THEORIE DER KONDENSIERTEN MATERIE:
Quantum mechanics on macroscopic scales

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Exercise 1: Cooper-pairs

a) Find the commutation- and anticommutation relations for the Cooper-pair creation and annihilation operators $b_k^\dagger = a_{k,\uparrow}^\dagger a_{-k,\downarrow}^\dagger$. Are Cooper-pairs bosons? **(3 points)**

b) Calculate the following expectation values

$$\langle \mathrm{BCS}|b_k^\dagger b_{k'}|\mathrm{BCS}\rangle \quad \text{and} \quad \langle \mathrm{BCS}|b_k^\dagger b_k b_{k'}^\dagger b_{k'}|\mathrm{BCS}\rangle\,,$$

denoting the occupation probabilities for one, respectively two Cooper-pairs to momentum k. The BCS ground state is

$$|\mathrm{BCS}\rangle = \left(\prod_k (u_k + v_k b_k^\dagger)\right)|0\rangle\,.$$

(3 points)

c) Find the relative variance $\sqrt{\langle N^2\rangle - \langle N\rangle^2}/\langle N\rangle$ of the numer of Cooper pairs in the BCS ground state.
Estimate, how the relative variance depends on the size of the superconductor. **(2 points)**

Exercise 2: Bogoliubov transformation

Consider two pairs of fermionic operators $a_1^{(\dagger)}$ and $a_2^{(\dagger)}$. We define new operators by

$$\tilde{a}_1^\dagger = u a_1^\dagger - v a_2 \quad \text{und} \quad \tilde{a}_2^\dagger = v^* a_1 + u^* a_2^\dagger$$

(and analogous for the corresponding hermitian conjugate operators) with complex numbers u and v. Find the conditions on u and v, so that the operators $\tilde{a}$ fulfill fermionic canonic commutation relations. **(4 points)**