The Dynamical Casimir Effect

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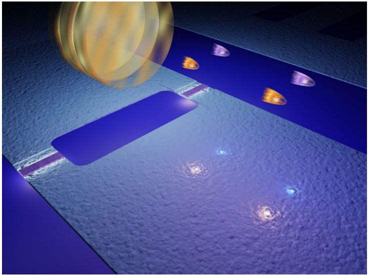
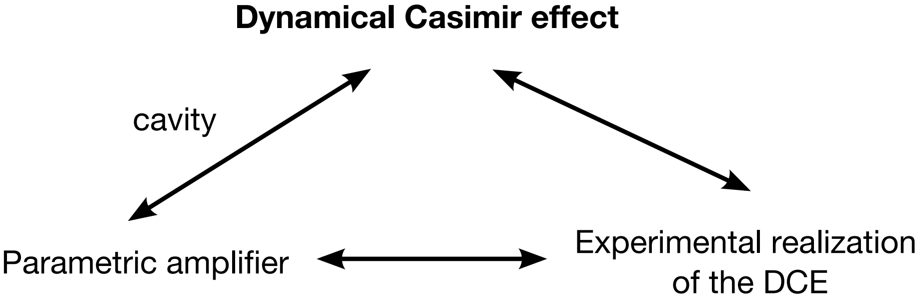
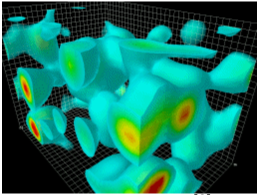


Moving mirror in the vacuum emits photons.[1]

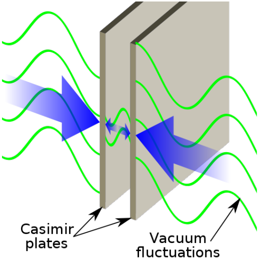
Outline


Dynamical Casimir effect
cavity
Parametric amplifier
Experimental realization of the DCE

The static Casimir effect



Vacuum fluctuations [2]



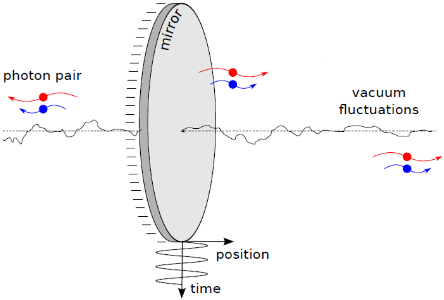
Casimir force between two metal plates [2]

- Two static mirrors in vacuum
- The vacuum is not empty!
  $\Rightarrow$ virtual particles: $\Delta E \Delta t \geq \frac{\hbar}{2}$
- Mismatch of vacuum modes **in space**

$$\Downarrow$$

Attracting force between two metal plates

The dynamical Casimir effect (1)



Moving mirror producing pair of photons.[3]

- Moving mirror in vacuum
- Non-uniform acceleration
- Mismatch of vacuum modes **in time**

$$\Downarrow$$

Creation of photon pairs from the vacuum

The dynamical Casimir effect (2)

Klein-Gordon equation for scalar field $\phi(x,t)$

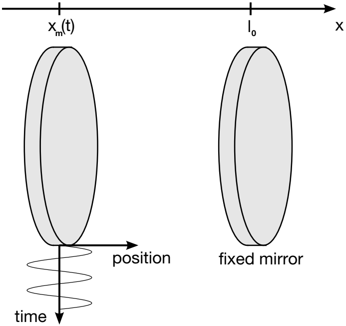
$$\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} - \frac{\partial^2\phi}{\partial x^2} = 0$$

Another setup for the DCE

Cavity with moving mirror.
Non-adiabatic adaption of the EM field to the boundary conditions if mirror velocity $v \lesssim c$.

