$19. \quad y = (a - \sqrt{2x})^3 \quad y' = -\frac{3(a - \sqrt{2x})^2}{\sqrt{2x}}$$

$$20. \quad y = \frac{1}{2x+3} \quad y' = -\frac{2}{(2x+3)^2}$$

4.2 Trigonometrische Funktionen

1. $y' = x \sin x + 2x + 4$
2. $y' = -2x \sin(1 - x^2)$
3. $y' = \frac{\sin 2x}{2 \cos^2 x} + \tan x \cos 2x$
4. $y' = 6 \sin^2(2x - 3) \cos(2x - 3)$
5. $y' = (x^2 - 2x + 2) \cos x - 2 \sin x$
6. $y = x \cos(1 + x) \quad y' = \cos(1 + x) - x \sin(1 + x)$
7. $y' = \frac{-\cot(x^2)}{x^2} - \frac{2}{\sin^2(x^2)}$
8. $y' = \frac{\cos x}{x} + \frac{1}{\sin x} - x \frac{\cos x}{\sin^2 x} - \frac{\sin x}{x^2}$
9. $y' = \frac{(1 + x) \sin x + (1 - x) \cos x}{(\sin x + \cos x)^2}$
10. $y' = \sin^2 x \cos x (1 - 2 \sin^2 x + \sin^4 x)$
11. $y' = -9 \sqrt{\frac{1}{3x^3}} \sin^2 \sqrt{\frac{3}{x}} \cos \sqrt{\frac{3}{x}}$
12. $y' = \frac{6 \cos x}{\sin^2 x} - \frac{9 \cos^3 x}{\sin^4 x} = \frac{-3(\cos^3 x + 2 \cos x)}{\sin^4 x}$
13. $y' = \frac{7 \sin^3 x}{\cos^8 x}$
14. $y' = \frac{2 \tan^3(\sqrt{x})}{\sqrt{x} \cos^2 \sqrt{x}}$
15. $y = \frac{1}{4 \sin x} + \frac{1}{4 \cos x} \quad y' = \frac{1}{4} \left(\frac{\sin x}{\cos^2 x} - \frac{\cos x}{\sin^2 x} \right)$
16. $y' = -5 \sin \frac{x}{4} \cos^4 \frac{x}{4}$

$$17. \quad y' = \frac{\cos^2 x (1 + \tan x)(\sin x + x \cos x) - x \sin x}{\cos^2 x (1 + \tan x)^2}$$

$$18. \quad y' = \sqrt{\frac{1}{x^3}} \cos \sqrt{\frac{1}{x}} \sin \sqrt{\frac{1}{x}}$$

$$19. \quad y' = \frac{1}{\cos^2 x} - \frac{1}{\sin^2 x} = \tan^2 x + \cot^2 x$$

$$20. \quad y' = 0$$

4.3 Logarithmische Ausdrücke, Exponentialfunktionen

$$1. \quad y' = \frac{3x^2}{x^3 + 2} + \frac{2x}{x^2 + 3}$$

$$2. \quad 2yy' = \frac{4}{x} - 2\frac{3}{3x-4} \quad y' = \frac{\frac{2}{x} - \frac{3}{3x-4}}{\pm \sqrt{\ln x^4 - \ln(3x-4)^2}}$$

$$3. \quad y' = \frac{3 \cos 3x}{\sin 3x} = 3 \cot 3x$$

$$4. \quad y' = \frac{1 + \frac{x}{\sqrt{1+x^2}}}{x + \sqrt{1+x^2}} = \frac{1}{\sqrt{1+x^2}}$$

$$5. \quad y = e^{1-x} \quad y' = -e^{1-x}$$

$$6. \quad y' = 18x$$

$$7. \quad y' = \frac{4a}{(e^{ax} + e^{-ax})^2}$$

$$8. \quad y' = -e^{-x} \ln x + \frac{e^{-x}}{x} \quad y'' = e^{-x} \ln x - 2e^{-x}(1/x + 1/x^2)$$

$$9. \quad y = e^{3x^2 \ln a} \quad y' = a^{3x^2} 6x \ln a$$

$$10. \quad y' = e^{-2x}(3 \cos 3x - 2 \sin 3x)$$

$$11. \quad y' = e^{-x^2 \sin x} (-2x \sin x - x^2 \cos x)$$

$$12. \quad y' = e^{5x^2-3x+1} (10x - 3)$$

$$13. \quad y = e^{x \ln x} \quad y' = x^x (\ln x + 1)$$

$$14. \quad y = e^{(\ln x)^2} \quad y' = x^{\ln x} \frac{2 \ln x}{x}$$

$$15. \quad y = e^{\ln x \exp(-x^2)} \quad y' = x^{\exp(-x^2)} \exp(-x^2) \left(\frac{1}{x} - 2x \ln x \right)$$

$$16. \quad y = 3x^2 \quad y'' = 6x$$

$$17. \quad y' = e^{2x} (2x \sin x + 2x^2 e^{2x} + x^2 \cos x)$$

$$18. \quad y' = \frac{1}{\ln(\ln x)} \cdot \frac{1}{\ln x} \cdot \frac{1}{x}$$

$$19. \quad y' = x^{(x^x)} (x^x \ln x (1 + x) + x^{x-1})$$

$$20. \quad y' = (x^x)^x (2x \ln x + x)$$

4.4 Kombinationen verschiedener Funktionen

$$1. \quad y' = x e^{-x^2} (2 \sin x - 2x^2 \sin x + x \cos x)$$

$$2. \quad y' = e^{-(x^2+1) \sin x} (-2x \sin x - (x^2 + 1) \cos x)$$

$$3. \quad y' = \frac{-2x^2 \cos x - \cos x + x \sin x}{e^{-x^2} x^2 \cos^2 x}$$

$$4. \quad y' = (2x^3 \sin^2 x - \cos x \sin x) e^{-x^4}$$

$$5. \quad y' = \frac{1 - 6x^2 \ln x}{x e^{3x^2}}$$

$$6. \quad y' = \cos x e^{-4x^6} (\cos^2 x - 2 \sin^2 x - 24x^5 \sin x \cos x)$$

$$7. \quad y' = -(\cos x)^{\cot x} \left(1 + \frac{\ln \cos x}{\sin^2 x}\right)$$

$$8. \quad y' = \frac{2ab}{a^a \cos^2 x - b^2 \sin^2 x}$$