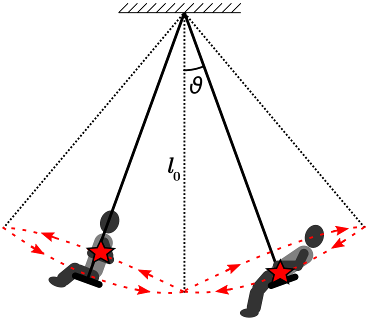
Boundary conditions

$$\phi(x_m(t),t) = 0 \ \text{ and } \ \phi(l_0,t) = 0$$



Cavity of two mirrors.

Classical example of a parametric amplifier (1)



★ center of mass

Child swing as a simple pendulum

$$\ddot{\vartheta} = -\frac{g}{l}\sin\vartheta \approx -\frac{g}{l}\vartheta$$

Now:

Sinusoidal modulation of the center of mass

Slightly varied effective length

$$l(t) = l_0 - \Delta l \sin(2\omega_0 t)$$

where $\omega_0 := \sqrt{\frac{g}{l_0}}$ and $\Delta l \ll l_0$

Classical example of a parametric amplifier (2)

Frequency $\omega(t)$ for modulated effective length

$$\omega^2(t) = \frac{g}{l(t)} = \frac{g}{l_0 - \Delta l \sin(2\omega_0 t)} \approx \omega_0^2\Big(1 + \epsilon \sin(2\omega_0 t)\Big), \quad \epsilon := \Delta l / l_0$$

Equation of motion for a parametric amplifier:

$$\ddot{\vartheta} = -\omega^2(t)\vartheta$$

Ansatz:

$$\vartheta(t) = A(t)\cos(\omega_0 t) + B(t)\sin(\omega_0 t)$$

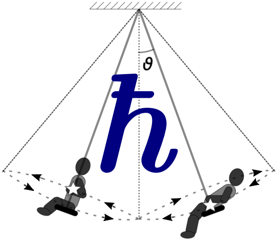
Approximate solution:

$$\vartheta(t) \approx \vartheta(0) \underbrace{e^{\epsilon\omega_0 t/4}}_{exp.\,amplification} \cos(\omega_0 t) + \frac{\dot{\vartheta}(0)}{\omega_0} \underbrace{e^{-\epsilon\omega_0 t/4}}_{exp.\,suppression} \sin(\omega_0 t)$$

“Quantum swinging child” (1)

QM 1D harmonic oscillator

$$\hat{H}(t) = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega^2 \hat{x}^2$$



Parametric amplifier:

$$\hat{H}_{\text{pa}}(t) \approx \frac{\hat{p}^2}{2m} + \frac{1}{2} m\hat{x}^2 \left(\omega_0^2 + \epsilon\omega_0^2 \sin(2\omega_0 t)\right)$$

Introducing ladder operators:

$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega_0}} \left(\hat{a} + \hat{a}^\dagger\right) \qquad \hat{p} = -i\sqrt{\frac{m\hbar\omega_0}{2}} \left(\hat{a} - \hat{a}^\dagger\right) \qquad [a, a^\dagger] = 1$$

QM parametric amplifier

$$\hat{H}_{\text{pa}}(t) \approx \hbar\omega_0 \hat{a}^\dagger \hat{a} + \frac{\hbar\omega_0}{4} \epsilon \sin(2\omega_0 t) \left(\hat{a}^\dagger \hat{a}^\dagger + \hat{a}\hat{a}\right)$$

“Quantum swinging child” (2)

Particle number operator $\hat{N} = \hat{a}^\dagger \hat{a}$
Initial state: system in groundstate $|0\rangle$ at $t = 0$ with $\langle \hat{N} \rangle(0) = 0$.
Ladder operators in **Heisenberg picture**: $\frac{\mathrm{d}}{\mathrm{d}t} a^{(\dagger)} = \frac{i}{\hbar} [H_{\mathrm{pa}}, a^{(\dagger)}]$

Number of quanta in the system at time t

$$\langle \hat{N} \rangle(t) = \langle 0 | \hat{a}^\dagger(t) \hat{a}(t) | 0 \rangle = \sinh^2 \left(\frac{\omega_0 \epsilon t}{2} \right)$$

The number of quanta depends on

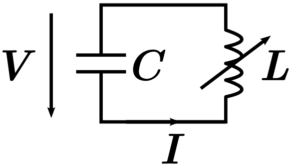
- frequency $\omega_0 = \sqrt{\frac{g}{l_0}}$ determined by geometric settings
- relative length change $\epsilon = \frac{\Delta l}{l_0}$ determined by the amplitude Δl
- time t

LC circuit with tunable frequency

Kirchhoff's laws

Capacitor $Q = CV \quad \Rightarrow \quad I = C\frac{\mathrm{dV}}{\mathrm{dt}}$

Inductance $V = -L\frac{\mathrm{dI}}{\mathrm{dt}}$



LC circuit diagram

$$\frac{\mathrm{d^2V}}{\mathrm{dt^2}} = -\frac{1}{LC}V$$

Parametric amplifier

Parametric amplifier with frequency $\omega(t) = \sqrt{\frac{1}{L(t)C}}$

Remember swing: $\omega(t) = \sqrt{\frac{g}{l(t)}}$

DC SQUID - Tunable inductance

Current through two JJs:

$$I = I_c\left(\sin\varphi_1 + \sin\varphi_2\right)$$

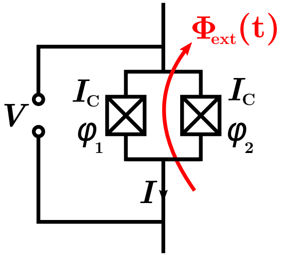
with $I \ll I_c$

Phase:

$$\varphi_1 - \varphi_2 = 2\pi\frac{\Phi_{\text{ext}}}{\Phi_0} + 2\pi n$$

Voltage:

$$V = \frac{\Phi_0}{2\pi}\frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\varphi_1 + \varphi_2}{2}\right)$$



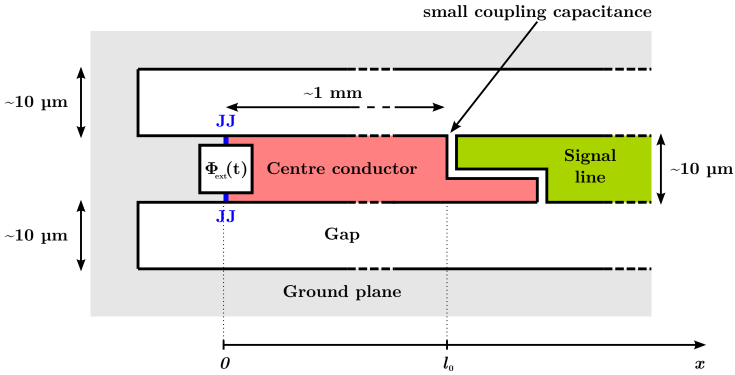
Circuit diagram of a DC-SQUID

Rewrite in the form $V = \frac{\mathrm{d}}{\mathrm{dt}}(LI)$

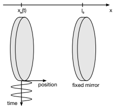
$\Rightarrow$ **Tunable inductance**

$$L(t) = \frac{\Phi_0}{4\pi I_C} \cdot \frac{1}{\cos\left(\pi\,\Phi_{\text{ext}}(t)/\Phi_0\right)}$$

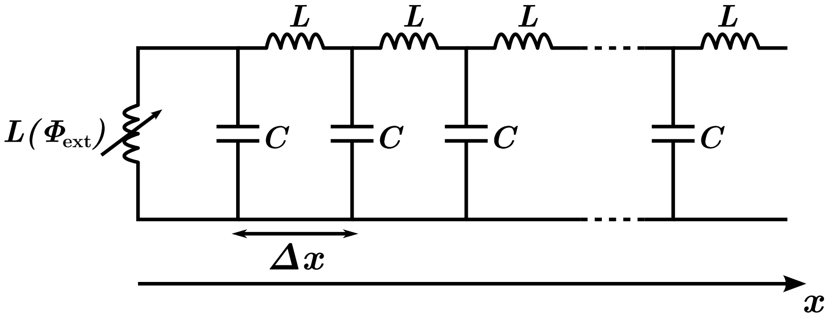
Stripline resonator (1)



Microwave resonator: analogue to two mirrors, where one is moving.



Stripline resonator (2)



Discrete model for a stripline resonator.

Stripline resonator (3)

Wave equation

$$\frac{1}{v^2}\frac{d^2\varphi}{dt^2} - \frac{d^2\varphi}{dx^2} = 0$$

inductance/length l

capacitance/length c

phase field $\varphi(x,t) = \frac{2\pi}{\Phi_0}\int^t dt'\, V(x,t')$

velocity $v = \sqrt{\frac{1}{lc}}$

Stripline resonator (3)

Wave equation

$$\frac{1}{v^2}\frac{d^2\varphi}{dt^2} - \frac{d^2\varphi}{dx^2} = 0$$

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phase field $\varphi(x,t) = \frac{2\pi}{\Phi_0}\int^t dt' V(x,t')$

velocity $v = \sqrt{\frac{1}{lc}}$

Boundary conditions

At $x = 0$: $\varphi(0,t) - \frac{L(\Phi_{ext})}{l} \cdot \frac{\partial\varphi(0,t)}{\partial x} = 0 \quad \Rightarrow \varphi\left(-\frac{L(\Phi_{ext})}{l}, t\right) = 0$

At $x = l_0$: $I(l_0) = 0 \quad \Rightarrow \frac{\partial\varphi(l_0,t)}{\partial x} = 0$

SQUID changes effective length of the cavity to $l_0 + \frac{L(\Phi_{ext})}{l}$

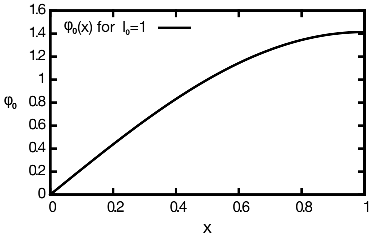
Solving the wave equation (1)

Boundary conditions (without external flux Φ_{ext})

$$\phi(0,t) = 0 \quad \text{and} \quad \frac{\partial \phi(l_0,t)}{\partial x} = 0$$

Solutions

$$\phi_n(x) = \sqrt{\frac{2}{l_0}} \sin\left(\left(n + \frac{1}{2}\right)\frac{\pi x}{l_0}\right)$$



Solving the wave equation (2)

Boundary conditions for external flux Φ_{ext}

$$\phi(0,t) - \frac{L(\Phi_{ext})}{l}\frac{\partial\phi(0,t)}{\partial x} = 0 \qquad \text{and} \qquad \frac{\partial\phi(l_0,t)}{\partial x} = 0$$

Ansatz

$$\phi(x,t) = \sum_n q_n(t)\phi_n(x)$$

Insert into wave equation (fundamental mode)

$$\ddot{q}_0(t) = -\omega_0^2\,(1+\epsilon\sin(2\omega_0 t))\,q_0(t)$$

for an appropiate choice of $\Phi_{ext}(t)$

Parametric amplifier!!!

Again the parametric amplifier...

Recall:

Number of quanta in the system at time t

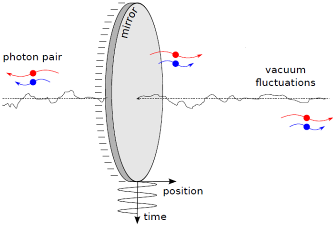
$$\langle \hat{N} \rangle (t) = \langle 0 | \hat{a}^\dagger (t) \hat{a}(t) | 0 \rangle = \sinh^2\left(\frac{\omega_0 \epsilon t}{2}\right)$$

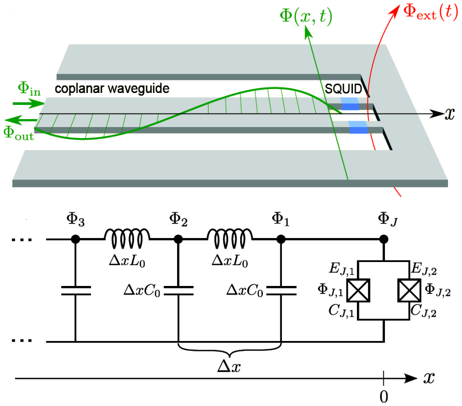
Properties of the system

- Typical velocities in a stripline resonator: $v = \frac{\Delta l \omega_0}{\pi} \approx 10^7 \,\frac{\text{m}}{\text{s}}$
- Resonance frequencies $\omega_0 \sim 2\pi \cdot 5$ GHz (microwaves).
- Number of photons $\langle \hat{N} \rangle (t)$ is limited by quality factor Q of the resonator.

Transmission Stripline (1)

open coplanar waveguide (CPW), sinusoidally driven boundary at frequency ω_d

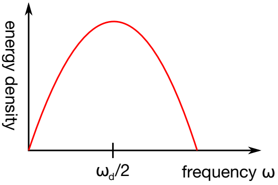




Transmission Stripline (analogue to one mirror)

Figures taken from [3].

Transmission Stripline (2)



Spectrum of emitted photons.[3]

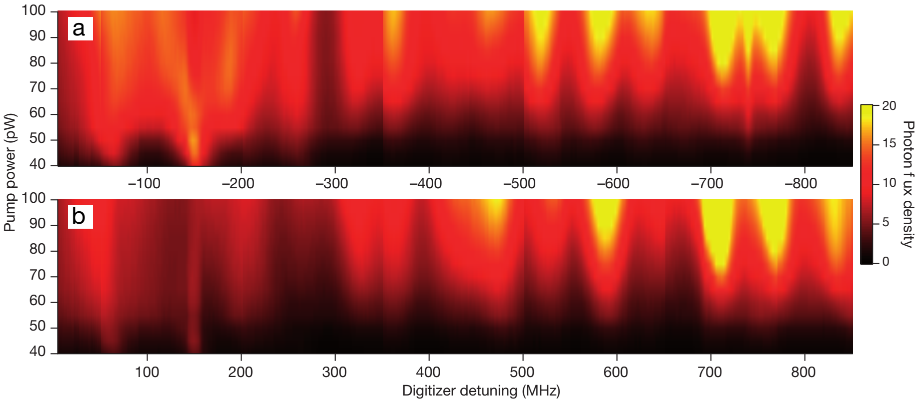
Production of pairs of correlated photons
frequency: $\omega_- + \omega_+ = \omega_d$

maximum effective velocity $v_e \approx 0.25 \cdot v_0$
v_0: speed of light in transmission line

Photons with frequencies of 4-6 GHz are generated (microwaves)

Transmission Stripline (3)

C. Wilson et. al, Nature 479 (2011)
Measurements with open Al waveguide terminated by a SQUID



Summary

The dynamical Casimir effect

- Virtual particles can be converted into real particles.

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- Parametric amplification occurs in many physical systems, e. g. dynamical Casimir effect in a cavity, Hawking radiation, Unruh effect,...

Summary

The dynamical Casimir effect

- Virtual particles can be converted into real particles.
- Parametric amplification occurs in many physical systems, e. g. dynamical Casimir effect in a cavity, Hawking radiation, Unruh effect,...
- SQUIDs and striplines can be used to observe these quantum electrodynamic effects.

THANK YOU FOR YOUR ATTENTION!

- [1] http://physicsworld.com/cws/article/news/2011/nov/17/how-to-turn-darkness-into-light
- [2] http://alemanow.narod.ru/vacuum.gif
- [3] P. D. Nation et al., *Rev. Mod. Phys.* **84** (2012)
- [4] C. Wilson et al., *Nature* **479** (2011)
- [5] M. P. Blencowe, *Mini Course on parametric amplifiers* (2012)